3g. Center of the enveloping algebra

See §3.1

Casimir element

```
In[•]:=  CasXW = HHr ** HHr - 4 HHr - (1 / 3) HHi ** HHi + 4 XX0 ** HHi -
            8 XX0 ** WW0 + 4 XX0 ** XX0 - 2 XX1 ** WW1 + XX1 ** XX1 - 2 XX2 ** WW2 + XX2 ** XX2
         CasZ = CasXW //. XWtoZsub // Expand
```

$$Out[•]= -4\ HHr - \frac{HHi ** HHi}{3} + HHr ** HHr + 4\ XX0 ** HHi - 8\ XX0 ** WW0 +$$
$$4\ XX0 ** XX0 - 2\ XX1 ** WW1 + XX1 ** XX1 - 2\ XX2 ** WW2 + XX2 ** XX2$$

$$Out[•]= 2\ i\ CKi + 2\ i\ WW0 - \frac{CKi ** CKi}{3} - WW0 ** WW0 - Z12 ** Z21 + 4\ Z13 ** Z31 + 4\ Z23 ** Z32$$

Checks of centrality

```
In[•]:=  Table[(XWlist[[j]] ** CasXW - CasXW ** XWlist[[j]] // Expand) /. nul → 0, {j, 1, 8}]
```

$$Out[•]= \{0, 0, 0, 0, 0, 0, 0, 0\}$$

```
In[•]:=  Table[(Zlist[[j]] ** CasZ - CasZ ** Zlist[[j]] // Expand) /. nul → 0, {j, 1, 8}]
```

$$Out[•]= \{0, 0, 0, 0, 0, 0, 0, 0\}$$

```
In[•]:=
```

Generator Δ_3

Only defined in the complex basis

```
In[•]:=  Dt3Z = -(I / 9) CKi ** CKi ** CKi + I CKi ** WW0 ** WW0 +
            I Z12 ** CKi ** Z21 + 2 I Z13 ** CKi ** Z31 - 6 I Z13 ** WW0 ** Z31 -
            6 Z13 ** Z21 ** Z32 + 2 I Z23 ** CKi ** Z32 + 6 I Z23 ** WW0 ** Z32 + 6 Z23 ** Z12 ** Z31 -
            2 CKi ** CKi + 2 CKi ** WW0 + 24 Z13 ** Z31 + 24 Z23 ** Z32 + 8 I CKi
```

$$Out[•]= 8\ i\ CKi - 2\ CKi ** CKi + 2\ CKi ** WW0 + 24\ Z13 ** Z31 + 24\ Z23 ** Z32 - \frac{1}{9}\ i\ CKi ** CKi ** CKi +$$
$$i\ CKi ** WW0 ** WW0 + i\ Z12 ** CKi ** Z21 + 2\ i\ Z13 ** CKi ** Z31 - 6\ i\ Z13 ** WW0 ** Z31 -$$
$$6\ Z13 ** Z21 ** Z32 + 2\ i\ Z23 ** CKi ** Z32 + 6\ i\ Z23 ** WW0 ** Z32 + 6\ Z23 ** Z12 ** Z31$$

Check of centrality

```
In[•]:=  Table[(Zlist[[j]] ** Dt3Z - Dt3Z ** Zlist[[j]] // Expand) /. nul → 0, {j, 1, 8}]
```

$$Out[•]= \{0, 0, 0, 0, 0, 0, 0, 0\}$$