4d. Multiplication relations

This concerns Table 2.1

```
In[ ]:= Clear[Phiprod, APhiprod, BPhiprod]
APhiprod[eps_, zt_, p_, r_] := (p + eps r + 2) / (2 (p + 1))
BPhiprod[eps_, zt_, p_, r_, q_] :=
 If[eps r == p, 0, eps (zt p - q) / (2 (p + 1)), eps (zt p - q) / (2 (p + 1))]
 (* in case of doubt keep expression *)
Phiprod[eta_, eps_, zt_, h_, p_, r_, q_, lst_] :=
 APhiprod[eps, zt, p, r] × Phi[h + eta, p + 1, r + eps, q + zt, lst] +
  BPhiprod[eps, zt, p, r, q] × Phi[h + eta, p - 1, r + eps, q + zt, lst]
Phiprod[eta_, eps_, zt_, h_, p_, r_, q_] := (* for symbolic computation *)
 APhiprod[eps, zt, p, r] × Phi[h + eta, p + 1, r + eps, q + zt] +
  BPhiprod[eps, zt, p, r, q] × Phi[h + eta, p - 1, r + eps, q + zt]
```

Check

```
In[ ]:= Clear[eta, eps, zt, h, p, r, q, al, bt, ztt, eq]
repeat = 5;(* we checked it with repeat equal to 20; that takes long *)
ksub = {Conjugate[zz_] :> Abs[zz]^2 / zz, Abs[bt]^2 → 1 - Abs[al]^2};
Do[eq = (Phi[eta, 1, eps, zt, {ztt, al, bt}] × Phi[h, p, r, q, {ztt, al, bt}] ==
       Phiprod[eta, eps, zt, h, p, r, q, {ztt, al, bt}] //. ksub // Simplify);
 If[eq, , Print[{p, r, q}, "  ", eq]], {p, 0, repeat}, {r, -p, p, 2},
 {q, -p, p, 2}, {eps, -1, 1, 2}, {zt, -1, 1, 2}]
```

No output means OK