5a. Automorphisms

See § 2.3

Automorphism given by element of GL_2

```
In[ ]:= Clear[aa, bb, cc, dd, A]
        A[ns[x_, y_, r_]] := ns[aa x + bb y, cc x + dd y, (aa dd - bb cc) r]

In[ ]:= Clear[x, x1, y, y1, r, r1]
        ns[x, y, r] ** ns[x1, y1, r1] //. Gsub
        A[%] == A[ns[x, y, r]] ** A[ns[x1, y1, r1]] //. Gsub // Simplify

Out[ ]= ns[x + x1, y + y1, r + r1 - x1 y + x y1]

Out[ ]= True
```

Interior automorphism

```
In[ ]:= n1 = ns[x1, y1, r1]
        n1 ** ns[x, y, r] ** iv[n1] //. Gsub // Simplify

Out[ ]= ns[x1, y1, r1]

Out[ ]= ns[x, y, r + 2 x1 y - 2 x y1]
```

Special cases

```
In[ ]:= Clear[t, th]
        A[ns[x, y, r]] /. {aa → t, dd → t, bb → 0, cc → 0}
        as[t] ** ns[x, y, r] ** iv[as[t]] //. Gsub

Out[ ]= ns[t x, t y, r t^2]

Out[ ]= ns[t x, t y, r t^2]

In[ ]:= nn1 = A[ns[x, y, r]] //. {aa → Cos[3 th], bb → -Sin[3 th], cc → Sin[3 th], dd → Cos[3 th]} /.
            ns[xx_, yy_, rr_] :→ ns[xx + I yy, rr] // Simplify
        nn2 = ms[E^(I th)] ** ns[x + I y, r] ** iv[ms[E^(I th)]] //. Gsub
        nn1 == nn2 // FullSimplify

Out[ ]= ns[(x + i y) (Cos[3 th] + i Sin[3 th]), r]

Out[ ]= ns[e^(3 i th) (x + i y), r]

Out[ ]= True
```

Comparison with action of Thangavelu

In[ ]:=
```
Clear[T, Ti, Ns, x, y, r]
T[Ns[xx_, yy_, rr_]] := ns[xx, yy / 2, -rr]
Ti[ns[xx_, yy_, rr_]] := Ns[xx, 2 yy, -rr]
Ti[T[Ns[x, y, r]]]
```

Out[ ]= Ns[x, y, r]

In[ ]:=
```
Clear[u, v, s]
Ns[x, y, r] ** Ns[u, v, s]
T[%] /. T[aa_ ** bb_] :> T[aa] ** T[bb] //. Gsub
Ti[%] // Simplify
```

Out[ ]= Ns[x, y, r] ** Ns[u, v, s]

Out[ ]= $\mathrm{ns}\left[u + x,\ \frac{v}{2} + \frac{y}{2},\ -r - s + \frac{v\,x}{2} - \frac{u\,y}{2}\right]$

Out[ ]= $\mathrm{Ns}\left[u + x,\ v + y,\ r + s - \frac{v\,x}{2} + \frac{u\,y}{2}\right]$