

## 5c. Theta functions

### Section 2.3.2

```
In[ ] := Clear[Th, sum, k, xi]
Th[ell_, c_, ph_, ns[x_, y_, r_], k_] :=
  sum[k] E^(2 Pi I ell (r - x (c / ell + 2 k + y))) (ph /. xi -> c / (2 ell) + k + y) // Simplify
Th[ell_, c_, ph_, n_] := Th[ell, c, ph, n, k]
```

### Relations

Relations for left translation in (2.49)

```
In[ ] := Clear[ell, c, ph, x, y, r]
Th[ell, c, ph[xi], ns[1 / (2 ell), 0, 0]** ns[x, y, r]] // . Gsub // Simplify
% / (E^(-Pi I c / ell) Th[ell, c, ph[xi], ns[x, y, r]]) // Simplify
```

$$\text{Out[ ]} = e^{-\frac{i\pi(c+2\text{ell}x+2\text{ell}(k-\text{ell}r+2\text{ell}kx+\text{ell}xy))}{\text{ell}}} \text{ph}\left[\frac{c}{2\text{ell}} + k + y\right] \times \text{sum}[k]$$

$$\text{Out[ ]} = e^{-2ik\pi}$$

```
In[ ] := Th[ell, c, ph[xi], ns[0, 1 / (2 ell), 0]** ns[x, y, r]] // . Gsub
% == Th[ell, c + 1, ph[xi], ns[x, y, r]] // Simplify
```

$$\text{Out[ ]} = e^{-2i\pi(-\text{ell}r+x+c x+2\text{ell}kx+\text{ell}xy)} \text{ph}\left[\frac{1+c+2\text{ell}(k+y)}{2\text{ell}}\right] \times \text{sum}[k]$$

Out[ ] = True

Differentiation relations in (2.54) and (2.55)

```
In[ ] := Clear[t]
Th[ell, c, ph[xi], ns[x, y, r]** ns[t, 0, 0]] // . Gsub
D[%, t] /. t -> 0 // Simplify
% == -4 Pi I ell Th[ell, c, xi ph[xi], ns[x, y, r]] // Simplify
```

$$\text{Out[ ]} = e^{2i\text{ell}\pi(r-t y-(t+x)\left(\frac{c}{\text{ell}}+2k+y\right))} \text{ph}\left[\frac{c}{2\text{ell}} + k + y\right] \times \text{sum}[k]$$

$$\text{Out[ ]} = -2i e^{2i\text{ell}\pi\left(r-x\left(\frac{c}{\text{ell}}+2k+y\right)\right)} \pi(c+2\text{ell}(k+y)) \text{ph}\left[\frac{c}{2\text{ell}} + k + y\right] \times \text{sum}[k]$$

Out[ ] = True

```
In[ * ]:= Th[ell, c, ph[xi], ns[x, y, r]** ns[0, t, 0]] // . Gsub
D[%, t] /. t -> 0 // Simplify
% == Th[ell, c, ph'[xi], ns[x, y, r]] // Simplify
```

$$\text{Out[ * ]} = e^{-2i\pi(-ell r + c x + ell x(2k+y))} \text{ph}\left[\frac{c}{2ell} + k + t + y\right] \times \text{sum}[k]$$

$$\text{Out[ * ]} = e^{-2i\pi(-ell r + c x + ell x(2k+y))} \text{sum}[k] \text{ph}'\left[\frac{c}{2ell} + k + y\right]$$

```
Out[ * ]:= True
```

```
In[ * ]:= Th[ell, c, ph[xi], ns[x, y, r]** ns[0, 0, t/2]] // . Gsub
D[%, t] /. t -> 0 // Simplify
% == Pi I ell Th[ell, c, ph[xi], ns[x, y, r]] // Simplify
```

$$\text{Out[ * ]} = e^{2iell\pi\left(r + \frac{t}{2} - x\left(\frac{c}{ell} + 2k + y\right)\right)} \text{ph}\left[\frac{c}{2ell} + k + y\right] \times \text{sum}[k]$$

$$\text{Out[ * ]} = i e^{2iell\pi\left(r - x\left(\frac{c}{ell} + 2k + y\right)\right)} ell \pi \text{ph}'\left[\frac{c}{2ell} + k + y\right] \times \text{sum}[k]$$

```
Out[ * ]:= True
```