# 6 Discrete subgroup and lattice

See (2.70) in §2.4

In[ ]:=
```
UFL = {{-I / Sqrt[2], 0, 1 / Sqrt[2]}, {0, 1, 0}, {-I / Sqrt[2], 0, -1 / Sqrt[2]}};
Conjugate[Transpose[UFL]].I21.UFL // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 0 \end{pmatrix}$$

In[ ]:=
```
Inverse[UFL].hm[ups].UFL // Simplify // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \text{ups} & 0 & 0 \\ 0 & \frac{\text{Abs}[\text{ups}]^2}{\text{ups}^2} & 0 \\ 0 & 0 & \frac{\text{ups}}{\text{Abs}[\text{ups}]^2} \end{pmatrix}$$

In[ ]:=
```
Inverse[UFL].nm[b, r].UFL // Simplify // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & i \sqrt{2}\, b & -2\, r - i\, \text{Abs}[b]^2 \\ 0 & 1 & -\sqrt{2}\, \text{Conjugate}[b] \\ 0 & 0 & 1 \end{pmatrix}$$

In[ ]:=
```
am[Sqrt[2]].nm[1, 0].am[1 / Sqrt[2]] == nm[Sqrt [2], 0] // Simplify
am[Sqrt[2]].nm[I, 0].am[1 / Sqrt[2]] == nm[I Sqrt [2], 0] // Simplify
am[Sqrt[2]].nm[0, 1 / 4].am[1 / Sqrt[2]] == nm[0, 1 / 2] // Simplify
```

Out[ ]= True

Out[ ]= True

Out[ ]= True

In[ ]:=
```
hm[E ^ (Pi I / 4)].nm[1, 0].hm[E ^ (-Pi I / 4)] == nm[-(1 - I) / Sqrt[2], 0] // Simplify
hm[E ^ (Pi I / 4)].nm[I, 0].hm[E ^ (-Pi I / 4)] == nm[- (1 + I) / Sqrt[2], 0] // Simplify
hm[E ^ (Pi I / 4)].nm[0, 2 / 4].hm[E ^ (-Pi I / 4)] == nm[0, 1 / 2] // Simplify
```

Out[ ]= True

Out[ ]= True

Out[ ]= True

In[ ]:=
```
hm[1 + I].nm[0, r].hm[1 / (1 + I)] == nm[0, 2 r] // Simplify
```

Out[ ]= True

```
In[ ]:= nm[0, 1/2] // MatrixForm
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 1+\frac{i}{2} & 0 & -\frac{i}{2} \\ 0 & 1 & 0 \\ \frac{i}{2} & 0 & 1-\frac{i}{2} \end{pmatrix}$$

```
In[ ]:= nm[(1 + I)^(-1), 0] // MatrixForm
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} \frac{3}{4} & \frac{1}{2}-\frac{i}{2} & \frac{1}{4} \\ -\frac{1}{2}-\frac{i}{2} & 1 & \frac{1}{2}+\frac{i}{2} \\ -\frac{1}{4} & \frac{1}{2}-\frac{i}{2} & \frac{5}{4} \end{pmatrix}$$

```
In[ ]:= Inverse[UFL].nm[b, r].mm[zt].UFL // Simplify // MatrixForm
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} zt & \frac{i\sqrt{2}\,b}{zt^2} & -zt\left(2\,r + i\,\text{Abs}[b]^2\right) \\ 0 & \frac{1}{zt^2} & -\sqrt{2}\;zt\,\text{Conjugate}[b] \\ 0 & 0 & zt \end{pmatrix}$$