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## A1 Modified Bessel functions

### Appendix A1

The functions  $I_\nu$  and  $K_\nu$  are known to Mathematica, as **BesselI** and **BesselK**.

To direct Mathematica in the simplification of expression with use of the contiguous relations the following macros are useful.

```
In[ ]:= Clear[Besselup, Besselndn]
Besselup[n_] :=
  {BesselI[n, x_]  $\rightarrow$  Simplify[(2 (n + 1) / x) BesselI[n + 1, x] + BesselI[n + 2, x]],
   BesselK[n, x_]  $\rightarrow$  Simplify[-(2 (n + 1) / x) BesselK[n + 1, x] + BesselK[n + 2, x]]};
Besselndn[n_] := {BesselI[n, x_]  $\rightarrow$  Simplify[(-2 (n - 1) / x) BesselI[n - 1, x] + BesselI[n - 2, x]],
  BesselK[n, x_]  $\rightarrow$  Simplify[(2 (n - 1) / x) BesselK[n - 1, x] + BesselK[n - 2, x]]};
```

Check of relations

```
In[ ]:= rel = {BesselK[nu - 1, x] - BesselK[nu + 1, x] == -2 nu x^(-1) BesselK[nu, x],
  BesselI[nu - 1, x] - BesselI[nu + 1, x] == 2 nu x^(-1) BesselI[nu, x]};
rel /. Besselup[nu - 1] // Simplify
rel /. Besselndn[nu + 1] // Simplify
```

```
Out[ ]:= {True, True}
```

```
Out[ ]:= {True, True}
```