

## A2b Exponential behavior

Some check, trusting that Mathematica has it right.

```
In[ ]:= Clear[kap, s, tau]
```

```
tau^(-kap) E^(tau/2) WhittakerW[kap, s, tau]
```

```
Series[%, {tau, Infinity, 2}]
```

```
Out[ ]:= e^(tau/2) tau^(-kap) WhittakerW[kap, s, tau]
```

$$\text{Out[ ]} = 1 + \frac{\left(\frac{1}{2} - \text{kap} + s\right) \times \left(-\frac{1}{2} + \text{kap} + s\right)}{\tau} + \frac{1}{32 \tau^2}$$

$$(-3 + 2 \text{kap} - 2 s) \times (-1 + 2 \text{kap} - 2 s) \times (-3 + 2 \text{kap} + 2 s) \times (-1 + 2 \text{kap} + 2 s) + O\left[\frac{1}{\tau}\right]^3$$

```
In[ ]:= -E^(Pi I kap) tau^kap E^(-tau/2) WhittakerV[kap, s, tau] /. Whsub
```

```
Series[%, {tau, Infinity, 2}] // FullSimplify
```

```
Out[ ]:= -e^(i kap pi - tau/2) tau^kap
```

$$\left( -\frac{i e^{-i \pi s} \pi \text{Csc}[2 \pi s] \text{WhittakerM}[\text{kap}, -s, \tau]}{\Gamma[1 - 2 s] \Gamma\left[\frac{1}{2} + \text{kap} + s\right]} + \frac{i e^{i \pi s} \pi \text{Csc}[2 \pi s] \text{WhittakerM}[\text{kap}, s, \tau]}{\Gamma\left[\frac{1}{2} + \text{kap} - s\right] \Gamma[1 + 2 s]} \right)$$

$$\text{Out[ ]} = e^{-\tau + i \text{kap} \pi + O\left[\frac{1}{\tau}\right]^3} \tau^{2 \text{kap}} O\left[\frac{1}{\tau}\right]^3 + \left( 1 + \frac{\frac{1}{4} + \text{kap} + \text{kap}^2 - s^2}{\tau} + \frac{1}{32 \tau^2} \right)$$

$$(1 + 2 \text{kap} - 2 s) \times (3 + 2 \text{kap} - 2 s) \times (1 + 2 \text{kap} + 2 s) \times (3 + 2 \text{kap} + 2 s) + O\left[\frac{1}{\tau}\right]^3$$