Differentiable manifolds – homework 1

Exercise 1. Show that the n-dimensional sphere

$$S^n = \{(x_0, \cdots, x_n) \in \mathbb{R}^{n+1} : x_0^2 + \cdots x_n^2 = 1\}$$

is a manifold.

Exercise 2. Show that with the coordinate charts found in lecture, S^2 becomes a complex manifold, i.e., the change of coordinates are holomorphic functions.

Exercise 3. A *diffeomorphism* between manifolds M and N is a smooth bijection $f : M \longrightarrow N$ whose inverse, $f^{-1} : N \longrightarrow M$ is also smooth. With this definition at hand, solve exercise 2 in Warner's chapter 1.

Exercise 4. Read exercise 6 in Warner. The content of this exercise states that the dimension of a (connected component of a) manifold is a well defined number.

Exercise 5. Fill out the details of/read the examples of manifolds on page 7 of Warner (Example 1.5)

Exercise 6. Show that $\mathrm{Gl}(n;\mathbb{R})$, the space of matrices with nonzero determinant, is a manifold and hence so is $\mathrm{Gl}(n;\mathbb{R}) \times \mathrm{Gl}(n;\mathbb{R})$. Show that matrix multiplication

$$m : \mathrm{Gl}(n;\mathbb{R}) \times \mathrm{Gl}(n;\mathbb{R}) \longrightarrow \mathrm{Gl}(n;\mathbb{R}); \qquad m(A, B) = AB;$$

and inversion

$$i : \mathrm{Gl}(n;\mathbb{R}) \longrightarrow \mathrm{Gl}(n;\mathbb{R}); \qquad i(A) = A^{-1}$$

are smooth maps.