

# Differentiable manifolds – hand-in sheet 4

hand in by: 10/Jan

## The orthogonal group

**Exercise 1.** Let  $M_{n \times n}$  be the set of  $n \times n$  matrices. Recall that the *group of orthogonal matrices* is

$$O(n) = \{A \in M_{n \times n} \mid AA^t = Id\},$$

where  $\cdot^t$  indicate matrix transposition. In this exercise we prove that  $O(n)$  is a Lie group.

1. (1 pt) Let  $Sym_n$  be the set of  $n \times n$  symmetric matrices, i.e.,

$$Sym_n = \{A \in M_{n \times n} \mid A = A^t\}.$$

Show that  $Sym_n$  can be given the structure of a manifold for which it is diffeomorphic to  $\mathbb{R}^{\frac{n(n+1)}{2}}$ .

2. (4 pt) Show that the map

$$\varphi : M_{n \times n} \longrightarrow Sym_n; \quad \varphi(A) = AA^t.$$

is smooth and that  $Id \in Sym_n$  is a regular value of this map. Hence conclude that  $O(n) \subset M_{n \times n}$  is an embedded submanifold. What is the dimension of  $O(n)$ ?

3. (2 pt) Show that  $O(n)$  endowed with matrix multiplication is a Lie group, i.e., show that multiplication and inversion are smooth maps on  $O(n)$ .
4. (3 pt) Describe the tangent space of  $O(n)$  at  $Id$ .