Differentiable manifolds – hand-in sheet 2

Hand in by 02/Oct

The tautological bundle

Exercise 1. Let $E_1 \subset \mathbb{R}^2 \times \mathbb{R}P^1$ be the set

$$E_1 = \{(x, l) \in \mathbb{R}^2 \times \mathbb{R}P^1 | x \in l\}.$$

- Find a natural set of coordinates for E which make it into a smooth manifold.
- Show that the following map is smooth and find its critical points

$$\pi_2 : E_1 \longrightarrow \mathbb{R}P^1, \qquad \pi_2(x, l) = l.$$

- Show that $\pi_2 : E_1 \longrightarrow \mathbb{R}P^1$ is indeed a line bundle over $\mathbb{R}P^1$. Is this bundle trivial?