

Differentiable manifolds – exercise sheet 7

Exercise 1. Show that no two surfaces in the list of classification of surfaces are homeomorphic to each other. A possible way to achieve this is

1. Compute the fundamental group of the surface obtained as 2-sphere with n perforations,
2. Compute the fundamental group of the surface obtained as a connected sum of g tori with n perforations,
3. Compute the fundamental group of the surface obtained as a connected sum of g projective planes with n perforations,
4. By considering the Abelianizations of the previous groups or otherwise, show that no two of them are the same.

Exercise 2. To which surface in the classification of surfaces do the surfaces described below correspond?

1. $aba^{-1}cbc^{-1}$
2. $abcd a^{-1}b^{-1}c^{-1}d^{-1}$
3. $abb^{-1}a^{-1}$.