Group theory – Hand in sheet 2

1– **1 point**) *Let $H < G$ be a normal subgroup and $K < G$ be a subgroup. Show that $H \cap K$ is a normal subgroup of K.*

2) *Given a group G and subgroups $H_i < G$, $i = 1, \cdots, k$ we denote by $\langle H_i : 1 \leq i \leq k \rangle$ the smallest subgroup of G which contains all the H_i and denote by $\cap_{i=1}^k H_i$ their intersection.*
a– **1 point**) *Show that $\cap_{i=1}^k H_i$ is a subgroup of G. Show that it is the biggest subgroup of G contained in all of the H_i.*

b– **1 point**) *Show that if all H_i are normal, so are $\langle H_i : 1 \leq i \leq k \rangle$ and $\cap_{i=1}^k H_i$.*

c– **1 point**) *Show that if the maximum common divisor of the orders of H_i is 1, then*

$$\cap_{i=1}^k H_i = \{e\}.$$

d – **1 point**) *Show that if the least common multiple of the orders of H_i is the order of G then*

$$\langle H_i : 1 \leq i \leq k \rangle = G.$$

3– **2 points**) *If $p_1, \cdots, p_k$ are distinct primes, show that an Abelian group of order $p_1 \ldots p_k$ is cyclic.*

4) *Let G be a group of order pq with p, q be primes $p > q$.*
a– **1 point**) *Show that any subgroup of G of order p is normal.*
Remark: This was done in lectures and actually holds as long as the index of a subgroup of order p is less than or equal to p.

b– **1 point**) *Show that if q does not divide $p - 1$ then G is Abelian.*

5) Find all the groups of order 15, 17, 19, 21 and 23.

6– **1 point**) *Find all groups of order 8.*