Group theory – Sheet 7

The exercises from the book are 8.8, 8.9, 8.10, 10.8, 10.11, 11.7, 11.8 and 14.1 to 14.6.

Exercises about previously covered material: 7.2, 7.9, 7.11, 10.7

(1) If $H < G$ is a normal subgroup and $K < G$ is a subgroup, show that $H \cap K$ is a normal subgroup of K. Conclude that A_n is the only normal subgroup of S_n for $n > 5$.

(2) Let $n > 4$ and $m < n$. Show that

a) If S_n acts on a set with m elements, then the size of any the orbits is either 1 or 2.

b) The only action of A_n on a set with m elements is the trivial one.