Group theory − Sheet 8

Exercises from the book: 14.2, 14.5, 14.6.

1) Let G be a finite group of odd order. Show that if G has only three conjugacy classes, then G is isomorphicto $\mathbb{Z}_3$. (Hint: solve first the hand-in exercise 6)

2) Let G be a finite group of even order. Show that if G has only three conjugacy classes, then G is isomorphic to D_3.

3) Let G be a group of order $2p$ where p is a prime greater than 2. Show that G is isomorphic to either $\mathbb{Z}_{2p}$ or D_p. (Hint: recall the proof of theorem 13.2 in Armstrong and the results from quiz 2)

4) Using exercise 6.10 or otherwise, show that the center of S_n for $n > 2$ and A_n for $n > 3$ is trivial.

5) Let G be a simple group of order n and let X be a set with m elements. Show that if n does not divide $m!$ then the only action of G on X is trivial. (Hint: a G action on a set X is a group homomorphism $\varphi : G \longrightarrow S_m$)