Vernieuwingsimpuls / Innovational Research

Grant application form 2003 — VIDI scheme

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Registration form (basic details)

1a. Details of applicant

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-Doctorate (date): April 1997
-Use of extension clause (see Notes): no

1b. Title of research proposal

Non-archimedean Geometry and Automorphic Forms

1c. Summary of research proposal

It is quite natural to count with numbers as their remainders after division by a fixed number (we say, *modulo* that number). For example, time is counted modulo 24 (s 19+8 "=" 3), and computer bits are counted modulo 2.

One can try to develop all the usual more advanced mathematics (geometry, analysis) in those calculating systems, and to a certain extent, this has been done in the 20th century, under the influence of number theory, where such counting has become a usual method.

Geometry in such calculating systems is a particularly rich topic, as it combines techniques from usual geometry over the real numbers with insights from the number theoretical side, and indeed, the analogues of many open problems about the integers (e.g., Riemann hypothesis) have been solved.

As a bonus, that geometry produces interesting discrete structures (like optimal networks – see for example picture 2 below) and has applications in coding theory and theoretical physics (string theory).

In this project, we are interested in studying low dimensional geometrical objects over such counting systems with a given set of symmetries. The novel method is to recursively deform the initial object with its symmetries (and interesting functions on it, called automorphic forms) to one that can be captured by a graph with infinitely many symmetries; the latter plays the modulo-*p* rôle of "uniformization" in usual complex geometry (a curve over **C** is quotient of a "flat" space by infinitely many symmetries). Five concrete goals are outlined below, one of which is to describe the maximal possible number of symmetries in terms of geometrical properties of the curve. We hope also that new exciting and practicable combinatorial structures (like the ones mentioned in the previous paragraph) will come out as a bonus.

KEYWORDS: non-archimedean fields, algebraic geometry, rigid analysis, automorphic forms

1d. NWO Council area

EW

1e. Host institution

Universiteit Utrecht

Vernieuwingsimpuls / Innovational Research

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Research proposal

2. Description of the proposed research: Non-Archimedean Geometry and Automorphic Forms

2a. Research Topic (Non-specialist outline).

This is a pure mathematics proposal, and as such motivated by internal questions of mathematics proper, but we try to indicate why these questions are timely and relevant for neighbouring fields and applications in sections that are labelled by a curly sidebar – developing these applications is not strictly part of the proposal, but could form a bonus.

In the first few sections I will try to outline in a not-too-specialist way what the words "non-archimedean", "degeneration" and "group action" mean, and how they are combined in the first principle of the project. After that, we introduce the " automorphic" aspect of the project.

(i) "Non-archimedean mathematics". Number theory has long since shown the need for not only the real numbers, but also its so-called *non-archimedean* companions such as the *p*-adic numbers or fields of positive characteristic. One should for example think of doing mathematics over the field *F* of formal power series in a variable *t* with coefficients modulo a prime *p*, say, $p=3$, and so the basic manipulations look something like

$$(1+t+2t^2+...)+(2+t+2t^2+...)=2t+t^2...$$

There certainly is algebraic and arithmetic geometry (=geometric study of solutions to polynomial equations) over such fields - it is actually the natural playground for discrete phenomena and applications abound to the construction of networks, codes, cryptography and whatnot in our digital world (1970 onward); cf. infra for a striking example.

What about non-archimedean calculus? For this, one needs a concept of distance, and the natural candidate for the above field *F* is to count two power series being close if their difference is divisible by a high power of *t*, so we take an absolute value of the form

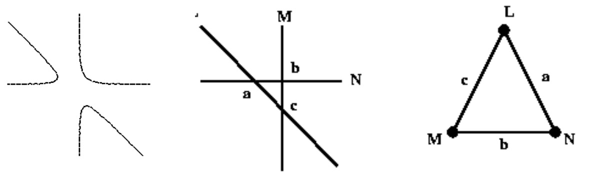
$$|a_r t^r+...|=p^{-r}$$

Such a distance behaves strangely (e.g., all triangles are isosceles), and this caused some obstacles to successful analysis, but they were removed by the recent introduction of so-called rigid analysis (cf. [1], culminating e.g. in the resolution of the Langlands programme [2]). Actually, there is a whole list of analogies between ``classical'' (real/complex) mathematics and these worlds, in which quite often the modulo *p* or *p*-adic concepts are better understood (as another striking example, the Riemann hypothesis is proven in such a context).

Recently, this non-archimedean calculus has gotten (partly speculative) relevance for string theory (partition function of the Polyakov string, non-archimedean geometry and mirror symmetry [3]), and in its turn has stimulated new developments in complex geometry (spectral triples [3]).

(ii) Degeneration and uniformization. Classical calculus becomes a powerful tool in the study of curves. These are (complex) one-dimensional solution spaces to sets of polynomial equations, and one of the most striking points is that they come with a *global uniformization* that allows one to describe their geometry completely by the (analytic) action of a group (=infinite set of symmetries) on a very easy space (albeit non-euclidean, as in general relativity), cf. [4] - more technically: by the action of its fundamental group on its universal covering space) - .

The point is that such a description is not always available in non-archimedean geometry. Consider for example the curve $xy(x+y+1)=t$ in (x,y)-space, or actually, family of curves, where *t* is a variable. Picture 1 displays some of these curves over the real numbers - it helps a bit to understand what is happening in the non-archimedean case. One sees a smooth curve for $t=1$, but for $t=0$, the curve degenerates into the union of three lines. A theory of Mumford says that non-archimedean curves can be uniformized (captured by an infinite group) exactly for *t* „close“ to zero *if* the degeneration is "totally split", i.e., exists of only rational curves (e.g., straight lines). The following description of such a degeneration will come in handy: in its *intersection dual graph*, each line is replaced by a dot and each intersection point of two lines by an edge joining the two dots corresponding to the lines (an example can be seen in the picture).



picture 1: a family of degenerating curves, left the curve for *t=1*, middle for *t=0* and right the intersection dual graph for *t=0*

The trouble is that, for *t* far away from zero, there is no such group. I will try to fix this by letting **the first principle of this project** be to study any curve in non-archimedean geometry by first putting it into a *family* of curves that degenerates and then studying a curve in this family close to the degeneration, for which a uniformization by a group exists. Of course, this principle should be applied depending on the information of the original curve one is interested in preserving through the process of deformation. It is meant to remedy the fact that there is not always a global uniformization in non-archimedean geometry.

(iii) Group action. Particularly interesting are the symmetries of such a curve. They form a group of so-called automorphisms of the curve. One can study symmetries of *X* by looking at the map from *X* to its „quotient“ *Y* (=orbit space) by those symmetries. This quotient naturally gets labels at certain points where the action is “strange” (above which there are fixed points).

If one now applies the philosophy of the previous section to the problem of studying such symmetries, the task becomes to understand how symmetries, so such a map from *X* to *Y*, degenerate(s) in families. The natural questions are then

(a) does there exist a family of curves with the same group of symmetries?
(b) does this family degenerate?
(c) what does the group action on the degeneration look like?
(d) if the degeneration is total, what can we say about its symmetries?

All of these questions are non-trivial, probably hopeless in general, but the above methodology is constructed to supply answers in the “generic” case (when *X* is what is called “ordinary” – this allows one to ignore particularly bad cases, but still provides answers in “almost all” cases). As for (d), joint work with Kontogeorgis and Kato ([5], 1999) provides a systematic approach. For (a) and (b), the question should be approachable by continuing work with Kato building on results of Pries (2002) [6]. For (c), the problem is that one should understand the symmetries that fix a whole component of the degeneration pointwise. Seminal work of Raynaud from the nineties should allow for some control (**the first task** is to expand this and consider it modulo *p* **-** cf. [7]). **The second task** is to study how a labelled point degenerates. A **concrete goal** is to control the maximal possible order of such a group of automorphisms in terms of the geometry of the curve (the so-called genus). Once this has been achieved, a very ambitious goal is to understand such maps by developing a *p*-adic or modulo *p* analogue of the theory of „dessins d'enfants“ (following a question of Yves André **-** the original dessins are combinatorial objects envisaged by Grothendieck to encode all information about a covering, cf. [8]).

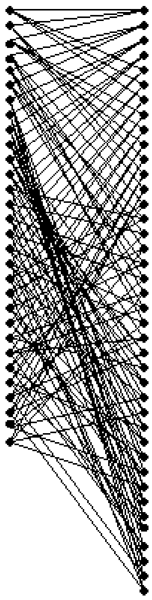
Another as yet hardly studied aspect of the theory is what happens when the dimension increases, i.e., one does not look at curves but *surfaces* instead. We know now fairly well how to construct a totally degenerate curve with given symmetries, but how can one construct such surfaces? This will boil down to understanding combinatorics of groups by their action on „higher dimensional graphs“ called buildings. Only a handful of examples is known, all of which reveal very peculiar behaviour: they tend to have special geometric behaviour compared to general surfaces, (cf. [9], where they produce the only known construction of “fake” projective planes). The **goal** is thus to construct more examples of uniformized surfaces expanding the known techniques for curves.

(iv) Functions versus geometry. It is a principle in geometry that functions on an object characterize the object (most objects look locally like the graph of a function). But how does one construct interesting functions on a curve? Once a uniformization is given, an answer can be obtained from a theory of *automorphic forms* , viz.,

functions that live on the easy space with a given (almost-)invariance for the uniformizing group (and hence become useful objects on the quotient space). Many basic questions are left open about these automorphic forms, such as:

- can one find an explicit finite set of generators for the ring they form?
- can one find all the relations between these generators?

One can say something about these problems using general geometrical features of the curves (cf. [10]), but they can also be translated into a combinatorial language since they degenerate together with the curve – a very natural approach in view of section *(ii)* above. Automorphic forms translate into „harmonic cocycles" on graphs (i.e., a fancy version of electric circuits). However, this translation has not been studied deeply enough. Hence **a first goal** is to collect computational data and order it in such a way that a theory can be formed.



picture 2: network constructed from a special curve

(v) Interactions with graph theory. It is striking that curves with particularly nice automorphic functions on them have led to asymptotically optimal bounds in coding theory (Vladut et. al. [11]) and networks with optimal transmission properties (Lubotzky, Morgenstern [12]) that could not be captured by less sophisticated methods: roughly speaking, the question is which network has at the same time the minimal number of connections and minimal average (disjoint) communication distance between the users. The surprising answer: take a very nice uniformizing group (called „principal congruence group"), let the corresponding curve degenerate, draw its intersection dual graph, and cut off irrelevant ends. An example is displayed in picture 2 (here, 40 inputs (on the right) connected to 30 outputs (on the left) by 120 edges such that every input set with less than 20 elements is connected to a set of at least the same size).

(vi) From functions to geometry. Another aspect of this line of research is how automorphic forms capture the geometry of curves over finite fields: as we have seen, the theory of automorphic forms for such curves has a natural geometrical/combinatorial flavour, and one could study in how far certain associated spaces of automorphic forms allow one to recover geometric information. The **goal** is to use the analogue of Zagier's spaces of toroidal forms (a special kind of automorphic forms, cf. [13]) to read off typical modulo p-geometric information (such as the „ p-rank" of the curve, which detects in a sense how „generic" or „special" the curve is) – compare with section *(iii)* for this notion of "generic",

*To **conclude**, there is a cluster of theories that interact and are situated somewhere in between number theory and complex geometry (so techniques from both sides can be applied): non-archimedean geometry (with group actions and degenerations), combinatorics and the theory of non-archimedean automorphic forms. The gain of studying them is on all three sides: an interesting curve often degenerates into a practicable circuit, an understanding of the graphs leads to properties of the curves (sometimes useful for coding) and their associated automorphic forms, and properties of automorphic forms reflect the special geometry of the curve.*

2b. Approach (Technical outline - please read 2a first)

The project will have two main directions (that interact), which come each in the form of two subprojects.

I. Non-Archimedean Uniformization

I.1. Equivariant theory of curves

The main focus is on how the action of a (finite) group on a curve X over a field k of positive characteristic degenerates in families. I believe that work of Cornelissen and Kato ([5]) that calculates the infinitesimal deformation space of such an action will allow me to decide when a genuine one-dimensional family of equivariant curves exists, and work of Pries ([5]) should be combined with this to almost always find such a family with a degenerate fibre at least if the number of automorphisms of X exceeds Hurwitz's bound. Thus, one can understand the group action on X if one can understand it on this degenerate curve. The method will be to study this object using and extending the theory of Raynaud from [6]: the degenerate fibre is a combinatorial object that is labelled by its decomposition- and inertia-groups. In the ordinary case, the degeneration of the action of decomposition groups should be studied in detail, especially the question of whether the conductor can only go down (local moduli). This should be easier to accomplish in the ordinary case (which is generic).

Any understanding of this situation is welcome, as it enhances our knowledge of the moduli space of curves in positive characteristic, especially of its stratification by automorphism groups. To set a concrete ultimate goal, I will try to prove inductively w.r.t. the genus g that an ordinary curve can have at most

$$max\{84(g-1), 2\sqrt{g}(\sqrt{g}+1)^2\}$$

automorphisms: this would be a sharp upper bound that improves upon existing formulae of S. Nakajima. The final induction step is the case where the degeneration is totally split, but this has been treated in [1]. Finally, one is still left to study the degeneration of curves with a rigid action of the full automorphism group by fixing a suitable subgroup for which the deformation space is not zero-dimensional.

One can even ask for a theory of p-adic or characteristic p „dessins d'enfants“, i.e., a combinatorial structure that describes a covering map of a moduli space of pointed curves. For this, the most recent point of view of non-archimedean geometry proposed by Berkovich and others (cf. [3]) should play a key role.

I.2. Higher-dimensional Non-Archimedean Uniformization

Like a Mumford curve X is uniformized by a free discrete subgroup Γ of $PGL(2,k)$, some surfaces S can be uniformized by a subgroup of $PGL(3,k)$. One can understand covers $X \to Y$ via groups contained in the normalizer of Γ. In this way, Y has labelled ramification points and the corresponding group comes with a combinatorial description via a graph (subtree of Bruhat-Tits tree, or its quotient graph) that captures its ramification subgroups as stabilizers of ends. Thus, the whole geometry of the cover can be understood from this graph of groups. The idea is to understand the analogy in a very concrete way for particular examples of covers of uniformized surfaces. The explicit combinatorics become much more fancy (Bruhat-Tits building, cf. [9]), but from earlier work in the one-dimensional case ([1]), I should be well-equipped to understand it. Every time such a nice structure is given, it comes (at least in characteristic zero) with an interesting theory of special functions (like hypergeometric functions arise in coverings of Riemann surfaces).

II. Automorphic Forms

II.1 Generation of Modular Forms and Harmonic Analysis on Trees (cf. [10]) – PhD project I

If A is the coordinate ring of an affine curve over a finite field, there is a meaningful theory of modular forms for subgroups of $GL(2,A)$. However, for general A (contrary to the affine line, or $A=\mathbf{Z}$), the ring structure of those modular forms is not well-understood. I would like to calculate it and understand it in case A comes from an elliptic curve. Natural generators seem to be Eisenstein series, but they cannot capture all modular forms. Natural relations seem to come from the fact that the image of a generic Drinfeld module of rank two over A is commutative, but maybe those relations are not all. The approach will be to calculate the normalization of the subring generated by Eisenstein series first (in some examples, using Groebner bases from computational algebraic geometry). A complementary approach is that of translating everything into the theory of harmonic cocycles on the degeneration graph of the corresponding modular curve (as in I.1). Here again, the computer can find bases for low weight spaces. Finally, one should compare the results from both approaches to get to the final result: a general theory that maybe allows one to read off the ring structure from the graph. The arithmetic of the elliptic curve should play a role in the final result. The problems naturally extend to the case of congruence subgroups (even for the affine line).

II.2 Toroidal forms for function fields – PhD project II

For any non-split torus T in $G=GL(2)$ over the function field F of a curve over a finite field, and can consider the space $V(T)$ of all L^2-automorphic forms $G(F)Z(\mathbf{A})\backslash G(\mathbf{A}) \to \mathbf{C}$ whose right-G translate integrals over $T(F)Z(\mathbf{A})\backslash G(\mathbf{A})$ vanish (here, $\mathbf{A}$ are the adeles of F and Z is the center of G). Let V, the so-called space of *toroidal forms*, denote the intersection of all $V(T)$ for different choices of T. Zagier [13] has suggested that the K-finite part of V (for K a maximal compact in G) should be a finite dimensional vector space of dimension $2g$ where g is the genus of F, but unpublished ideas of A.J. de Jong and G. Kings suggest that this is wrong. The original concept of Zagier was that the corresponding (infinite dimensional) spaces for $F=\mathbf{Q}$ contain the principle series representation $P(s)$ with parameter s precisely when s is a non-trivial zero of the Riemann zeta function. Now since $P(s)$ is unitarizable precisely when s is real in the interval $(0,1)$ or has real part 1/2, the Riemann hypothesis (and the simplicity of the zeros) is equivalent to the unitarizability of V.

Our idea is to understand the problem for function fields a little better: once translated into the language of rank two vector bundles on the curve, one should be able to understand the dimension defect between dim V^K and $2g$ in terms of the geometry of the curve, e.g., in terms of its p-rank. One of the ultimate goals is a formula that expresses this difference purely in terms of geometry. On the other hand, in each given case, one can calculate these spaces by harmonic analysis on trees (much as in II.1), and this gives another approach to getting a better understanding of the situation.

2c. Innovation

I. A systematic use of degeneration in combination with rigid analytic uniformization is new, although there are some traces of it in works of Raynaud et. al. In fact, Kato and I try to combine the quite specialized knowledge and techniques we have developed over the last few years (trees of groups, deformation theory computations) with those insights. We are the first to tackle the equicharacteristic full ordinary case (the case mostly considered is that of a cyclic group of order p in mixed characteristic).

II. The projects in II involve also less speculative components/calculations – quite natural as they are meant to be PhD-projects.

II.1 No-one has tried to calculate any example at all of such rings of modular forms over elliptic curves; hence whatever methods are used, they are clearly uncovering unknown territory. It fits well with the recent revival of the classical theory of modular forms in the works of Borisov and Gunnells on generating ring of modular forms by methods of toroidal geometry (and links with elliptic genera).

II.2 Toroidal forms have been largely neglected in the literature, although there are some unpublished thoughts circulating; I have received requests by people interested in dynamical aspects of the Riemann zeta function to carry out this project. In short: a timely topic that should be revived.

2d. Plan of work

Composition of the research team:

- the PI (principle investigator – current teaching load of 50% should be reduced) ;
- 2 PhD-candidates (“aio’s”) both for 4 years. If it is difficult to recrute two aio’s, we propose to change one position to a 2 year post-doc instead. That post-doc could be involved in any of the subprojects.

However, the generic plan does not involve a post-doc – the plan is rather to use the overhead to invite more than the usual number of guests for slightly longer than usual stays. This is due to the fact that mathematical research gets most from several denser contacts throughout the whole period, rather than one long-term visitor (compare with the success of research institutes that use this construction as their main format, like IAS, IHES, MPI) – to create a miniature version of this successful atmosphere will also create a good climate for the PhD-candidates.

Timeline for research:

- Part I of the research is by the PI together with F. Kato (Kyoto). The plan is to study these topics at regular intervals during mutual visits (possibly involving consultation of some of the people mentioned below). It seems useless to make a precise time-table for this research, as it involves several speculative concepts and hence its focus will keep on changing (as it did in our collaboration over the last three years).
- Part II.1 and part II.2 are meant to lead to two PhD's (probably with F. Beukers). For the candidates, the first year should involve reading about Drinfeld modular forms (II.1) or automorphic forms (II.2) and developing a plan for computer calculations; the second year should be spent on the actual computations, the third year is meant to interpret the results theoretically and the fourth year is for finishing up. The PI will be also be actively involved in the research. The two projects are intimately linked, also with Part I of the proposal, and the idea is that there is as much interaction (also with visitors) as possible.

Collaboration/Contacts:

- Local contacts could involve F. Beukers (*p*-adics, computer algebra), F. Oort (arithmetic geometry), E. Looijenga (moduli, curves), R. Bruggeman (automorphic forms), W. van der Kallen (computer algebra). Arithmetic geometry is one of the main research topics at Utrecht, with currently even two local seminars ("geometry" and "algebra and number theory").
- (Inter)national collaboration:
  I: F. Kato (Kyoto); II.1 possibly P. Gunnells (Amherst).
- Possible visitors (this is just a tentative list; none of the people has been contacted):
  I. A. Kontogeorgis (Samos), F. Herrlich (Kaiserslautern), P. Lochak (Paris), A. Mézard (Paris), M. van der Put (Groningen), M. Raynaud (Paris), M. Saïdi (Durham), S. Wewers (Bonn).
  II.1 E.-U. Gekeler (Saarbrücken), M. van der Put (Groningen).
  II.2 D. Zagier (Paris/Bonn) and G. Kings (Regensburg).

2e. Literature references

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Cost estimates

3a. Budget

This purely mathematical research does not involve high non-staff costs. These will most probably only consist of buying a portable computer (estimated cost 3 k€) with relevant computer algebra software to perform the calculations in part II of the proposal and some books (estimated 1k€ per year).

We expect to spend some money on travel (est. below 5 k€ per year), and a larger amount of money for inviting visitors in the style described in 2d. above (but this cannot be predicted precisely as it also depends on the guests).

In agreement with the institute, these minor amounts have simply been incorporated into the overhead on the staff costs. This leads to the following simplified budget model:

	200y	200y+1	200y+2	200y+3	200y+4	TOTAL
Staff costs: (in k€)						
Applicant	71	73	74	77	79	374
Ph D student (AIO)	I: 24	I:26 II:24	I:28 II:26	I:35 II:28	II:35	226
TOTAL	95	123	128	140	114	600

3b. Have you requested any additional grants for this project either from NWO or from any other institution?

no

Curriculum vitae

4a. Personal details
Title(s), initial(s), first name, surname: Dr. Gunther L.M. Cornelissen
Male/female: Male
Date and place of birth: July 4th 1971, Gent (Belgium)
Nationality: Netherlands

4b. Master's ('Doctoraal')
University/College of Higher Education: Universiteit Gent
Date: July 1993
Main subject: arithmetic of elliptic curves (algebra, number theory, geometry)

4c. Doctorate
University/College of Higher Education: Universiteit Gent
Date: April 1997
Supervisor ('Promotor'): Jan Van Geel (Gent) and Ernst-Ulrich Gekeler (Saarbrücken)
Title of thesis: Geometric properties of modular forms over rational function fields

4d. Work experience since graduating

1997-2001	Post-doctoral fellow of the Belgian National Science Foundation (1fte, fixed term)
1997-2001	Gastforscher at the Max-Planck-Institut für Mathematik, Bonn (Germany) (1 fte, fixed term)
2000-2001	Guest Professor at the KULeuven (0.2 fte, fixed term)
2001-	Docent/Onderzoeker at Utrecht University (1 fte, tenured)

4e. Man-years of research
5

4f. Brief summary of research over last five years
- Relation between projective normality of modular curves and generation in low degree for rings of Drinfeld modular forms [2,10,11]
- Arithmetic study of zeros of Eisenstein series (Newton polygons, Galois groups), including relations to supersingular reduction of Drinfeld modules [1,4,5]
- Divisibility of class numbers of hyperelliptic curves over finite fields by powers of two, including a geometric approach via the 2-primary torsion group of the Jacobian of the curve [5,7,8]
- Relations between model theory and problems in arithmetic geometry, including the abc-hypothesis and (with K. Zahidi) a conjecture of Mazur on the topology of rational points, also in the non-archimedean case [3,12]
- (with F. Kato and A. Kontogeorgis) Study of the automorphism group of a Mumford curve in positive characteristic by the Bruhat-Tits tree, including a sharp upper bound and the determination of all curves that attain it (analogue mod p of the determination of all Hurwitz groups) [6,13,14,15,16]
- (with F. Kato) Determination of the pro-representable hull of the infinitesimal deformation functor of an ordinary curve with a group action [9]
- (with F. Kato) Determination of the pro-representable hull of the infinitesimal deformation functor of the rigid analytic embedding of the normalizer of the Schottky group of a Mumford curve into *PGL(2)*, including a comparison to the previous functor [9]
- (with M. Tripolitaki) Applications of Drinfeld moduli schemes to the inverse problem of Galois theory over function fields [17]

4g. International activities
- (1994/95) 3 months assistantship at the Universität Saarbrücken, visiting E.-U. Gekeler
- (1997-2001) 4 years stay at the MPI in Bonn
- (2000, 2002) two longer visits to Japan (visiting F. Kato in Kyoto and invitations to Hokaido, Osaka, Tokyo), one to Crete (visiting M. Tripolitaki and T. Pheidas)
- further invited talks in Ireland, Sweden, France, Germany (multiple), Belgium (multiple), Netherlands (multiple)

4h.Other academic activities
Organisation of conferences:
- Number Theory Seminar and Oberseminar at the MPI in Bonn from 1998 to 2001 - organizer
- Conference “Drinfeld modules, moduli schemes and applications” (Alden-Biezen 1996) – collaborator
- Conference “Hilbert’s 10^{th} Problem: relations with arithmetic and algebraic geometry” (Gent 1999) – co-organizer
- Conference “200 Years of Number Theory after Gauss” (Gent, 2001) – co-organizer
- Session on “Arithmetic Geometry” at the first joint meeting of the Belgian and German Mathematical Societies (Liege, 2001) – co-organizer
- Mathematical Colloqium Utrecht (2002-) – co-organizer
- Intercity Seminar “Automorphic Forms that admit an infinite Series Expansion” (Amsterdam-Utrecht-Nijmegen, 2002) – co-organizer
- Mini-workshop on “Hilbert’s 10^{th} Problem, Mazur’s Conjecture and Divisibility Sequences” (Oberwolfach, 2003)– co-organizer

Other: degree in Mathematical Education (Gent, 1993), member of the AMS (2000-), reviewer for Zentralblatt (since 1999), responsable for answering layman’s requests at the MPI (1998-2001), referee for several journals and scientific organizations.

4i. Scholarships and prizes
- (1993-1997) PhD-grant (“aspirant”) of the Belgian National Science Foundation (FWO-Vlaanderen), approx. scale 9 CAO-universiteiten
- (1997) Wuytack Travel Fund, lump sum approx. € 1500
- (1997-2001) multiple offer of a stipend as “Gastforscher” at the MPI (Bonn), replaced by a post-doc stipend of FWO-Vlaanderen, approx. Scale 10/11 CAO-universiteiten
- (1997-1999) Research Grant of FWO-Vlaanderen (Universiteit Gent joint with KULeuven) “Algebraic Geometry with applications to algebra and number theory” together with J. Van Geel (Gent); Gent budget approx. € 6500
- (1999-2001) Research Grant of FWO-Vlaanderen (Universiteit Gent joint with KULeuven) “Algebraic Geometry with applications to algebra and number theory” together with J. Van Geel (Gent); Gent budget approx. € 5000
- (2002) NWO-visiting scholarship for one month visit of F. Kato to Utrecht, approx. € 1000

List of publications

5. Publications:

-International (refereed) journals

[1] Sur les zéros des séries d'Eisenstein de poids q^k*-1* pour $GL_2(\mathbf{F}_q[T])$, C. R. Acad. Sci. Paris Sér. I Math. **321**, 817--820, 1995.

[2] Drinfeld modular forms of weight one, J. Number Theory **67**, 215--228, 1997.

[3] Stockage diophantien et hypothèse abc généralisée, C. R. Acad. Sci. Paris Sér. I Math. **328**, 3--8, 1999.

[4] Deligne's congruence and supersingular reduction of Drinfeld modules, Arch. Math. (Basel) **72**, 346--353, 1999.

[5] Zeros of Eisenstein series, quadratic class numbers and supersingularity for rational function fields, Math. Ann. **314**, 175--196, 1999.

[6] (with F. Kato and A. Kontogeorgis) Discontinuous groups in positive characteristic and automorphisms of Mumford curves, Math. Ann. **320**, nr. 1 (2001), 55-85.

[7] Two-torsion in the Jacobian of hyperelliptic curves over finite fields, Arch. der Math. **77** (2001), 241-246.

[8] The 2-primary class group of certain hyperelliptic curves, J. Numb. Th. **91**, nr. 1 (2001), 174-185.

[9] (with F. Kato) Equivariant deformation of Mumford curves and of ordinary curves in positive characteristic, to appear in Duke Math. J.

[10] (with F. Kato) Mumford curves with maximal automorphism group, 2000, to appear in Proc. A.M.S.

[11] (with F. Kato) Mumford curves with maximal automorphism group II: Lame type groups in genus 5-8, 2002, to appear in Geom. Dedicata.

-Books, or contributions to books

[12] Drinfeld modular forms of level T, in: Drinfeld modules, modular schemes and applications (Alden-Biesen, 1996), pp. 272--281. World Sci. Publishing, River Edge, NJ, 1997.

[13] A survey of Drinfeld modular forms, in: Drinfeld modules, modular schemes and applications (Alden-Biesen, 1996), pp. 167--187. World Sci. Publishing, River Edge, NJ,1997.

[14] (with K. Zahidi) Topology of Diophantine sets: remarks on Mazur's conjectures, in: Hilbert's Tenth Problem: Relations with arithmetic and algebraic geometry, Contemp. Math. **270**, pp. 253--260, AMS, Providence, 2000.

[15] Nichtarchimedische Geometrie, in: Max-Planck-Gesellschaft: Jahrbuch 2000, pp. 566--571, Vandenhoeck-Ruprecht, Göttingen, 2000.

[16] Diangle groups, proceedings van het „Symposion on Algebraic Geometry (Kinosaki)", Osaka Univ., 2000.

-Other

[17] (with M. Tripolitaki) Torsion of Drinfeld modules and equicharacteristic unimodular Galois covers, preprint, math.NT/0209023, 2002.