ENGLISH TRANSLATION

This is a clumsy translation of the lecture, only to be used to follow the lecture in the Aula. It will destroy itself immediately after reading.

Dear Rector, most honored audience,

This poem is entitled:

Housekeeping Error

This morning there he was again,
my unfaithful friend,
the empty sheet.

I am sitting leaned against him,
and together, we awaiten silence.

In the meanwhile, in the kitchen, life is
being charred.

Ladies and gentlemen,

The above sentence could refer to various activities, such as mathematics, philosophy, poetry or composition: the empty page could be filled with formulas, wisdom, verse or a musical score. All these creative activities have one thing in common: it appear, at least according to a widely spread opinion, that one is wasting one's time. According to mathematical giant Alain Connes, the best mathematics is done laying down on a sofa at dusk. This romantic picture is far removed from the daily reality of email, students, teaching, writing and board meetings, but it does point to the crux of the matter, which is: thinking.

Today, I want to talk to you about one such kind of waste of time. Of course, I don't consider it a waste of time, but you might, and of course it isn't a waste of time. Let me again present the usual proof: at the heart of the modern computer, Google, medical scans, safe wireless communication, GPS and the railway schedule is modern math.

In the mathematics that lays invisibly behind these applications, we attach old, hidden names to them, such as recursion theory, linear algebra, Radon transform, elliptic curve and graph theory. These theories have really not been conceived in order to get an application; and I don't expect you to know any of them. I even secretly find it unsatisfactory that pure mathematics nowadays has to keep it's back straight by invoking the argument of sudden application.

To quote my mathematical idol Yuri Manin: 'The point is that there is an inherent weakness in trying to justify one's concerns by saying that they are useful. Useful is a word of engineering.'

The distance from pure mathematics to its application is huge. Not so much in time, I mean in visibility. Pure mathematics does it's magic because it is not an application and often only arises from reality at a great distance.

Society really runs behind pure science, except of course the kind of science that runs behind society.

I do not know whether you personally value the kind of mathematics I will talk about. Probably, you have a neutral attitude, since the topics are far away from your bed side; or you think so, although maths is actually standing invisibly next to your bed, in the mobile phone on your night desk.

When mathematicians are laying down sofas, their thoughts are free and wild. I would like to share this passion with you. But this distance isn't helping us out. Mathematics has experienced a century long densification of its language, that will only allow for effective communication after years of study. Without this language of dense mathematical symbols, maths cannot do its job, but exactly because of this, it creates a distance.

In mathematics, relevance is coupled with distance: stepping back and looking at things abstractly. With mathematics and other activities that mainly consist in Oblomovian laying on sofas, I want to promote this taking distance.

So let us now turn to mathematics. Let the outside world char in the kitchen.

We thus arrive at the title of the lecture - 'Measuring ambiguity'.

Life is full of ambiguity. Since you are now attending a maths talk, you can imagine that I mean something else by this than you are thinking. To immediately remove all doubts and reveal myself as a mathematical nerd, I will present to you four examples of ambiguity in mathematics.

The first example is algebraic. The square root of 15 is a number whose square is 15. But there are two of those. If I call one 'square root of 15', then the other one is 'minus square root of 15', since -1 times -1 is 1. How can I distinguish these roots? I can say that one is positive and talk about 'the' square root as name for the positive root, as I was just now secretly doing already.

Now we try the same in a similar situation. The square root of -15 is a number whose square is -15. Please don't start doubting the existence of this number, it has been with us for several centuries. I will call it r, of root. But then, there is a second square root of -15, namely, -r. How can I distinguish these roots? The answer is that I cannot. Only is I measure one, that is, have assigned a name to one, then do I know the other one in a precise way, too.

Compare: in the current interpretation of the theory of observation in quantum mechanics, it is a measurement itself that pins down the value of what is being measured.

Things find their place when they have been given a name. If this is r, than that is -r.

Living with this unavoidable ambiguity is called 'Galois Theory'. Or, to do more justice to the subject: a theory that measures in a very precise way how ambiguous a situation is. Thanks to Galois theory, I can postulate that the solutions to $X^4 = 15$ are in a very precise way three times less ambiguous than those of $X^4 - X = 15$.

It might at first seem strange that there is a quantitative difference between these two equations, since they both have four solutions. The crux of Galois Theory is that the number of solutions is less important than the difference in freedom in assigning names to the solutions.

Évariste Galois (1811-1832), the mathematician after which this theory has been named, died in a duel. The night before his death, he wrote his 'last letter', where we read: Lately, I have been thinking about the application of transcendental analysis to the theory of ambiguity. This is the motto for todays lecture.

In connection with solving polynomial equations, I would like to finish with an anecdote that I often present to the algebra class. My father-in-law wanted to construct his own gate. It consisted of a long horizontal metal bar, attached to a shorter vertical hollow pipe. This pipe should be placed over a thinner vertical bar that is fixed to the ground, so the gate could open and close. The trouble is the imprecise construction: the inside of the hollow pipe is too thick for thin bar. Thus, the large horizontal bar will hang down a bit. The question is: how much at the end of the gate? We quickly found the relevant fourth degree equation and solved it by computer. The solution was 5.9 inches. We proudly present the solution at dinner. And then, my brother-in-law mumbled something and said, so 6 inches.

I like to use this example to illustrate the kind of mathematics that I do. For the kind of mathematics I am going to talk about today, the theory matters; the algebra of the first solution. The second one, estimating, is no good for seeing a general theory about the solution of equations. Theory needs distance and universality.

Of course, these are just expensive words to cover up our incompetence with respect to my brother-in-law. By assuming the inner rod had zero diameter, he could use congruent triangles.

The second example stems from geometry. Imagine a circle. You may for example consider the stars on the European flag that is hanging down above my head so prominently. If I attach this flag with the wrong side to the post, you wouldn't notice (except of course for the attachment material, but I am again talking in theory). If I draw a vertical line through the flag, the stars are arranged in a mirror symmetric way w.r.t. this line. I may call one side left and the other right, but by turning around the flag, I can create ambiguity in what I mean.

Now really replace the stars by a circle. Points of the circle come in pairs w.r.t. the line. Algebraically, I can choose coordinates to the line is the y-axis, the x-axis is orthogonal to this, through the center of the circle, and the circle has radius four. Then next to the point on the y-axis with coordinate one are two points with x-coordinates $\sqrt{15}$ and $-\sqrt{15}$. The second example has turned into the first, but in geometrical disguise.

The third example occurs in mathematical physics. For many computations with elementary particles, one needs to renormalize. This is a physical computational recipe, that can be performed in various ways. The results, however, the constants of nature, should be independent of the chosen calculation. The reason for this is again a Galois group, that Pierre Cartier called 'The cosmic Galois group'. Again, this allows me to make quantitative statements, such as: a four dimensional field theory with third degree potential is ambiguous, but infinitely less so than a three dimensional with sixth degree potential. In mathematics, we are not afraid of infinite Galois groups.

The fourth example is an experiment. I put two screens on this stage, to cover up two drummers, invisible to the audience. Each of them plays a drum, that however needn't have the form of a regular round drum; it might as well be triangular, star shaped or cloud shaped. Then can you hear whether or not these drums have the same shape?

In 1910, the Dutch physicist Hendrik Lorentz delivered a lecture in Göttingen, at that time the Mecca of modern mathematics.

As an aside, I want to mention that the lecture was made possible by a donation of Paul Wolfskehl, that left 100.000 Marks to the Göttingen mathematical institute, to be presented to the solver of Fermat's Last Theorem; but as long as such a solution was not presented, the interest could be used to organize a distinguished series of lectures. The Wolfkehl lecture of Lorentz was at the origin of the theory of integral kernels, and as such practically the whole first volume of Methods of Mathematical Physics by Courant and Hilbert, and in this sense one might say that failure to solve a number theoretical puzzle is responsible for this kind of mathematical physics. This might be the actual meaning of my chair, which is about the relation between number theory and mathematical physics.

Anyhow, Lorentz asked whether it is possible to determine the area of a drum by merely listening to it. The answer turns out to be yes. The pope of maths, Hilbert, however, who was present at the lecture, said that he would probably never hear the answer to this question during his lifetime. This conjecture of Hilbert turned out to be false: he lived through the 1940s, but his brilliant student Hermann Weyl solved the problem a year after the lecture, in 1911.

With all these high speed developments, one tends to forget that a female Ph. D. student of Lorentz, Johanna Reudler, verified the statement for circles, quadrangles, spheres, parallelepipeds and cylinders, in her beautiful thesis 'On the black radiation in spaces of different form'. The only reference to 'Fräulein Reudler' can be found in a footnote in Weyl's paper, and Lorentz, in his lecture, mentions her anonymously.

In the 1990s, Gordon, Webb and Wolpert constructed the first examples of drums that don't look alike, but sound alike. A recent study of Koen Thas and his collaborators actually shows that all known examples of this phenomenon can be captured via symmetries of certain geometries, so called Galois-geometries.

In this way, we are back at the first example, via the second example.

In summary, we now have four examples of ambiguity: polynomial equations and root extraction, symmetry in figures, invariance in renormalization and the drum shape problem. There is always some kind of Galois theory that is responsible for our understanding of the situation, or rather: the measurement of our inability to understand the situation.

I also want to remark that these examples run straight through the so-called boundaries between algebra, number theory, geometry, analysis and mathematical physics. Frans Oort once entitled a lecture 'Algebra of geometry?', and answered 'Algebra and geometry!'. I would go further and quote Dieudonné: 'It would be absurd and go against the gist of our science to want to divide it into rigid parts, in the manner of the traditional cutting of boundaries between algebra, analysis, geometry etc. that nowadays is completely fallacious.

Before I start making music with mathematics, I would like to use the first two examples to tell you something about my research during the past ten years. Those of you who know me mathematically also know that I love to jump from A to B, which I myself experience rather positively, but some people find tiring. So let me try to draw a line in this potpourri.

The first example was about ambiguity in solving polynomial equation. There are amazingly few equations for which the ambiguity is explicitly known. In some sense, we have a mathematical theory about measuring this ambiguity, but determining the Galois groups in an explicit manor is much harder. And mathematician are notoriously lazy when it comes to computing. By the way, the finite Galois group of the superrenormalizable ϕ_4^3 has also never been computed, though it is known in practice how to do that. Here, physicists have exceptionally copied a lazy habit of the mathematicians.

I started my career by trying to come to grips with this laziness and compute some Galois groups in the theory of modular forms. The background is the work of Issai Schur on Galois groups of classical orthogonal polynomials: they are, in traditional terminology, without emotion 'Ohne Affekt'.

I proved the same holds true for the zeros of Eisenstein series on the modular curve. The moral is that Eisenstein series are some kind of orthogonal polynomials in the hyperbolic plane. This was revealed almost simultaneously in work of Kaneko and Zagier, building on unpublished results of Oliver Atkin, a great name in the world of modular forms that passed away just three weeks ago.

In the second example, we spoke about ambiguity in geometry, due to symmetries. Think back of the flag. In my own research, I am concerned with deformations of object that keep their symmetries. The flag, for example, is symmetric by interchanging left and right.

I a long project with Fumiharu Kato, Ariane Mézard and Jakub Byszewski (1984-) I investigated the structure of the 'deformation space' of curves with symmetry.

The most symmetrical object in the world are probably the regular convex 4-, 6-, 8-, 12- and 20-faced solids: tetrahedron, cube, octahedron, icosahedron and dodecahedron. They are the miracles of everyday geometry. One may now ask the question: are there such things in other geometries? This is the question I tried to answer in work with Kato and Aristides Kontogeorgis in the case of so-called non-archimedean geometry. It is a different way of measuring than in everyday life: in these geometries, a number is small if it is highly divisible by a given prime number.

There was amazing physical relevance, e.g., in the computation of the partition function of the Polyakov string by Manin, and now in the work of Kontsevich and Soibelman on mirror symmetry. Manin illustrates the connection between prime numbers and physics by the equality: the product of 'one minus the inverse squared' over all prime numbers equals $6/\pi^2$. The left side is number theory, the right is physics or analysis.

We could use our work as a kind of 'chemical' building blocks in answering the question what kind of symmetries can occur at all in non-archimedean geometry.

Finally, I briefly want to mention my research in provable impossibility. A strong point for mathematics is its ability to establish its own knowledge frontier: it can sometimes, rather precisely, measurably, make clear what she itself cannot accomplish.

This ambiguity is Gödel's theorem. Yuri Matijasevich gave it the following formulation: there is a game with two players, such that whatever move the second player makes, the first will have a winning move, but this winning strategy cannot be determined by a computer. You can always win, but can never compute how to win. If you replace 'game' by 'mathematics' and 'compute' by 'prove' in the above, you are close to Gödel's theorem.

In mathematics, it is possible to measure how impossible something is. The question how hard it is to solve diophantine equations I have addressed in work with Karim Zahidi, Thanases Pheidas and Sasha Shlapentokh. This work is a quantification of the undecidability in Gödel; probably a formulation he would have chosen in later life, see an undated manuscript from his hand from the 1930s.

Summarizing, and looking back at the past years, I see the following pattern emerge.

In the mathematics of the 20th century, we have learn to quantify ambiguity, and to turn our incompetence or impossibility to measure something itself into an object of measurement.

Die Vermessung des Unvermögens zu Messen. Also this is a kind of Galois theory.

We shouldn't give up too soon: a method that doesn't lead to results often has as reason an interesting phenomenon, that we should study instead.

I am reminded of Hendrik Lenstra's Bernoulli-lecture, where he proposed 'not understanding' as an important mechanism in the creation of new mathematics. To his toolbox for research, I would like to add: measuring misunderstanding.

Maybe we can even draw some conclusion out of this for the real world, that is now all charred up in the kitchen. Disability can be a quality, if understood. If you know you cannot do something, measure why you cannot do it.

The ambiguity in the drum form problem I want to use in order to tell you something about my current research. We will soon see how an individual musical tone is built up. But music is a sequence of tones. This sequence turns out to display interesting statistical behavior, as was discovered by Voss and Clarke in the 1970s. If I throw dice to determine the pitch of consecutive notes, 'white noise' appears: it doesn't sound like music, since there is no correlation between consecutive notes. If I instead use dice to determine the difference in pitch between consecutive notes, there will be too much correlation, and I get boring 'brown noise'. Actual music turns out to follow the pattern of 'pink noise', of 1/f-noise. One can use dice to approximate this process. I used it many years ago with Karim Zahidi to write music for a teacher symposium about Art and Music in Belgium. We chose some rhythm and harmonies, and these are the little pieces that were played on the organ by Gert Oost before. He will now repeat one of those for you.

Unfortunately (or maybe lucky?), we misread the dicing recipe for pink noise and lost a lot of correlation. What we did was basically throw consecutively with 3, 1, 2 and 1 dice, so tones are in consecutive intervals 3 to 18, 1 to 6, 2 to 12 and 1 to 6. This suggest a kind of natural harmony. I don't know what you think, but we found the sound quite agreeable, and secretly even better sounding that pink noise. Those of you who know Karim know that there can be only one name for this noise: dark red noise. It has sinusoidal correlation, so its correlation structure itself takes on the form a a pure tone - a nice kind of self reference.

This leads us to the mathematical structure of the individual tone, the color of an instrument. What you hear when you hit a string, beat a drum, or blow air through an organ pipe consists of a combination of pure tones. The vibrations are described by the wave equation, and pure tones are so-called eigenfunctions of the Laplace operator. You can see them by taking a picture of a vibrating string: they are combinations of the sines and cosines of high school.

How they are combined precisely determines the color of the instrument, from spiky sawtooth to lovely recorder. The audible information is called 'spectral data' in mathematics. There are then two kinds of problem: inverse problems, i.e., what information is audible? and direct problem, i.e., what do I know about the spectrum given the instrument?

Remember: Weyl told us that area in audible, Gordon, Webb and Wolpert that shape isn't.

The inverse spectral question can be traced to a report of the physicist Arthur Schuster for Brittish Association in 1882. Schuster, who came up with the terminology 'spectroscopy', wrote: 'It would baffle the most skillful mathematician to solve the inverse problem and to find out the shape of a bell by means of the sounds which it is capable of sending out. [...] In the meantime we must welcome with delight even the smallest step in the desired direction.'

We do not know the chemical composition of the sun by traveling back and forth and bringing a slice of helium. We watch and do spectroscopy.

An unexpected application of the inverse spectral problem in gravitation theory arises from the work of Japanese physicist Masafumi Seriu: the spectrum is used as a measure of the distance between different solutions to the Einstein equations for the universe. As you might know, the universe we live in is curve and space and time are indivisibly connected on a grand scale. The equations of Einstein provide us with possible models of the universe, but there are many of those. How can we compare these models and reality? And, 'Is cosmology possible', i.e., will there be a model that matches the cosmos? Seriu wants to measure this by looking at spectra. But the problem of 'isospectrality' remains: that the shape of a universe cannot necessarily be deduced from its sound. So what do we do? We turn to Riemann.

As a Ph. D. student when I had to do surveillance at exams I used to take along a volume of the collected works of a Great Mathematician, to browse through it from time to time. I remember finding Kronecker nog so hard to read, but Riemann I didn't understand a word of. Last year, I tried reading in his collected works again, a mere 500 pages of which only 200 published during his lifetime. I still don't understand. But I take up the challenge to add 'epsilon' to it. 'Epsilon' is math talk for the 'negligibly small'.

Riemann brought is fundamental concepts such as the Riemann zeta function and Riemannian geometry, so me say a bit about those.

First the Riemann zeta function and the Riemann hypothesis, the most famous open problem in mathematics. The spectrum, the overtones, are packaged together by mathematicians into a zeta function. The Riemann zeta function belongs in this way to a circular string, which by the way is uniquely determined by its spectrum.

Number theorists in the audience might hover at my definition of the Riemann zeta function. For them (and for me) it measures the distribution of prime numbers. When we look at where in the natural numbers we find the prime numbers, so numbers whose only divisors are itself and unity, then this appears to be quite random. But large computation by Gauss revealed a certain regular distribution, and the primes are least 'deviant' from this distribution precisely if the Riemann hypothesis is true. By the way, with Oliver Lorscheid I looked at a method proposed by Don Zagier to attack this question, obviously with epsilon success.

Anyhow, the number theoretical zeta function is the same as that of the circular string, and this is the beginning of the great mystery. Looking at the title of the popular book by Marcus du Sautoy, 'The music of the primes', it ought to be possible to listen to the prime numbers. Sir Michael Berry indeed once produced a tone based on the prime numbers, but opinions tend to differ. I once heard 'Better than Wagner'.

Another fundamental concept of Riemann is so-called Riemannian geometry, a flexible model geometry, e.g. in relativity theory. There is not one geometry, but many, and life on a sphere is different from that on a plane: walking on a sphere you will wander back to where you came from, but an a finite plane, you are bound to fall into a pit full of monster sooner or later.

Floris Takens (1940-) asked me last year how to compare different Riemannian geometries. The reason for this question was a misunderstanding: I was interested in a notion of 'being the same' in noncommutative geometry, let alone comparing anything to anything. But it did get me thinking.

Riemannian geometries produce sound. Spectral geometry, Riemannian geometry from the viewpoint of the Laplace-operator, the zeta function, so the sound it produces, has experienced a new incarnation during the past 20 years in the works of Alain Connes called noncommutative geometry. I will spare you the details, but let me tell you that it has firm roots in quantum mechanics. A problem with this extremely young branch of mathematics is nicely formulated by Yuri Manin (I could keep on quoting him): 'Noncommutative geometry nowadays looks like a vast building site [but it] lacks common foundations: for many interesting constructions of `noncommutative spaces' we cannot even say for sure which of them lead to isomorphic spaces ...'. Whence my question about 'being the same'.

Now in noncommutative geometry, one tends to look at families of zeta functions instead of zeta function itself. The families are indexed by function on the geometry that is being used as musical instrument. This immediately calls for the question whether this family of zeta function does determine the geometry, the central question of this line of research. With Matilde Marcolli, I already verified that a closed surface can be reconstructed from such a Connes-style geometry. I am now working on the general case with Jorge Plazas.

We construct, for each pair of diffeomorphic Riemannian geometries, a zeta-field. A constant zeta-field should give the same geometries; a holomorphic field should point to a covering, and in general this field is some kind of measure of the difference between the Riemannian geometries, thus answering Takens's question. We don't know yet whether this is going to work out; we know that 'constant zeta field' determines the geometry in a residual set of metrics.

Instead of elaborating further on this, I want to close by showing you a movie in which you see an object that is very important in the proofs: nodal sets. They were first revealed for square glass plates by Robert Hooke in 1680, and the experiments where repeated and popularized by Ernst Chladni. In the meanwhile, a New Age company will turn your voice into a psychedelic pattern. Anyhow, Hooke and Chladni threw sand on a glass or metal plate and put that into vibration. What happens you can now see with salt thrown onto a loudspeaker, and experiment from Wake Forrest University. This doesn't only display the nodal lines of the Laplacian eigenfunction.

Those of you who cannot stand high pitches should close their ears.

These patterns combine symmetry and drums in a heady way.

I will summarize our research program as follows: by listening quantum mechanically, i.e., using the zeta field from noncommutative geometry, to classical instruments, we will reveal their shapes. We listen in some sense by covering up consecutive parts of the drum: drumming with two hands, so to speak.

But isn't the zeta function something number theoretical? What kind of implications and analogies does this theory have in number theory? Sir Michael Atiyah said that the fortress of number theory would be the next to be conquered by methods of mathematical physics. Very intriguing.

One takes the universe, and two very big hand. With one hand, one covers up bits and pieces of the universe. With the other hand, one hits the universe like a drum. We see wonderful

Chladni-patterns. In this way, we can hear in which universe we are.

If you ever feel shaken, the reason might be that my research team is probing the universe again.

It is customary to finish with some words of thanks. But those people that I want to thank most would not appreciate being mentioned here by name. That they may, however, know how much I thank them.

This is it. I deliberately made the two errors of a young professor: to say too many different things, and to talk too much about ones own work. Maybe therefore, you feel the need to lay down on a sofa. Alas, I can only offer you a drink and some snacks.

I have spoken.