STANDARD MODEL

① Vector space V (r-dimensional) $\overset{①}{\Longrightarrow}$ $Fr(V)$ = set of all $\underbrace{\text{frames } e=(e_1,\dots,e_r) \text{ of } V}$

$\Longleftrightarrow$ linear isomorphisms $\hat{e}: \mathbb{R}^r \xrightarrow{\sim} V$

– a GL_r-torsor ….. nice action $Fr(V)\times GL_r \to Fr(V)$

② Vector bundle $\begin{matrix} E \\ \downarrow \\ M \end{matrix}$ (rank r) $\Longrightarrow$ $\begin{matrix} Fr(E) = \{(x,e) : x\in M, e\in Fr(E_x)\} \\ \downarrow \\ M \end{matrix}$ principal GL_r-bundle

③ E can be recovered from $Fr(E)$: $\left(Fr(E)\times\mathbb{R}^r\right)/GL_r \xrightarrow{\sim} E$, $[(\underset{M}{x},\underset{Fr(E_x)}{e}),\underset{\mathbb{R}^r}{v}] \longmapsto \underset{E}{\hat{e}(v)}$

④ $\begin{matrix} E \\ \downarrow \\ M \end{matrix}$ & inner product g $\Longrightarrow$ $\begin{matrix} Fr(E,g)^{\subseteq Fr(E)} \\ \downarrow \\ M \end{matrix}$ principal $O(r)$-bundle … an $O(r)$-reduction of $Fr(E)$.

similarly for complex structures

(parallelizations)	$(\varphi_1,\dots,\varphi_r)$		
Symplectic structures $(r=2k)$	$\omega: V\times V\to \mathbb{R}$ bilinear, skew symmetric, non-degenerate	(V,ω) ? $(e_1,\dots,e_k;\ f_1,\dots,f_k)$ with $\omega(e_i,e_j)=0$, $\omega(f_i,f_j)=0$, $\omega(e_i,f_j)=\delta_{ij}$	$\mathbb{R}^r=\mathbb{R}$… $\omega_{can}=\sum$… $(\sum dy_i$…

[-2-]

SLOGAN: GEOMETRIC STRUCTURES CAN BE ENCODED BY THE "COLLECTION OF FRAMES THAT ARE ADAPTED TO THE STRUCTURE"

On V:

$$\hat{e}: \mathbb{R}^r \xrightarrow{\sim} V$$

(*) $\mathbb{R}^r$ has a canonical structure; $\hat{e}$ preserves the structure

Example:

⓪ The structure: innerproducts

① On a vector space V: $g: V\times V\to\mathbb{R}$, ...

② Try to guess "frames adapted to g": orthonormal frames of (V,g).
(OR: Follow ③ & ④ and use (*))

③ Standard model: $(\mathbb{R}^r, g_{can})$ $\quad g_{can}(u,v)=\sum_{i=1}^r u_i v_i$

④ Isomorphisms: $A:(V,g)\to(V',g')$ isometries (linear, $g'(A(u_1),A(u_2)) = g(u_1,u_2)$ $(\forall\, u_1,u_2\in V)$.

Check ② (if a guess was made): $\left(g(\hat{e}(e_i^{can}),\hat{e}(e_j^{can})) = g_{can}(e_i^{can},e_j^{can});\ g(e_i,e_j)=\delta_{ij}\right)$

isomorphism $\hat{e}:(\mathbb{R}^r,g_{can})\to(V,g) \iff e=$ orthonormal.

⑤ The…

⑥ Ada…

⑦ On ve…
structu…
of st…
$g=$…

⑧ Fram…

$A(\gamma)=\psi'$

$\mathbb{R}^r=\mathbb{R}^{2k}$

$A:(V,\omega)\to(V',\omega')$

Sp_k

Sp_k-torsor

$\omega(f_i,f_j)=0$

δ_{ij}

$\omega_{can}=\sum_{i=1}^{n}(e_i^{can})^*\wedge(e_{i+k}^{can})^*$

$(\sum dy_i\wedge dx_i)$

$\omega'(A(v_1),A(v_2))=$
$=\omega(v_1,v_2)$

$G=\{(d_1 \dots d_r)\}$

$V=W_1\oplus\dots\oplus W_r$

THE

"...RUCTURE"

...onical
...tructure
...the structure

$(A(u_1),A(u_2))=$
$=g(u_1,u_2)$
$(\forall u_1,u_2\in V)$

⑤ The symmetry group $G\subseteq GL_r$ associated to the given structure: automorphism of the standard model from ③

$(\mathbb{R}^r, g_{can})$

$\Rightarrow$ in this case the group is $G=O(r)$

⑥ Adapted frames all together:

$Fr(V,g)=O(r)$-torsor $\subseteq Fr(V)$

Lemma: Any $Q\subseteq Fr(V)$ which is an $O(r)$-torsor comes from an inner product g.

⑦ On vector bundles: such a structure is a collection of structures, one on each E_x, "varying smoothly w.r.t. x"

$g=\{g_x\}_{x\in M}$ discussed before.

⑧ Frame bundles: put together ⑥ applied to fibers

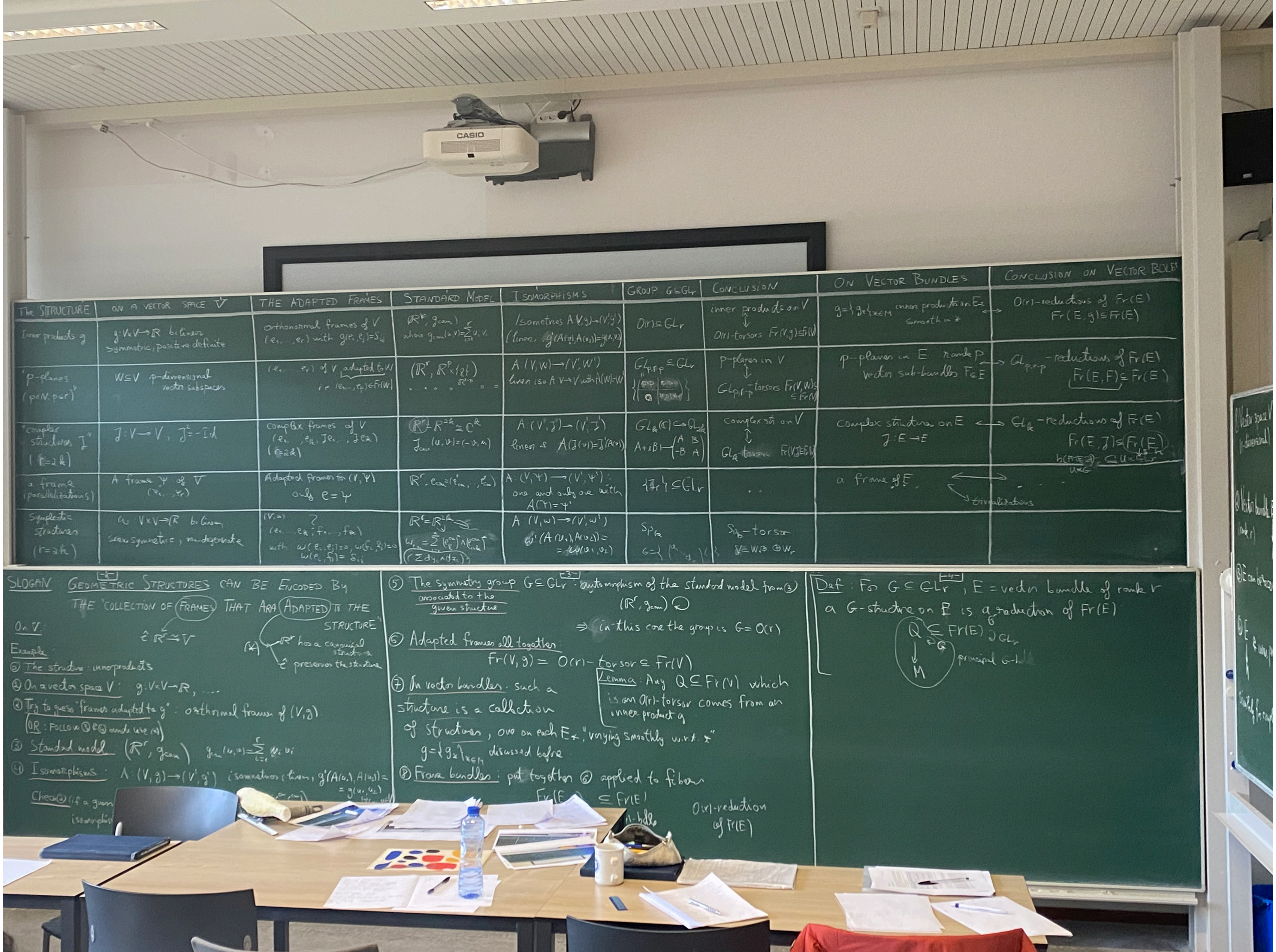
$Fr(E,g)\subseteq Fr(E)$ — $O(r)$-reduction of $Fr(E)$

$\downarrow$ princ. $O(r)$-bdle

M

Def:

a G-


CASIO
The STRUCTURE
ON A VECTOR SPACE V
THE ADAPTED FRAMES
STANDARD MODEL
ISOMORPHISMS
GROUP G⊆GLr
CONCLUSION
ON VECTOR BUNDLES
CONCLUSION ON VECTOR BDLES
Inner products g
g: V×V→ℝ bilinear symmetric, positive definite
orthonormal frames of V (e1, …, er) with g(ei, ej)=δij
(ℝ^r, gcan) where gcan(u,v)=Σ ui vi
Isometries A:(V,g)→(V',g') (linear, g'(A(v1),A(v2))=g(v1,v2))
O(r)⊆GLr
inner products on V ↕ O(r)-torsors Fr(V,g)⊆Fr(V)
g={gx}x∈M inner products on Ex smooth in x
O(r)-reductions of Fr(E) Fr(E,g)⊆Fr(E)
"p-planes (p∈N, p≤r)"
W⊆V p-dimensional vector subspaces
(e1, …, er) of V adapted to W i.e. (e1, …, ep)∈Fr(W)
(ℝ^r, ℝ^p×{0})
A:(V,W)→(V',W') linear iso A:V→V' with A(W)=W'
GLp,r-p ⊆ GLr
p-planes in V ↕ GLp,r-p-torsors Fr(V,W)⊆Fr(V)
p-planes in E rank p vector sub-bundles F⊆E
GLp,r-p-reductions of Fr(E) Fr(E,F)⊆Fr(E)
"complex structures J" (r=2k)
J: V→V, J²=−Id
complex frames of V (e1, …, ek, Je1, …, Jek) (r=2k)
ℝ^r = ℝ^2k ≅ ℂ^k Jcan(u,v)=(−v,u)
A:(V,J)→(V',J') linear & A(J(v))=J'(A(v))
GLk(ℂ) ↪ GL2k A+iB ↦ (A B / −B A)
complex str on V ↕ GLk-torsors Fr(V,J)⊆Fr(V)
complex structures on E J: E→E
GLk-reductions of Fr(E) Fr(E,J)⊆Fr(E)
a frame (parallelizations)
A frame ψ of V (ψ1, …, ψr)
Adapted frames for (V,ψ) only e=ψ
ℝ^r, ecan=(e1can, …, ercan)
A:(V,ψ)→(V',ψ'): one and only one with A(ψ)=ψ'
{Ir}⊆GLr
a frame of E
trivializations
Symplectic structures (r=2k)
ω: V×V→ℝ bilinear skew symmetric, non-degenerate
(V,ω) ? (e1, …, ek; f1, …, fk) with ω(ei,ej)=0, ω(fi,fj)=0 ω(ei,fj)=δij
ℝ^r=ℝ^2k
(Σ dyi∧dxi)
A:(V,ω)→(V',ω') ω'(A(v1),A(v2)) = ω(v1,v2)
Spk
Spk-torsor
-2-
SLOGAN: GEOMETRIC STRUCTURES CAN BE ENCODED BY "THE COLLECTION OF FRAMES THAT ARE ADAPTED TO THE STRUCTURE"
On V:
ê: ℝ^r ⥲ V
(*) ℝ^r has a canonical structure
ê preserves the structure
Example:
⓪ The structure: inner products
① On a vector space V: g: V×V→ℝ, ….
② Try to guess "frames adapted to g": orthonormal frames of (V,g)
(OR: FOLLOW ③ & ④ and use (*))
③ Standard model (ℝ^r, gcan) gcan(u,v)=Σ ui vi
④ Isomorphisms: A:(V,g)→(V',g') isometries (linear, g'(A(u1),A(u2)) = g(u1,u2)
Check② (if a given
-3-
⑤ The symmetry group associated to the given structure G⊆GLr: automorphism of the standard model from ③ (ℝ^r, gcan)
⇒ in this case the group is G=O(r)
⑥ Adapted frames all together
Fr(V,g) = O(r)-torsor ⊆ Fr(V)
⑦ On vector bundles: such a structure is a collection of structures, one on each Ex, "varying smoothly w.r.t. x"
g={gx}x∈M discussed before
Lemma: Any Q⊆Fr(V) which is an O(r)-torsor comes from an inner product g
⑧ Frame bundles: put together ⑥ applied to fibers
⊆ Fr(E)
O(r)-reduction of Fr(E)
-4-
Def: For G⊆GLr, E = vector bundle of rank r
a G-structure on E is a reduction of Fr(E)
Q ⊆ Fr(E)
M
principal

The STRUCTURE	ON A VECTOR SPACE V	THE ADAPTED FRAMES	STANDARD MODEL	ISOMORP…
Inner products g	$g: V\times V \to \mathbb{R}$ bilinear symmetric, positive definite	orthonormal frames of V $(e_1, \dots, e_r)$ with $g(e_i, e_j) = \delta_{ij}$	$(\mathbb{R}^r, g_{can})$ where $g_{can}(u,v) = \sum_{i=1}^{r} u_i v_i$	Isometric… (linear, g'…
"p-planes" $(p \in \mathbb{N}, p \leq r)$	$W \subseteq V$ p-dimensional vector subspaces	$(e_1, \dots, e_r)$ of V adapted to W i.e. $(e_1, \dots, e_p) \in Fr(W)$	$(\mathbb{R}^r, \mathbb{R}^p \times \underbrace{\{0\}}_{\mathbb{R}^{r-p}})$	$A:(V,W)\to$… linear iso A…
"complex structures J" $(r = 2k)$	$J: V \to V$, $J^2 = -Id$	complex frames of V $(e_1, \dots, e_k, Je_1, \dots, Je_k)$ $(r=2k)$	$\mathbb{R}^r = \mathbb{R}^{2k} \cong \mathbb{C}^k$, $J_{can}(u,v) = (-v, u)$	$A:(V,J$… linear &
a frame (parallelizations)	A frame ψ of V $(\psi_1, \dots, \psi_r)$	Adapted frames for (V, ψ): only $e = \psi$.	$\mathbb{R}^r$, $e_{can} = (e^1_{can}, \dots, e^r_{can})$	$A:(V,\psi)\to$… one and … $A(\psi)=$…
Symplectic structures $(r=2k)$	$\omega: V\times V \to \mathbb{R}$ bilinear skew symmetric, non-degenerate	(V,ω) ? $(e_1, \dots, e_k; f_1, \dots, f_k)$ with $\omega(e_i, e_j) = 0$, $\omega(f_i, f_j) = 0$, $\omega(e_i, f_j) = \delta_{ij}$	$\mathbb{R}^r = \mathbb{R}^{2k}$, $\omega_{can} = \sum_{i=1}^{n} (e_i^{can})^* \wedge (e_{i+k}^{can})^*$ $(\sum dy_i \wedge dx_i)$	$A:(V,\omega)$… $\omega'(A$…

-2-

SLOGAN: GEOMETRIC STRUCTURES CAN BE ENCODED BY THE "COLLECTION OF FRAMES THAT ARE ADAPTED TO THE STRUCTURE"

On V: $\hat{e}: \mathbb{R}^r \cong V$

(*) $\mathbb{R}^r$ has a canonical structure
$\hat{e}$ preserves that …

Example:

⓪ The structure: innerproducts

⑤ The symmetry group $G \subseteq$… associated to the given structure

⑥ Adapted frames all toget…

…EL	ISOMORPHISMS	GROUP $G \subseteq GL_r$	CONCLUSION	ON VECTOR BUNDLES	CONCLUSION ON VECTOR BDLS
	Isometries $A:(V,g)\to(V',g')$ (linear, $g'(A(v_1),A(v_2))=g(v_1,v_2)$)	$O(r)\subseteq GL_r$	inner products on V $\updownarrow$ $O(r)$-torsors $Fr(V,g)\subseteq Fr(V)$	$g=\{g_x\}_{x\in M}$ inner products on E_x smooth in x $\longleftrightarrow$	$O(r)$-reductions of $Fr(E)$ $Fr(E,g)\subseteq Fr(E)$
o	$A:(V,W)\longrightarrow(V',W')$ linear iso $A: V\to V'$ with $A(W)=W'$	$GL_{p,r-p}\subseteq GL_r$ $= \left\{\begin{pmatrix} p\times p & p\times(r-p) \\ 0 & (r-p)\times(r-p)\end{pmatrix}\right\}$	p-planes in V $\updownarrow$ $GL_{p,r-p}$-torsors $Fr(V,W)\subseteq Fr(V)$	p-planes in E rank p vector sub-bundles $F\subseteq E$ $\longleftrightarrow$	$GL_{p,r-p}$-reductions of $Fr(E)$ $Fr(E,F)\subseteq Fr(E)$
)	$A:(V,J)\to(V',J')$ linear & $A(J(v))=J'(A(v))$	$GL_k(\mathbb{C})\hookrightarrow GL_{2k}$ $A+iB\longmapsto\begin{pmatrix}A & B\\ -B & A\end{pmatrix}$	complex str on V $\updownarrow$ GL_k-torsors $Fr(V,J)\subseteq Fr(V)$	complex structures on E $J:E\to E$ $\longleftrightarrow$	GL_k-reductions of $Fr(E)$ $Fr(E,J)\subseteq Fr(E)$ $h(Fr(E,J))\subseteq U\times GL_r$ $U\times G$
…)	$A:(V,\Psi)\longrightarrow(V',\Psi')$: one and only one with $A(\Psi)=\Psi'$	$\{I_r\}\subseteq GL_r$	…	a frame of E $\longleftrightarrow$ trivializations	…
	$A:(V,\omega)\longrightarrow(V',\omega')$ $\omega'(A(v_1),A(v_2))=\omega(v_1,v_2)$	Sp_k $G=\left\{\begin{pmatrix}a_1 & & \\ & \ddots & \\ & & a_r\end{pmatrix}\right\}$	Sp_k-torsor $V=W_1\oplus\dots\oplus W_r$		

[3]

…group $G\subseteq GL_r$: automorphism of the standard model from (3)

…the

…ture

$(\mathbb{R}^r, g_{can})$ ↺

$\Rightarrow$ in this case the group is $G=O(r)$

…es all together:

… torsor $\subseteq Fr(V)$ … which

[4]

Def: For $G\subseteq GL_r$, E = vector bundle of rank r

a G-structure on E is a G-reduction of $Fr(E)$

$$\begin{array}{l} Q \subseteq Fr(E) \circlearrowleft GL_r \\ \circlearrowleft G \\ \downarrow \\ M \end{array}$$

principal G-bdle

... A)	GL_k-torsors $Fr(V,J) \subseteq Fr(V)$	$J: E \to E$	GL_k-reductions of $Fr(E)$ $Fr(E,J) \subseteq Fr(E)$ $h(Fr(E,J)|_U) \subseteq U \times GL_r$ $U \times G$
$\{I_r\} \subseteq GL_r$	...	a frame of E $\longleftrightarrow$ trivializations	...
Sp_k $G = \left\{ \begin{pmatrix} d_1 & & \\ & \ddots & \\ & & d_r \end{pmatrix} \right\}$	Sp_k-torsor $V = W_1 \oplus \dots \oplus W_r$		

...ism of the standard model from (3)

$(\mathbb{R}^r, g_{can})$ ⟲

...this case the group is $G = O(r)$

...sor $\subseteq Fr(V)$

...ma: Any $Q \subseteq Fr(V)$ which

...an $O(r)$-torsor comes from an

...inner product g.

... smoothly w.r.t. x"

...lied to fibers

$Fr(E)$ $\qquad$ $O(r)$-reduction of $Fr(E)$

$O(r)$-bdle

-4-

Def: For $G \subseteq GL_r$, E = vector bundle of rank r

a G-structure on E is a G-reduction of $Fr(E)$

$$\begin{array}{ccl} Q & \subseteq & Fr(E) \circlearrowleft GL_r \\ \downarrow & \circlearrowleft G & \\ M & & \end{array}$$

principal G-bdle

Symplectic structures ($r = 2k$)

$\omega: V \times V \to \mathbb{R}$ bilinear

skew symmetric, non-degenerate

structure $\omega \in \Omega^2(M)$

$\boxed{-5-}$

Def: Given $G \subseteq GL_n(\ldots)$, a G-structure on an n-dimensional manifold M is any G-structure on its tangent bundle

$$\mathcal{S} \subseteq Fr(TM)$$

$\dim M = n$

$r = \operatorname{rank} TM = n$

G/T "

$E_e \subseteq \ \subseteq E_1 \subseteq E$

$k_e \qquad k_1$

$Gr_{k_e, \ldots, k_1}(E)$

Given $\mathcal{S}$, a chart (U, x) of M is said to be adapted to $\mathcal{S}$ if

$$\left(\left(\frac{\partial}{\partial x_1}\right)_x, \ldots, \left(\frac{\partial}{\partial x_n}\right)_x \right) \in \mathcal{S} \qquad (\forall)\, x \in U$$

Say that $\mathcal{S}$ is $\boxed{\text{integrable}}$ if $\boxed{\text{around any } x \in M \text{ one can find a chart adapted to } \mathcal{S}.}$

$\Updownarrow$

$\boxed{\exists \text{ an atlas of } M \text{ made of charts adapted to } \mathcal{S}.}$

Ex 1: When

It is int

Rk: thi

For your info

Ex 2: For

Requirement

$\mathcal{S}^G_{can} = \{$

Rk: A

$\Longleftrightarrow x$ i

Ex 1: When $G = \{I_r\}$, $\boxed{G\text{-}}$ an $\{I_r\}$-structure on $M \Leftrightarrow$

$\Leftrightarrow$ a global frame $(X^1, \dots, X^n)$ of TM $\left(\begin{array}{c}\text{parallelization} \\ \text{of } M\end{array}\right)$

It is integrable $\Leftrightarrow$ locally. $X^i = \frac{\partial}{\partial x_i}$

Rk: this would imply: $[X^i, X^j] = 0$

For your info: the converse true.

Ex 2: For general $G \leq GL_n \Rightarrow$ we do have a canonical G-str $\mathcal{S}^G_{can}$ on $\mathbb{R}^r$

Requirement on $\mathcal{S}^G_{can}$: $\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n}$ is in $\mathcal{S}^G_{can} \Rightarrow$ determine $\mathcal{S}^G_{can}$ uniquely.

$$\mathcal{S}^G_{can} = \left\{ (x, e) \in Fr(T\mathbb{R}^n) \,/\, e_i = \sum_j g_{ij} \cdot \left(\frac{\partial}{\partial x_j}\right)_x, \text{ the matrix } (g_{ij}) \in G \right\}$$

Rk: A chart $\chi : \underset{\underset{M}{\cap}}{U} \to \underset{\underset{\mathbb{R}^r}{\cap}}{\Omega}$ is adapted to a G-str. $\mathcal{S} \Longleftrightarrow$

$\Longleftrightarrow \chi$ is an "isomorphism" between $(U, \mathcal{S}|_U)$ and $(\Omega, \mathcal{S}^G_{can}|_\Omega)$.

Ex: define this & solve exercise

Def

a

The STRUCTURE	ON A VECTOR SPACE V	Group	On a manifold M	Adapted charts	When are they integrable?	Are the integrable ones interesting?
Inner products g	$g: V\times V \to \mathbb{R}$ bilinear symmetric, positive definite	$O(r)$	a Riemannian metric on M	(U,x) s.t. $\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n}$ orthonormal frame		NO (too restrictive)
"p-planes ($p\in\mathbb{N}$, $p\leq r$)"	$W\subseteq V$ p-dimensional vector subspaces	$Gl_{p,r-p}$	p-dimensional distributions $\mathcal{F}\subseteq TM$	(U,x) s.t. $\left(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_p}\right)$ - frame of $\mathcal{F}$	$\xleftrightarrow{\text{Frobenius}}$ $\mathcal{F}$ is involutive i.e. $\mathcal{F}$ = foliation ($X, Y\in\Gamma\mathcal{F} \Rightarrow [X,Y]\in\Gamma\mathcal{F}$)	YES
"complex structures J" ($r=2k$)	$J: V\to V$, $J^2=-Id$ $GL_k(\mathbb{C}) \subset$	GL_k	almost complex str. $J: TM\to TM$	$n=2k$ $\frac{\partial}{\partial x_{k+i}} = J\left(\frac{\partial}{\partial x_k}\right)$	$\xleftrightarrow[\text{Nirenberg}]{\text{Newlander}}$ J is a complex structure	YES
a frame (parallelizations)	A frame Ψ of V $(\psi_1, \dots, \psi_r)$	$\{I_r\}$	Parallelization $(X^1, \dots, X^n)$	(U,X) s.t. $X^i = \frac{\partial}{\partial x^i}$ (on U)	$\xleftrightarrow{\text{Prop}}$ $[X^i, X^j]=0$ $\forall i,j$	Not so
Symplectic structures ($r=2k$)	$\omega: V\times V\to\mathbb{R}$ bilinear skew symmetric, non-degenerate		An almost symplectic structure $\omega\in\Omega^2(M)$		$\xleftrightarrow{\text{Darboux}}$ $d\omega=0$ (i.e. ω = symplectic)	Yes

Def: Given $G\subseteq GL_n(\dots)$, $\boxed{-5-}$ a G-structure on an n-dimensional manifold M is any G-structure on its tangent bundle
$$\mathcal{S}\subseteq Fr(TM)$$

$\dim M = n$
$r = \operatorname{rank} TM = n$

$E_p \subseteq \dots \subseteq E_1 \subseteq E$
$k_p \quad k_1$
$Gr_{k_p,\dots,k_1}(E)$
G/T

Given $\mathcal{S}$, a chart (U,x) of M is said to be adapted to $\mathcal{S}$ if
$$\left(\left(\frac{\partial}{\partial x_1}\right)_x, \dots, \left(\frac{\partial}{\partial x_n}\right)_x\right) \in \mathcal{S} \quad (\forall)\, x\in U$$

Say that $\mathcal{S}$ is $\boxed{\text{integrable}}$ if $\boxed{\text{around any } x\in M \text{ one can find a chart adapted to } \mathcal{S}.}$
$\Updownarrow$
$\boxed{\exists \text{ an atlas of } M \text{ made of charts adapted to } \mathcal{S}.}$

Ex1: When $G=\{I_r\}$, $\boxed{-6-}$ an $\{I_r\}$-structure on M $\Leftrightarrow$
$\Leftrightarrow$ a global frame $(X^1, \dots, X^n)$ of TM (parallelization of M)

It is integrable $\Leftrightarrow$ locally $X^i = \frac{\partial}{\partial x_i}$

Rk: this would imply: $[X^i, X^j]=0$
For your info: the converse true.

Ex2: For general $G\subseteq GL_n$ $\Rightarrow$ we do have a canonical G-str. $\mathcal{S}^G_{can}$ on $\mathbb{R}^r$

Requirement on $\mathcal{S}^G_{can}$: $\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n}$ is in $\mathcal{S}^G_{can}$ $\Rightarrow$ determine $\mathcal{S}^G_{can}$ uniquely.
$$\mathcal{S}^G_{can} = \left\{(x,e)\in Fr(T\mathbb{R}^n) \,/\, e_i = \sum_j g_{ij}\left(\frac{\partial}{\partial x_j}\right)_x, \text{ the matrix } (g_{ij})\in G\right\}$$

Rk: A chart $x: \underset{\underset{M}{\cap}}{U} \to \underset{\underset{\mathbb{R}^r}{\cap}}{\Omega}$ is adapted to a G-str. $\mathcal{S}$ $\Leftrightarrow$
$\Leftrightarrow$ x is an "isomorphism" between $(U, \mathcal{S}|_U)$ and $(\Omega, \mathcal{S}^G_{can}|_\Omega)$.
Ex: define this & solve exercise

Def: For $G\subseteq GL_r$ $\boxed{-4-}$, E = vector bundle
a G-structure on E ... a reduction of Fr(...
$$Q \circlearrowleft G \text{ principal}$$
$\downarrow$
M