Vernieuwingsimpuls /Innovational Research
Grant application form (detailed proposal) 2013 **Vici scheme**
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Registration form (basic details)

1a. Details of applicant

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1b. Title of research proposal: ***Poisson Geometry Inside Out***

1c. Scientitic summary of research proposal: Poisson Geometry lies at the cross-road of Geometry and Mathematical Physics; it originates in Hamiltonian mechanics; as part of Geometry, it can be seen as an amalgam of three classical geometric theories: Symplectic, Lie and Foliation theory. The PI's philosophy/inspiration was based on Poisson Geometry's built-in potential to provide a beautiful and powerful interplay between such theories.

Poisson Geometry saw spectacular developments over the last 15-20 years. Most prominently, on the Mathematical Physics side, Kontsevich solved the "quantization problem", one year before he received the Fields Medal. On the geometry side, as people say, the main breakthrough was the solution of the PI with Fernandes on the "integrability" and the "linearization" problems (all these were open for 20-30 years).

The title indicates the two sides of the project and the metamorphosis between them:

Inside: For a long time research in Poisson Geometry concentrated mainly on local or formal aspects-themselves difficult or subtle (e.g. the quantization and the integrability problem). Global aspects (Poisson Topology) were unravelled in the last years (e.g. the work of Fernandes and the PI on linearization and stability). The time has arrived to address the most fundamental but difficult questions regarding existence, classification and constructions of invariants. Which spaces admit interesting Poisson structures? How many? What obstructs their existence? Such central questions give perfect directions for interesting research; they form the roots/motivating aims of this project.

Outside: While our starting questions are not new, our approach/philosophy certainly is. Actually, this projects aims also at showing that such basic Poisson Geometry questions give rise to completely new and unexpected (no longer built-in!) interplays with areas like Partial Differential Equations, Integral Geometry, hyperkähler and contact geometry. In particular, using this interplaying bridge, we aim at important applications outside Poisson Geometry- e.g. Markus' conjecture from affine geometry or even existence results for PDEs.

1.d Keywords: *Poisson Geometry, Lie groupoids, Lie pseudogroups, Exterior Differential Systems, Contact Geometry.*

1e. Host institution: Utrecht University

1f. NWO domain: Bèta

1g. NWO Division: EW

1h. Main field of research: 11.50.00, Wiskunde/Meetkunde, topologie.

Research proposal

2a. Description of the proposed research

I. General introduction

The general aim/direction: This is a pure mathematics proposal, rooted in the field of PG (Poisson Geometry), built on the PI's philosophy that the power and the beauty of PG comes from its potential to provide interplays between other fields.

Here we address some of the most fundamental but difficult questions: which spaces admit interesting Poisson structures? how many? what obstructs their existence? Our approach to such questions makes the project grow rapidly over other parts of Geometry, proposing completely new interplays. In fact, revealing such unexpected connections and applications of ideas *inside* PG to fundamental or long-standing open problems *outside* PG is part of our overall aim.

Poisson Geometry (PG): PG emerged from the mathematical formulation of Hamiltonian mechanics and had an exciting historical development. Very briefly: it all started with Poisson's work on classical mechanics and his hundreds of pages of computations which lead him to the discovery of the so-called Poisson brackets (1809). Their main properties- the Leibniz and Jacobi equations (which reduced Poisson's computations to … a few pages)- were fully understood after the work of Jacobi (1884). Poisson Geometry is about the geometric study of general Poisson spaces, i.e. spaces M endowed with an operation (bracket) which associates to any two observables f, g on M a new observable $\{f, g\}$, satisfying the Leibinz and Jacobi equations. Note that these equations dominated mathematics ever since. They played a central role already in Lie's fundamental work (end of the 19-th century) on symmetries of PDEs (partial differential equations) which lead him to the discovery of Lie groups as mathematical formulation of symmetry. Then came the relationship with quantum mechanics, the formulation of the "deformation-quantization problem" by Bayen- Flato-Lichnerowicz-Sternheimer and Lichnerowicz's first geometric studies. But probably the "birth-certificate" of Poisson *Geometry* was Weinstein's work [We1] which went deeper by unraveling the geometry behind the classical works of the 19-th century and formulating some clear exciting problems for the decades to come. In particular, it addressed the so called "linearization problem" which, next to the "deformation-quantization problem", became a driving force.

As mentioned in the summary, PG knew a huge development especially over the last 15-20 years, with Kontsevich's solution to the quantization problem [Kon] and with various contributions of the PI and collaborators, which finally led us to a geometric understanding of the linearization problem (bringing to an end the several efforts made since Weinstein's paper). It is impossible to describe here even briefly the modern development of PG. However, it remains a very lively field which, because of its many interactions with other fields, continues to offer basic/attractive/exciting open problems and to attract a large mathematical community (e.g. the biennial Poisson conferences attract more than 200 participants).

But what is Poisson Geometry? The answer, as the PI likes to formulate it (and which serves as his motivation for working in this field) is: PG is a beautifully subtle amalgam of three classical fields, interacting inside PG, with a huge potential for providing new interactions between various fields of geometry. The three fields are:

- *Lie theory (LT)* mentioned above, which deals with mathematical structures that encode symmetries, such as

    - Lie groups = the abstract formulation of symmetry, which nowadays pervade mathematics and mathematical physics.

    - Lie-pseudogroups = *local* symmetries of geometric objects- whose study by Cartan went beyond the original scope, shaping modern geometry (with concepts like differential forms, EDS or tools like the Cartan-Kähler for analytic PDEs).

    - Lie groupoids = point symmetries, which originated in Ehresmann's global formulation of PDEs and came with the discovery of jets (used ever since).

- *Foliation Theory (FT)* which originates in the global study of differential equations and studies the intricate geometry of partitions of spaces into subspaces of a given dimension (e.g. the Moebius band by the circles).

- *Symplectic Geometry (SG)* which studies the non-degenerate Poisson structures, started earlier than PG and, through its huge developments, conquered an independent and richer territory.

Again, for all these three fields, the historical development is very exciting, not independent of each other, with motivations coming from Differential Equations and Mathematical Physics and with deep connections with Topology.

Returning to the original question, the brief answer is:

PG=LT+FT+SG.

More precisely, a Poisson structure on M induces (and can be seen as):

- FT: a partition of M into leaves,

- SG: on each leaf a symplectic structure

- LT: transversal to each leaf, a Lie algebra

all interacting through the object encoding the global symmetry- itself a symplectic (!), Lie (!) groupoid. And this is our (complete) global understanding of PG, after various works over the last 15-20 years [27,23,24,CaFe,MkXu,Wein,Xu1,etc,etc].

II. The project

Each subproject starts from a fundamental aspect/open problem *inside* Poisson Geometry and then, based (I believe) on several new and original ideas, it reveals some completely new (and unexpected, I believe) interplays and directions *outside* Poisson geometry, with applications on both inside/outside component. In particular, along the way we will reveal several new research directions and their inside/outside importance.

I try to "narrate" the project, to make it as self-contained (and non-technical) as possible, linearly describing the subprojects, main directions and problems, the "warm-up problems", some of the ideas/methods to be used, as well as the needed man-power.

II.1. Subproject 1: Linearization (Poisson Geometry)/smooth Cartan-Kähler (PDEs)

Linearization phenomena inside PG are quite well understood by now. However, this sub-project proposes a new and more fundamental viewpoint/interplay:

> ☞ *General direction: Study the link between Poisson Geometry and the Geometry of Partial Differential Equations. More precisely: between the linearization theorems (PG) and the Cartan-Kähler theorem (PDEs).*

This would bring new insight and powerful analytic to attack some open problems inside PG (one is mentioned below). However, the main gain of this subproject would clearly be on the outside component (the geometry of PDEs): a smooth version of the Cartan-Kähler theorem, which would be of independent interest, completely new, with a huge potential for applications. To appreciate this, and to explain the insight of the PI that makes the proposed link plausible, recall:

1. *On the PDE side*: the Cartan-Kähler theorem is one of the main tools used for proving existence of solutions of analytic PDEs. The main condition in the theorem is an "involutivity condition", which can be phrased in terms of vanishing of certain Spencer cohomology groups. It is notorious however that analyticity is absolutely necessary (as shown by a famous example of Levy), and this seriously limits the range of applications.

2. *On the PG side*: the linearization around a fixed point x holds in the following cases:
   a. the analytic case provided the isotropy Lie algebra $\mathfrak{g} = \mathfrak{g}_x$ at x is semisimple. The proof [Wein,Conn2] reveals that the result is controlled by the vanishing of:
   $$H^2(\mathfrak{g}, S^k(\mathfrak{g})) = H^2\left(\mathfrak{g}, C^{\infty}_{k-polyn}(\mathfrak{g}^*)\right),$$
   (Lie algebra cohomology) where, to indicate the relationship with the next case, we interpret the elements of $S^k(\mathfrak{g})$ as degree k polynomial functions on $\mathfrak{g}^*$.
   b. also (!) in the smooth case provided $\mathfrak{g}$ is also of compact type. Again, we know (from the proof) that this phenomenon is controlled by the "tame vanishing" of:
   $$H^2(\mathfrak{g}, C^{\infty}(\mathfrak{g}^*))$$
   where "tame" refers to the existence of homotopies satisfying certain "tame estimates" in the sense of Hamilton [Ham].

With these in mind, the main steps to take are the following:

> ☝ *Problem 1: Prove that the analytic linearization of PG follows from (a version of) Cartan-Kähler.*

More details are given below. The study of this problem should clarify the setting necessary to take the next step- moving to the smooth setting.

Problem 2: Investigate why smooth linearization works while the (naïve) "smooth Cartan-Kähler" fails. Extrapolate the findings to more general PDEs.

And then the final step (before applications):

Problem 2- the ambitious version: Find/prove a smooth version of Cartan-Kähler for PDEs.

A particular case which is of particular interest is that of Lie pseudogroups. Indeed, this would suffice for many applications to Geometry (think also of the historical development!). Moreover, the statement and the proof are expected to become simpler and could be treated independently (see below).

Problem 3: Find/prove a smooth version of Cartan-Kähler that applies to Lie pseudogroups.

Here are more details on the statements, as well as an indication of our methods/plan of attack. The insight into the correct conditions (tame-vanishing of cohomology) and methods (Nash-Moser techniques) comes from our work on linearization. To test the link between PG and Cartan-Kähler we propose to first look at another basic result in PG: existence of symplectic realization [Wein,7]. Indeed, due to the analogy with similar results for Lie algebras [BCG], the PI strongly expects that the following is a good "warm-up problem" for a starting PhD (where the main work is to get acquainted with Cartan-Kähler):

Warm-up problem: Prove existence of analytic symplectic realizations of Poisson manifolds using Cartan-Kähler.

Problem 1: The first step is to reinterpret the linearization problem as a PDE system. This should be done carefully (the "differential consequences" should be included in the equations). Then, as hinted by the general discussion above, one should identify the relevant Spencer cohomology with the groups $H^2(\mathfrak{g}, S^k(\mathfrak{g}))$. This may require some homological algebra machinery. In principle, this will allow us to apply Cartan-Kähler right away; however, one may need an improvement of the standard Cartan- Kähler because, most probably, the resulting PDEs will not satisfy the standard regularity conditions. Again, a PhD student may start with a particular case (with smaller such technical issues, if at all):

Warm-up problem: Solve problem 1 when $\mathfrak{g}_x = so(3)$.

If the regularity issues turn out to be too difficult to solve directly, then one proceeds by checking that the method of proof (rather than the statement) of Cartan- Kähler applies. Then one looks back and identifies the improvement that emerges from the proof.

Note also that the insight that we gain here, combined with the framework from our previous work (the PI and Marcut) on normal forms, should allow us to solve the following:

Problem 4 (application to PG): Extend the analytic Conn-linearization to arbitrary leaves.

Problem 2: Here we will aim at the ambitious version (assuming an excellent PhD student). The approach will consist of two parts. One consists of setting up the formalism- but this will probably be achieved while solving (at least partially) problem 1. The main novel issue here is to formulate the correct cohomological conditions. With the mind at the hidden conditions that appear in the linearization theorems from PG, and the difference there between the analytic and smooth cases (2a and 2b above), one expects a smooth version of the Spencer cohomology for PDEs. With this, the expected condition for a general smooth Cartan-Kähler is: tame vanishing of the smooth Spencer cohomology groups. A nice/instructive exercise at this point is to check that the well-known "Levy counterexample" fails this condition.

With the formalism in place, the second part of our approach is the analytic one, in which the actual existence result is proven. Here the main analytic tool will be the Nash-Moser technique, i.e. a Newton-type iteration process, in which the loss of derivatives is corrected by smoothing operators. This is where the tameness condition will come into play. Our inspiration will be Hamilton's framework [Ham] as well as the one from the PhD thesis of Marcut (former student of the PI).

Problem 3: In principle, this is a particular case of problem 2, but one for which the conditions of the theorem simplify (like in [Ham], the existing group-like structure and the induced left-translations should allow us to reduce the number of points at which the cohomology vanishing is satisfied). However, even if problem 2 (the ambitious version) fails, problem 3 would still have a good chance of success and can be treated separately. The reason is very simple: since now we deal with diffeomorphisms instead of sections of arbitrary bundles, we can use flows of vector fields (and their compositions) to produce the required diffeomeorphisms (similar to Marcut's thesis and [Conn1]).

I would also like to further indicate the importance of problem 3. Briefly: it comes from the relevance of Lie pseudogroups to geometric structures (think also of the historical development!) and the fact that most of the existing works make, right from the start, an analyticity assumption. However, this assumption:

- is made precisely because the standard Cartan-Kähler requires analyticity
- is very unnatural and unfortunate, since geometry abounds of examples of pseudogroups which are only smooth.

Therefore, problem 3 would allow us to take another interesting:

☞ *General direction*: Develop the (global) theory of Lie pseudogroups in the smooth context.

This last direction is also related to the project of giving a global (modern) geometric formulation of Cartan's work on (analytic) Lie pseudogroups- a PhD project (for Yudilevich) that is very nicely developing under the supervision of the PI. Here we expect some very nice interactions inside the team.

The entire subproject is work for the PI plus at least one excellent PhD-student and/or a PostDoc with expertise in the geometry of PDEs.

II.2. Subproject 2: Classification of PMCT /integral affine, hyperkahler geometry

Next, I pass to a type of fundamental questions inside PG that are wide open: classification results. The most natural class of Poisson structures that are susceptible of classification are, like in Lie Theory, the ones that have a "compact behaviour". They are the most interesting ones also from a topological viewpoint, hence their study may be viewed as an essential step in the "Poisson Topology" program. Emphasize however that these are just the first hints of the beautiful geometry hidden behind the "compact behaviour" in PG; as we shall explain, the richness of the outcome clearly exceeds our original expectations and cross the boundaries of PG to some unexpected territories. These indications were recently discovered by the PI together with Fernandes and Torres (the level of these findings is that of "blackboard discussions"). Here we describe some of the resulting research directions. Note that, although all these directions are motivated by our Poisson geometric problem, some belong to different fields, are of independent interest and of a rather fundamental nature. Alltogether, they give rise to a quite complex sub-project, with very interesting questions for at least 2 PhD-students + one PostDoc (next to the PI and his collaborators).

The setting: The precise meaning of "compact behaviour" is that the associated symplectic groupoid is compact; Poisson manifolds with this property will be called

PMCT = Poisson manifolds of compact type.

Given a PMCT M, its compact behaviour reflects into a similar behaviour of the various geometries that underlies it: the foliation, the symplectic leaves and the isotropy Lie algebras. In particular, the symplectic foliation $\mathfrak{F}$ has compact leaves with finite fundamental groups and the leaf space

$$B := M/\mathfrak{F}$$

is an orbifold. We will first concentrate on the regular case. Furthermore, we make the simplifying assumption that the leaves are 1-connected; this is just for presentation purposes, since it ensures that the leaf space B is smooth; in general, B will still be quite manageable: since it is an orbifold, the entire discussion below extends using some rather standard techniques (e.g. using proper etale groupoids- a technique that the PI is familiar with already from his PhD years).

Examples: There is one aspect that I would like to emphasize from the beginning: the role of the search for interesting examples (even when not successful, or especially then!) for the development of the theory. The first attempts to build such examples (besides the obvious ones) may seem disappointing: they fail. However, they all fail in a very interesting way- often because of various topological conflicts. In other words, there is something to learn from each such failure, paving the way to beautiful examples and an exciting theory. It is clear now that all these are due to the fact that a PMCT requires the presence of several rigid geometries, all interacting in a non-trivial way. Here is a "baby illustration" of such attempts- as we have found it through "blackboard discussions". Looking for examples with $B = S^2$ fails mainly because the (flat!) bundle consisting of the second cohomology groups of the symplectic leaves would have to be trivial, and then the variation of the symplectic areas would give rise to a smooth function with no critical points; we learn that, in general, there are no 1-connected examples. However, the argument that excludes $B = S^2$ can be refined to exclude also the non 1-connected case $B = S^1 \times S^2$; looking carefully at the argument, we find the conceptual explanation: $S^1 \times S^2$ does not admit an integral affine structure. Hence, already the (unsuccessful) search for such simple examples reveals one of the hidden, but central, geometric structures (integral affine structures). On the other hand, the case $B = S^1$ produces no such contradictions- and, indeed, one can find such examples; however, even this (simple and positive) case brings us to the world of symplectic topology and the geometry of K3 surfaces (see below), giving rise to several interesting problems/directions of this project.

Back to the general theory: The PMCT M comes with a locally trivial fibration

$$p: M \to B$$

with symplectic fibers. Hence one can try to understand/study M as made out of the base B, the generic fiber S and a family $\{\omega_b: b \in B\}$ of symplectic forms on S. The study of these pieces and their interaction gives rise to several distinct phenomena.

1. Integral affine geometry

From PMCT to IA-geometry: Although our fibration p has symplectic fibers, its behaviour is very different than the one of the standard symplectic fibrations: the variations of the symplectic forms is not zero, but of "compact type". More precisely, this variation gives rise to a maximal rank lattice

$$\Lambda \subset T^*B$$

which is locally spanned by closed 1-forms; giving such a lattice Λ is equivalent to giving an IA= integral affine structure on B, i.e. a smooth atlas with coordinate changes coming from the IA-group:

$$Aff_{\mathbb{Z}}(n) = \mathbb{R}^n \rtimes GL_n(\mathbb{Z}).$$

As part of the general theory of IA-structures, TB inherits a flat torsion-free connection which we interpret as a representation of the algebroid TB on TB; it integrates to an action (parallel transport) of the homotopy groupoid $\Pi(B)$ of B on TB (preserving the dual lattice $\Lambda^{\vee}$). The identity map of TB may be interpreted as an algebroid cocycle hence it integrates to a groupoid cocycle

$$Dev: \Pi(B) \to TB, \ \ \gamma \mapsto Dev(\gamma) \in T_{\gamma(0)}B.$$

Together with the linear action, this gives an IA-action of $\Pi(B)$ on $(TB, \Lambda^{\vee})$. This is the PI's global interpretation of the standard developing map from affine geometry. The more familiar description is obtained by fixing a point $b \in B$ and an (integral) basis of T_bB: the action groupoid $\Pi(B)$ on TB induces at b the linear holonomy

$$\rho_l: \pi_1(B, b) \to GL_n(\mathbb{Z});$$

combined with the restriction of Dev to $\pi_1(B, b)$, it gives the affine holonomy

$$\rho_a: \pi_1(B, b) \to Aff_{\mathbb{Z}}(n);$$

the restriction of Dev to the universal cover of B with base-point b gives the standard developing map

$$dev: \tilde{B} \to \mathbb{R}^n.$$

This is $\pi_1(B, b)$-equivariant (w.r.t. the affine action ρ_a) and a local isomorphism of IA-structures. The IA-manifold B is called complete if dev is an isomorphism (see also below).

We return now to our PMCT, the induced IA-structure on B and point out the symplectic nature of Dev in this case. The family $\omega = \{\omega_b: b \in B\}$ of symplectic forms on the fibers S_b can be interpreted as a section of the bundle $\mathcal{H}$ over B with fibers:

$$\mathcal{H}_b := H^2(S_b).$$

This is a flat bundle, hence one can consider the differential of ω, denoted

$$I: TB \to \mathcal{H}.$$

One can show that I is an embedding of IA-representations of $\Pi(B)$ and, modulo I, the developing map is simply the groupoid boundary of ω:

$$I\big(Dev(\gamma)\big) = \gamma^*\big(\omega_{\gamma(1)}\big) - \omega_{\gamma(0)}.$$

This indicates that the IA-manifold B should enjoy rather special properties.

Integral affine geometry- the general theory: Let us move now to the general discussion of IA-geometry. IA-manifolds appear as some of "the simplest" manifolds one may expect; looking at the definition, one may be even tempted to discard them right away as being trivial. Now, a search through the existing Differential Geometry literature comes with quite a surprise: while one finds various examples here and there (especially related to integrable systems), while there is a considerable amount of work on affine manifolds ($\mathbb{Z}$ replaced by $\mathbb{R}$) or on IA-structures *with singularities*, one does not find any systematic study of IA-structures; in particular, even the most fundamental problems are open. Therefore:

> ☞ *General Direction: Study/develop the field of IA-geometry (integral affine geometry). Of course, this direction comes with its own problems heading towards classification results*.

IA-Geometry sits inside the larger world of Affine Geometry- which, in turn, is much better understood and was studied much more systematically (with a considerable amount of related literature!). Therefore it is natural (and wise) to first go through the existing study of Affine Geometry (*reading seminar*!). Even there, some of the most basic questions are long-standing open problems. The first one is the Markus conjecture- for affine structures which admit invariant volume forms. Such forms come for free in the IA-case, hence one faces:

> *Problem (the Markus conjecture for IA-structures*): *Any compact IA-manifold B is complete (see above) i.e. of type $\mathbb{R}^n/\Gamma$ for some discrete subgroup*
> $$\Gamma \subset Aff_{\mathbb{Z}}(n)$$
> *which acts freely, properly and co-compactly on $\mathbb{R}^n$.*

There are various cases in which the conjecture is known to be true- e.g. when the fundamental group is nilpotent [FGH]. I would like to make two comments:

- this problem may very well turn out to be equivalent to the full Markus conjecture (we are not aware of any affine structure that admits an invariant volume form but does not admit a compatible IA-structure).
- this IA-geometric version (or just reformulation) provides us new structure/tools for attacking the problem. Indeed, our own point of view (originating from PMCTs!) is to encode the IA-structure into a bundle of tori
  $$\mathcal{S} = T^*B/\Lambda$$
  over B, which comes with a multiplicative *symplectic* structure (these are also the same thing as compact integrations of B, viewed as a Poisson manifold with the zero Poisson tensor!). Dually, one has a *complex* torus bundle
  $$\mathcal{S}^\vee = TB/\Lambda^\vee ,$$
  (a natural "compact complexification" which contains B as a CR submanifold). The developing map becomes holomorphic. These bundles fit in the general framework of T-duality. It is tempting to make several further comments here, but they are probably too speculative at this point; let me only point out that this allows us to bring in some powerful tools of complex and symplectic geometry.

Note that the Markus conjecture shifts the attention to the possible groups Γ. Adapting the existing terminology, by IA-crystallographic groups we mean discrete subgroups Γ of $Aff_{\mathbb{Z}}(n)$ s.t. the induced action on $\mathbb{R}^n$ is proper and co-compact. Such groups are automatically finitely-generated. The extra-condition that the action is free (giving rise to the smooth $\mathbb{R}^n/\Gamma$) is easily seen to be equivalent to the intrinsic condition that Γ is torsion-free; this can always be achieved by passing to a subgroup of finite index (Corollary 6.13 of [Ra]).

> ☞ *Problem/Direction: Which groups arise as fundamental groups of (complete) IA-manifolds? Which manifolds admit IA-structures? How many?*

Although a complete answer even to the first question is probably extremely difficult, it is still a very good motivating problem and research direction. A milder version of this problem is that of finding interesting classes of examples (besides the obvious, abelian, ones). And, as for PMCTs, the search for examples reveals interesting general phenomena. For instance, one finds immediately two non-isomorphic IA-structures on the 2-torus; hence classifying the manifolds that admit IA-structures is easier than classifying all IA-structures. A small variation gives an example on the Klein bottle K, corresponding to the IA-transformations on the plane:

$$a(x,y) = (x+1,-y), \qquad b(x,y) = (x, y+1).$$

This simple example can be looked at from three different viewpoints, depending on whether we think of K as being finitely covered by the 2-torus, being an S^1-bundle or a fibration over S^1:

- The first viewpoint reveals the group-theoretic notion of "virtuality": recall that, whenever a certain property "P" of groups is defined (e.g. abelian, solvable, etc), a group is said to be "virtually-P" if it admits a finite index subgroup satisfying P.
- The second viewpoint brings us to the so-called real Bott-manifolds and the problem of studying the existence of IA-structures on more general iterated S^1-bundles [LM]. Working this out would be a very good <u>*warm up problem*</u> for a PhD student.
- The third viewpoint is less restrictive, hence more interesting; applying it inductively, we bump into the famous conjecture of Auslander, open since 1964. More precisely, we come across the notion of (strongly) polycyclic groups= groups Γ with the property that they admit a subnormal series

  $$1 = \Gamma_0 \subset \Gamma_1 \subset \cdots \subset \Gamma_{n-1} \subset \Gamma_n = \Gamma$$

  s.t. each factor Γ_i/Γ_{i-1} is (infinite) cyclic. Regarding the various group-theoretic concepts that enter the discussion, one should keep in mind here:
  - one has a sequence of strict implications that applies to finitely generated groups:

    abelian ⇒ nilpotent ⇒ polycyclic ⇒ solvable
  - for discrete subgroups of connected Lie groups the last "⇒" becomes "⇔".

  With these, the Auslander conjecture says that the fundamental group of any compact, complete affine manifold is virtually-polycyclic. Again, it is tempting to investigate the Auslander conjecture for IA-geometry since, as indicated above, more structure/tools are available.

Another indication of the relationship of polycyclic groups with IA-geometry is Mostow's result [Ra] that any such group Γ admits a faithful representation

$$\rho: \Gamma \to GL_n(\mathbb{Z}).$$

It is interesting to remember here also Milonor's early work who shows that any torsion-free polycyclic group does arise as the fundamental group of a complete affine manifold; his expectations that this can be realized by *compact* manifolds was disproved in 1983 by Margulis' famous free non-abelian rank-2 (hence not virtually-polycyclic) example. Even more dramatically (for our problem), we now know that there exist (torsion-free) polycyclic groups which are actually nilpotent but which cannot be realised as fundamental groups of compact complete affine manifolds [Ben,Bu-Gr].
On the positive side however, note that the world of nilpotent groups and of nilmanifolds is a potential source for general classes of examples of IA-structures. Compact nilmanifolds arise as quotients G/Γ of 1-connected nilpotent Lie groups G modulo lattices Γ (discrete co-compact subgroups). It is well-known [Ra] that any such G is algebraic; moreover, it admits a lattice iff it admits a rational form $G_{\mathbb{Q}}$ or, equivalently, iff the Lie algebra $\mathfrak{g}$ of G admits a basis for which the corresponding structure constants are rational. It is natural to expect that the resulting lattice in $\mathfrak{g}$ (spanned by the rational basis) often gives rise to an IA- structure on the resulting nilmanifold- another <u>*warm-up problem*</u>! Here we also have to bring in the notion of left-invariant affine structures on Lie groups (see [Bur] and the references therein) which is worth investigating from the IA-geometry viewpoint. Note also that this procedure also works to produce some examples of IA-solvmanifolds which are not

nilmanifolds (a phenomenon that is worth further investigation). Relevant to our discussion are also the following results of Auslander and Mostow [Ra]: a discrete group

- can be realised as a lattice in a 1-connected nilpotent Lie group iff it is nilpotent, torsion free and finitely generated.
- is polycyclic iff it can be realised as a lattice in a solvable Lie group with a finite number of components, and iff it admits a finite index normal subgroup that can be realised as a lattice in a 1-connected solvable Lie group.

Our discussion slowly brings us to the next:

> ☞ *Problem: Study integral affine manifolds via their fundamental groups. E.g.: is any integral affine manifold uniquely determined by its fundamental group?*

To indicate the type of phenomena we have in mind, recall here that solvmanifolds are determined by their fundamental groups [Aus,Ra,Bau] (again, the nil-case is more special because, due to the algebraic nature, the lattices determine not only the nilmanifold but also the actual Lie group- called the Malcev completion) . Another result relevant here is that for virtually-solvable affine crystallographic groups $\Gamma_1, \Gamma_2 \subset Aff(n)$, any isomorphism f between Γ_1 and Γ_2 induces a (polynomial) diffeomorphism of $\mathbb{R}^n$ compatible with f. In particular, in the torsion-free case, the resulting affine manifolds $\mathbb{R}^n/\Gamma_1$ and $\mathbb{R}^n/\Gamma_2$ are diffeomorphic (Theorem 1.20 in [FrGo]).

Similarly, we expect that IA-manifolds can be understood/studied via their fundamental groups. Note that the results mentioned above show that a positive answer to the (IA) Markus and Auslander conjectures (e.g in the nilpotent case) would give a positive answer to the last part of the problem; however, even then, one is left with the question of understanding how the actual IA-structures relate to the group. To take the next steps, it is clear that we have to look at/further study the various "algebraic completions (hulls)" of groups and their interaction with IA-geometry- the The theory is by now quite well-developed and ready to be used (see [Bau,etc]): the theory started with the case of (torsion-free) nilpotent groups Γ and their Malcev completions, followed by generalisations to more general groups, such us polycyclic groups (Mostow), or virtually-polycyclic groups (Baues). As before, left-invariant IA-structures on (the resulting) Lie groups should come into play. The symplectic/complex viewpoint, specific to IA-structures (see above), should be relevant as well.

Back to PMCT: Back to the theory of PMCT (Poisson manifolds of compact type), as we pointed out in our discussion on the developing map, the IA-structures that arise may be more manageable. Hence, for PMCTs, the problem to constantly keep in mind is:

> ☞ *Problem*: Study the previous problems for the IA-structures coming from PMCTs. Which IA-structures arise from PMCTs?

2. *Symplectic gerbes; q-Hamiltonian spaces*

From PMCT to symplectic gerbes: We have seen that a PMCT induces an IA-structure on the leaf space B, encoded into the symplectic torus bundle over B:

$$\mathcal{S} = T^*B/\Lambda$$

Here we explain that there is yet another structure induced on B: a gerbe with band $\mathcal{S}$ which is compatible with the symplectic geometry- therefore called a symplectic gerbe.

Gerbes are well-understood from several viewpoints (each one with its own advantages), ranging from more categorical approaches to "down to earth" ones using Cech cocycles.

Roughly speaking, gerbes over a space B are higher order versions of S^1-principal bundles which represent elements in $H^3(B,\mathbb{Z}) \cong H^2(B,S^1)$; the class associated to a gerbe (the Diximier-Douady class), is a higher version of the Chern class.

The viewpoint that is most convenient for us is that of bundle-gerbes [Mur] and groupoid extensions [BeXu1,Moe2,etc]. Indeed, returning to our PMCT, the symplectic groupoid Σ of M induces an extension

$$p^*\mathcal{S} \to \Sigma \to M \times_B M$$

Hence we immediately obtain a bundle gerbe over B and an associated cohomology class

$$d \in H^2(B,\mathcal{S})$$

(cohomology with coefficients in the sheaf of sections of $\mathcal{S}$). Indeed, it is not a surprise that the existing theory extends if one replaces S^1 (or, better, the trivial bundle $B \times S^1$) by a general torus bundle $\mathcal{S}$ over B (however, this may serve as a good warm-up problem for a starting PhD student). The key remark here is that the construction is compatible with the symplectic form of Σ, hence one strongly expects:

Problem: Show that u comes from a similar element ("the symplectic Diximier-Douady class") in cohomology with coefficients in the sheaf $\mathcal{S}_{Lagr}$ of Lagrangian sections of

$$\mathcal{S}: d_{Lagr} \in H^2(B,\mathcal{S}_{Lagr}).$$

It becomes clear that this discussion fits into a general (missing) theory; it gives a doable but laborious problem which could become a solid chapter of a PhD thesis.

Problem: Develop the theory of symplectic gerbes; apply it to PMCTs.

Relations with q-Hamiltonian spaces: The relationship with the theory of q-Hamiltonian spaces arises already when looking for the simplest (nontrivial) types of PMCT: with $B = S^1$- when we come across q-Hamiltonian S^1-spaces. Recall that such a space is an S^1-symplectic manifolds (Q,ω) (with infinitesimal generator V), together with an S^1-valued moment map, i.e. a smooth map $\mu: Q \to S^1$ satisfying

$$\mu^*(d\vartheta) = i_V(\omega),$$

Indeed, it is not difficult to see that if Q is compact, the action is free and the μ-fibers are 1-connected, then the resulting quotient

$$M := Q/S^1$$

is a PMCT (the corresponding symplectic groupoid is simply the fusion product $(Q \times_{S^1} Q)/S^1$). Conversely, any PMCT with $B = S^1$ arises this way [FGM] (this should follow also from the previous problem and the relationship with gerbes described below, since the relevant cohomology vanishes for dimensional reasons). Conclusion: finding such PMCTs= finding spaces (Q,ω) as before (with free S^1-action with contractible orbits). Interesting enough, this reformulation was an open problem in Symplectic Topology- solved in [Kot], using rather deep properties of Kähler metrics.

Back to the general discussion, it is now clear that, as before, for a symplectic torus bundle $\mathcal{S}$ on a manifold B (equivalently: an IA-structure Λ on B), one can extend the previous discussion and talk about q-Hamiltonian $\mathcal{S}$-spaces, with moment map $\mu: Q \to B$ satisfying

$$\mu^*(\alpha) = i_{V_\alpha}(\omega) \text{ for all } \alpha \epsilon \Lambda.$$

We call it free if the action is free. In this case, if the μ-fibers are 1-connected then $M = Q/\mathcal{S}$ is a PMCT with B as the associated leaf space.

Problem: Fill in the details; show that the PMCTs constructed in this way are precisely those whose Diximier-Douady class is trivial. Show that the resulting theory of q-Hamiltonian $\mathcal{S}$-spaces fits in the general theory of Poisson-valued moment maps [MiWe,BuCr,Xu3]; exploit this to construct new examples of PMCT (e.g. using fusion products).

3. Geometry of K3-surfaces; hyperkähler geometry

The problem of constructing PMCT can be approached slightly differently, by first searching for the typical fiber S of p. Our discussion on the developing map associated to a PMCT implies that, at least when p is flat, we are looking for: a compact 1-connected manifold S, together a group action

$$\Gamma \ni \gamma \mapsto \varphi_\gamma \in Diff(S)$$

plus an Γ-invariant IA-subspace

$$W \subset H^2(S)$$

(where IA is w.r.t. the integral cohomology), s.t.:

- The induced action of Γ on W is free, properly discontinuous and co-compact.
- The elements $\xi \in W$ can be represented by symplectic forms ω_ξ on S, smooth w.r.t. ξ.

With these, one obtains the PMCT:

$$M := (W \times S)/\Gamma, \qquad (\text{with } B = W/\Gamma).$$

When such an action and W exist, we say that S is (symplectically) Γ-crystallographic.

> ☞ *General Direction: Study/find the Γ-crystalographic manifolds. For which Γ does there exist at least one? (conjecture: for all torsion-free crystallographic groups).*

It is clear that such manifolds must have a very rich and intricate cohomology. Even for $\Gamma = \mathbb{Z}$, spaces like spheres, projective spaces, etc do not work. The next attempt is flag manifolds; a good *warm-up problem* is to exclude these as well (for all non-trivial Γ). Next, 4-manifolds are especially appealing because of the power of the intersection form Q which is clearly very suited for our problem. For instance, one can show that the vector space corresponding to W is isotropic and is orthogonal to W (w.r.t. Q), hence we immediately exclude the 4-manifolds with definite intersection form. The K3-surfaces, with their rich $3H + 2(-E_8)$ intersection form, are the next natural candidates and, indeed, it seems that they do work for $\Gamma = \mathbb{Z}$ or $\mathbb{Z}^2$. The argument would make full use of the understanding of the period map for marked K3-surfaces, the (strong) Torelli theorem, the understanding of the Kähler classes (cone), of the (effective) Hodge symmetries, etc. The details are rather technical (and still to be written down carefully), not appropriate to describe here- especially because a more geometric/conceptual explanation for the fact that they "miraculously work" is missing at this point. An intriguing/exciting problem is:

> *Problem: Clarify (geometrically) the fact that the K3-surfaces are Γ-crystalographic for $\Gamma \in \{\mathbb{Z}, \mathbb{Z}^2\}$. Generalize to other 4-manifolds.*

It is clear that the main structure of K3 surfaces that is relevant here is the hyperkähler one. Therefore, the more ambitious:

> *Problem: Show that hyperkähler manifolds are Γ-crystalographic (for higher rank Γs).*

A good starting point is provided by the Hilbert schemes associated to K3-surfaces (this is a bit more than a "warm-up", but certainly doable). It is interesting mentioning here that, although for K3-surfaces one can use both the Kähler as well as the holomorphic symplectic viewpoint, when moving to higher dimensions (already for the Hilbert schemes of K3s), it appears that the second viewpoint is technically more suitable. Note also that the direction towards which we are slowly moving (the interaction between lattices in Lie groups and hyperkähler geometry) is clearly outside PG.

4. Resolutions of singularities/Lie theory

Let us very briefly indicate another direction that emerges from the theory of PMCTs. So far we have restricted to *regular* PMCTs. However, it is likely that regularity must always hold- as a consequence of a "Poisson desingularization" operation. Again, this seems to be a more general/fundamental phenomenon, which brings us towards the outside component of the sub-project. The model that one should have in mind comes from Lie theory. There, the study of a compact G is very much related to the standard map

$$G \times_W \mathfrak{t} \to \mathfrak{g}, \quad (g, v) \mapsto Ad_g(v),$$

that we interpret as "desingularization" of the Lie algebra $\mathfrak{g}$ of G. Our main remark here is very simple: this map is, at its heart, Poisson geometric. The remark is (conceptually!) new, not so obvious, but rather easy to check once the answer is given: to any Poisson manifold M, one can associate its "desingularization":

$$\widetilde{M} \coloneqq \{ (x, \mathfrak{t}) : x \in M, \mathfrak{t} - \text{maximal torus in the isotropy Lie algebra } \mathfrak{g}_x \}.$$

When applied to the linear $M = \mathfrak{g}^*$, one recovers $G \times_W \mathfrak{t}$!

When applied to a PMCT, this produces a regular Dirac manifold which keeps the regular symplectic leaves of M but blows up the non-regular ones; the result is regular, but some of the leaves are no longer symplectic. The compact behaviour (and the structures previously discussed) lifts up to $\widetilde{M}$, where an argument based on the volume of the leaves and its IA-nature implies that the volume is constant- therefore excluding the non-regular leaves.

The complete (nontrivial) arguments still have to be written down in detail by the PI and his collaborators. However, they clearly mark the start of some more important/fundamental questions. One comes from the striking similarity with phenomena from group actions [DK,AM], which bags for a general approach via proper groupoids. There is also the close relationship with the Grothendieck simultaneous resolution, therefore the idea of exploiting the PG-viewpoint to "resolutions of singularities" [EvLu,LG]. I state here another problem, more fundamental I believe, closer in spirit to the remark we started with above: the investigation of a deeper, more direct bridge between PG and Lie theory. We use the framework that provides the correct compactness assumption from a Lie-theoretical viewpoint (in particular, it brings PMCTs and duals of compact Lie algebras together): proper Poisson manifolds, in the sense that their symplectic groupoid is proper. The following is a summary of a very nice possible PhD project:

> *Problem: Investigate the "desingularization" for proper Poisson manifolds. Show that standard theory such as Weyl's integration formulas, Duistermaat-Heckmann polynomials, Delzant polytopes (clearly related to IA-structures!) are Poisson geometric (i.e. extend to general proper manifolds). Exploit this! (e.g. give a new, completely Poisson geometric, proof to Delzant's theorem).*

II.3. Subproject 3: Existence results in PG/Contact Geometry

Here we are inspired by:

- Foliation Theory, where the question of existence of codimension-1 foliations on a given space was a driving force for the field: the question on S^3 marked its birth (and came with the famous Reeb foliation), for S^5 was treated by Lawson and brought singularity theory and open book decompositions into the field, etc etc, up to the fundamental work of Thurston who settled the problem for arbitrary manifolds.

- Symplectic Geometry/Topology, where the similar basic question of existence of symplectic structures on manifolds played an even more important role, bringing into the field fundamentally new, deep ideas such as Gromov's h-principle (with important applications outside Symplectic Geometry, including foliations), the theory of pseudo-holomorphic curves [Grom] and the various related invariants such as Seiberg-Witten.

Passing to PG, the corresponding central problem is:

> *Motivating problem: Study the existence of codimension-1 SFs (symplectic foliations) on compact manifolds. Concentrate on existence results.*
> (recall: codimension-1 symplectic foliations = corank-1 Poisson structures)

Note that the first interesting case (of S^5) was solved only recently by Mitsumatsu [Mit]. Inside the world of codimension-1 symplectic foliations one finds a very interesting class: for which the leafwise symplectic structures "do not vary"; they are the foliated generalization of the well-known symlpectic fibrations; we call them tame.

> *Motivating problem: Study the existence of codimension-1 tame SFs on compact manifolds. Concentrate on finding obstructions.*

As indicated, the two problems are of different nature: while for the first one we expect many interesting examples, for the second we expect deep topological obstructions.

These problems seem to be extremely difficult. Mitsumatsu's techniques (as he observed himself) brake down completely in higher dimensions. Moreover, even for the case of S^5 (!!!), the answer to the second problem is wide open. In particular, it is clear that one needs some rather new ideas/insight.

I am now slowly moving to the core of this sub-project, heading towards its "outside component". The PI has studied in detail Mitsumatsu's construction, extracting some conceptual methods (which may be useful for proving more general existence results) and, more importantly, tracing some of the core (implicit) ideas/structures back to their origins and to earlier literature. With PG geometry in mind, one notices striking "coincidences" that indicate very exciting and deep connections with Contact Geometry, bringing us to a completely new:

> *General direction: Understand in depth the interaction between the:*
> *- codimension-1 symplectic foliations of Poisson Geometry*
> *- contact structures of Contact Geometry*
> *Is there a duality/mirror phenomenon? Exploit it!*

Here are some details. Contact structures may be seen as "odd analogues" of symplectic structures, which were intensively studied, with deep connections with Topology, with several beautiful results and applications. The literature reveals various other structures that come with a similar label ("odd symplectic"), but none of those gets even close in importance to contact structures. Our claim/expectation is that codimension-1 SFs provide not only another such "odd analogue", but a very important/fundamental one, which seem to mirror contact structure. The PI is surprised that this was not noticed/called for before; the explanation may be the missing Poisson geometric motivation/insight (provided by this project).

Note that a complete understanding of such a relationship would probably solve the previous motivating problems, as well as other hard problems in Contact Geometry. For instance, for existence results on S^{2n+1}: while the contact question is clear, the SF one is hard. For a product $S \times S^1$ with S symplectic, the situation is reversed.

✍ To indicate the duality/analogy that we have in mind, we mention here that the contact forms of *Boothby-Wang type* (arising from S^1-principal bundles $P \to S$ over symplectic manifolds) mirror the *symplectic fibrations* $P \to S^1$ over the circles. The *fillability* of the first ones is probably related to the *tameness* of the second ones. The *regular* contact forms (which, after a small change, can always be transformed into ones of Boothby-Wang type) seem to correspond to symplectic structures which are of *cosymplectic* type (and which, similarly, after a small deformation, can be transformed into symplectic fibrations). Mitsumatsu's work gives rise to a general *symplectic turbulization/gluing* procedure: for symplectic W with cosymplectic boundary M, one can find a symplectic foliation on $W \times S^1$ with $M \times S^1$ (with its canonical symplectic structure) as symplectic leaf; quotients of such can then be glued to obtain the desired SF. This is strikingly similar to the (simpler) contact construction of [GS] (Theorem 1); etc. One should also have in mind here the *confoliations* of [ET], the process of *deforming contact structures* into foliations or, more recently (cf. [Mor] for Mitsumatsu's construction), into SFs. Contact structures relate to (some) SFs also through *symplectic cobordisms* similar to [EGH]; actually, a step in Mitsumats's construction consists of building such a cobordism to replace a simple contact boundary by one which is cosymplectic (needed to perform symplectic turbulization). Such operations are clearly important for this sub-project. Other ideas for relating the two worlds may come from Jacobi geometry or the philosophy of generalized complex geometry. Regarding the formal framework, the most natural one is that of pairs (ω, α) (two-form, one-form) on M^{2n+1}, satisfying $\omega^n \wedge \alpha \neq 0$; for some purposes (e.g. cobordisms), one may forget about α and just keep in mind the maximally non-degenerate ω and the induced orientation on its kernel line-bundle. There are various types of "integrability conditions" that are natural here, giving rise to a plenitude of structures and names. The most appealing ones- flexible enough and still sufficiently rigid- are the ones for which ω is closed; indeed, as an application of Gromov's principle, this can always be arranged; on top, important tools such as adapted *open book decompositions* and *approximately holomorphic geometry* are still available [MMP, MT2]. One has to check how these adapt to the weaker (but relevant) case $d\omega \wedge \alpha = 0$. In the search for the best framework there are indications that the dual, *more Poisson geometric*, picture of bivectors and vector-fields may be more appropriate. For instance, for symplectic turbulization, one can reinterpret the cosymplectic structure on M as a corank-1 Poisson structure π together with a transverse Poisson vector field V which extends to a b-Poisson structure $\tilde{\pi}$ on W; the symplectic turbulization is simply $W \times S^1$ with

$$\Pi := \tilde{\pi} + \tilde{V} \wedge \frac{d}{d\vartheta} .$$

I have indicated above some of the interesting questions, key-words and techniques for this subproject. However, formulating clear (non-trivial but still workable) problems would be too risky, since each step here would depend dramatically on the previous ones. Hence this project may not be so appropriate for a PhD student, but for the PI together with a PostDoc with solid background in Contact Geometry.

III. Planning

I tried to "narrate" the project, linearly describing the

- subprojects
- main directions
- central problems
- "warm-up problems" (for the junior researcher)+ semester-long reading seminars.

Along the way I also mentioned some of the ideas and techniques that we plan to use, as well as the needed man-power (for timing, see also 3a). I hope that the linear presentation of the project makes the resulting planning as clear/realistic as long-time research planning can be. As further indication, here is the rough plan for handling PhD students:

- The PI introduces them into the main directions of the project, concentrating on the appropriate subproject/main problem (statements, motivation, main ideas).
- They have about one year to get acquainted with the problem: take active part in the reading seminars, look at the relevant literature, fill in missing background, then start working out some of the "warm-up problems".
- Then they go deeper into their problem, discuss more intensively with the PI, encounter the first real difficulties … doing "real research".
- By the start of the third year one expects first serious results- to be explained in our seminars (a very useful – clarifying and inspiring- process for the PhD student!). At this point, longer (1-3 months) visits abroad are often helpful.
- In the fourth year one takes the last serious steps, then puts everything together in research papers (main chapters of the thesis).

Moreover, based on previous experience, I will aim at forming a team of researchers closely interacting with each other through

- the "Friday Fish" (weekly, full-day) seminars with talks ranging from the reading seminar to shorter (2-4 hours) expositions of research papers related to the project or talks by the visitors (please see: http://www.projects.science.uu.nl/poisson/ for an indication).
- Weekly meetings in which the more advanced PhD students or postdocs lecture about their research to the younger PhD students.
- Weekly or biweekly (1-2 hours) meetings of each member of the team with the PI for more in-depth discussions.
- More ad-hoc meetings (e.g. during the coffee breaks of the team- in front of the blackboard).

Regarding collaborations, I plan to continue the ongoing ones- most notably with Fernandes and Martinez-Torres (related to subproject 2). I regularly meet researchers with expertise related to various parts of the project; in particular, I expect to consult with:

- E. Meinrenken and A. Alekseev (q-Hamiltonian spaces, localization)
- P. Olver (Lie-pseudogroups and PDEs)
- C. Laurant-Gengoux, M. Stienon (gerbes, resolutions of singularities)
- A. Haefliger and Y. Mitsumatsu (symplectic foliations)
- N. Zung (Nash-Moser techniques)

I also plan to start new collaborations related to EDSs (hopefully with the school of R. Bryant) and Contact Geometry. At the national level, I expect discussions with:

- Cavalcanti and Ziltner (UU, symplectic/hyperkähler geometry).
- Moerdijk (RUN, gerbes).
- Pasquoto (VU, Contact Geometry).
- Posthuma (UvA, proper groupoids).

2b. Knowledge utilisation

This project concerns fundamental, curiosity driven research. For such research, the road to concrete "real life applications " is rather long, unpredictable, through a chain of interactions of type

pure mathematician → applied mathematician → engineer

or

pure mathematician → theoretical physicist → experimental physicist

Also, it often takes decades rather than years to reach the end of the road. One should also be aware of the psychological boundary that the "abstract mathematical concepts" have to cross; often however, after the appropriate period of time, they become jargon in other parts of Science. With these facts in mind, it is clear that the comments that the PI can make here are highly speculative.

Poisson Geometry (and the related geometries appearing in the project such as symplectic and contact geometry, the integral affine geometry and its relationship to integrable systems) arise from Classical Mechanics and have found various applications back in Physics (geometric mechanics and fluids, control, imaging, geometric optics thermodinamics, hydrodynamics, etc). Actually, there are quite a few applications-oriented people that are close to the Poisson Geometry community (e.g. the school of Ratiu/Marsden, of Cushman/Bates/ Sniatycki, of Manuel de Leon, etc) and the PI occasionally interacts with some of them. These may be the next link in a chain of interactions which originates in this project, and we will actively search contact with some of these groups to explore such applications. Also the Poisson Geometry conferences which, as I mentioned before, attracts a large and varied mass of participants (including Mathematical Physicists), may provide new opportunities for establishing such new contact.

In the "outside component" of our project one also finds several theoretical aspects related to the theory of PDEs (Partial Differential Equations)- e.g. the general smooth Cartan-Kahler we are aiming at, or the planned revisit of Cartan's theory on symmetries of PDEs. It is well-known that many "real-life" phenomena are modelled by PDEs (actually, many PDEs carry very suggestive names such as "the heat equation", "the wave equation", etc); hence the results we are aiming at would be natural starting points for chains of interactions towards such applications. As a first step in that direction, the PI is planning to establish new contacts with more applications-oriented people (e.g. Peter Olver, whose PhD students are natural candidates for one of the PostDoc positions on the project), creating the premises for such interactions. Note for instance the efforts to bring together various specialists on the geometry of PDEs, the theory of EDS and Lie groupoids- such as the next meeting at the Fields Institute (December 2014), which will have the PI as well as P. Olver and R. Bryant as main lecturers.

In summary, it is unrealistic to assume that such applications could occur during the life-time of the project but we do intend to actively place some pointers from our work to more applied fields.

2c. Number of words used

section 2a1 and 2a2 _____1800_____ (max. 1800 words)
section 2b ______475________ (max. 1000 words)

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<u>Lie groupoids</u> (relevant to subprojects 1 and 2)

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Foliation theory (relevant to subprojects 2 and 3)

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<u>Related to Contact Geometry</u> (relevant to subproject 3)

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Related to Moment maps/localization formulas (relevant to subprojects 2 and 3)

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Cost estimates

3a. Budget

Staff costs per calendar year in k€ incl. surcharge								
Scientific Staff	Intensity (fte)	Duration (months)	Year 1 k€	Year 2 k€	Year 3 k€	Year 4 k€	Year 5 k€	TOTAL k€
Applicant	0.5	60	65	65	65	65	65	325
Postdoc	1.0	36	65	65	66			196
	1.0	24	65	65				130
Postdoc	1.0	24			65	65		130
PhD student	1.0	48	44	48	52	56		200
PhD student	1.0	48		44	48	52	56	200
PhD student	1.0	48		44	48	52	56	200
Non-scientific staff (NWP)	Intensity (fte)	Duration (months)	Year 1 k€	Year 2 k€	Year 3 k€	Year 4 k€	Year 5 k€	TOTAL k€
Academic level								
HBO/Bachelor-level								
MBO/Foundation Degree-level								
Non-staff costs in k€			Year 1 k€	Year 2 k€	Year 3 k€	Year 4 k€	Year 5 k€	TOTAL k€
Give a description of the non staff cost, as detailed as possible								
Travel (3 k€ per year per person)			9	21	18	15	9	72
Foreign guests					10		10	20
Consumables			5	6	5	6	5	27
TOTAL			188	355	445	311	201	1500

Our intention is to hire:

- 1x(PhD student) for subproject 1, in the Fall of 2014
  (when two of my present PhD students will be done)

- 1x(3 years PostDoc) for subproject 2+ 1x(2 years PostDoc) for subproject 3, in the Fall of 2014 (when 4 of my present PostDocs will be gone)

- 2x(PhD student) for subproject 2, in the Fall of 2015
  (when the last two of my present PhD students will be done)

- One 2 years PostDoc (or, if possible, a PhD student) in the Fall of 2016
  (when all my present PostDocs and the 2 years Postdoc on the project will be gone)

3b. Indicate the time (in fte) you will spend on the research: 0.5.

3c. Intended starting date: September 2014.

3d. Have you requested any additional grants for this project either from NWO or from any other institution? no.

3e. Has the same idea been submitted elsewhere? no.

Curriculum vitae

4a. Personal details

Title(s), initial(s), first name, surname: Marius Crainic

Male/female: male

Date and place of birth: February 3, 1973/Aiud (Romania)

Nationality: Romanian

Birth country of parents: Romania

4b. Master's ('doctoraal')

University/College of Higher Education: Mathematical Research Institute,The Netherlands

Date (dd/mm/yy): 30/06/1996

Studies: Master Class

Main subject: Algebraic Topology, K-theory, Number Theory

4c. Doctorate

University/College of Higher Education: Utrecht University

Date (dd/mm/yy): 03/04/2000

Supervisor ('promotor'): Ieke Moerdijk

Title of thesis: Cyclic cohomology and characteristic classes for foliations

4d. Work experience since completing your PhD

Employement: in the following, all are 1fte and, since 2002, permanent at UU:

2002- present: Utrecht University (UD in 2002, UHD in 2008, Professor in 2012).

2002- 2007: Research Fellow of the Dutch Royal Academy (KNAW).

2007, February: Invited Professor, Institute Henri Poincare, Paris, France.

2003, May: Invited Professor, Université Blaise Pascal (Clermont-Ferrand), France

2001- 2002: Miller Research Fellow, University of California at Berkeley.

2000- 2001: Post Doc at the Mathematical Institute, Utrecht University.

Months spent since completing the PhD

Experience	Number of months
Research activities	129.72
Education	19.2
Care or sick leave	-
Management tasks	7.8
Other, please specify:	-

(Calculation:

1/7/2000- 1/4/2001: PostDoc at UU (1fte, 75% research+ 25% education):
0.56 years research+ 0.18 years education.
1/4/2001- 1/7/2002: Miller Research Fellow (1fte, 100% research):
1.3 years research
1/7/2002 -1/7/2007: KNAW-fellow (1fte, 100 % research):
5 years research
1/7/2007 -1/9/2007: 'normal' UD (1fte, 50 % research+ 50% education):
0,1 years research + 0.1 years education.
1/9/2007 -1/9/2012: 0,5 VIDI; 0.65 research, 0.25 education, 0.10 administration, hence:
3.25 years research + 1.25 years education+ 0.5 years administration
1/9/2012 -1/9/2013: 0,5 ERC; 0.60 research, 0.25 education, 0.15 administration, hence:
0.6 years research, 0.25 years education, 0.15 years administration

TOTAL: 0.56 + 1.3+ 5+ 0.1+ 3.25 + 0.6 = 10.81 years of research
0.1+ 1.25+ 0.25= 1.60 years education
0.5+ 0.15= 0.65 years administration)

4e. Academic staff supervised

Below one finds references to various grants of the PI, such as: "the Vidi project" refers to the NWO-Vidi project "Poisson Topology", while "the ERC project" refers to the ERC-Starting Grant project "New-Poetry".

PhDs	Promotor	Co-promotor	
Ongoing:			
1. Roy Wang, Dec. 2011- Dec. 2015 (supported by the ERC project)	M. Crainic	X	
2. Ori Yudilevich, Sept 2011- Sept 2015 (supported by the ERC project)	M. Crainic	X	
3. Joao Nuno, Oct. 2010- Oct. 2014 (supported by Portuguese Sci. Found)	M. Crainic	X	
4. Boris Torres, Sept. 2010-Sept. 2014 (75% supported by the Vidi project)	M. Crainic	X	
Succesfully completed:			
5. Maria Salazar, May 2009- May 2013 (supported by the GQT cluster (¾) and UU (1/4))	M. Crainic	X	
6. Ionut Marcut, 2008- 2012 (supported by the Vidi project)	M. Crainic	X	
7. Camilo Arias Abad, 2004-2008 (supported by my Open Compet. project "Symm. and Deform. in Geometry")	I. Moerdijk	M. Crainic	
8. Niels Kowalzig, 2006-2009 (supported by the GQT cluster)	I. Moerdijk, K.Landsman	M. Crainic H.Posthuma	
			Subtotal: 8

<u>Post Doc's</u> 1. Pedro Frejlich, November 2012- November 2015, supported by the NWO-Open Competitie project "Flexibility and Rigidity of Lie brackets" 2. Daniele Sepe, January 2013- June 2014, supported by the ERC project 3. Matias del Hoyo, January 2013- June 2014, supported by the ERC project 4. David Martinez Torres, March 2013- January 2014, supported by the ERC project 5. Ioan Marcut September 2012- September 2013, supported by the ERC project 6. Florian Schaetz, September 2011- September 2013, supported by the ERC project 7. Ivan Struchiner, June 2009- September 2012 Supported by the Vidi project 8. Sergey Igonin, September 2009- September 2012 supported by the NWO-Open Competitie (NWO) project "Geometry of PDEs and Poisson Structures" 9. David Martinez Torres, September 2005- September 2008, supported by the NWO-Open Competitie project "Symmetries and Deformations in Geometry" 10. Henrique Bursztyn September 2006- September 2007 supported by the NWO-Open Competitie project "Symmetries and Deformations in Geometry"			
			<u>Subtotal: 10</u>

4f. Brief summary of research over the last five years (max 450 words)

In general terms, my research is driven by the main questions "why" and "how" and the search for the simplest answers/phenomena; if asked correctly, they provide the guarantee for interesting fundamental research and for solving hard open problem. Much of my research over the last 5 years was motivated by questions in Poisson Geometry and was stimulated by interactions within my Poisson Geometry team (see 4e) or by continuation of older collaborations. Here is a brief summary:

- *Linearization theorems*: in our Annals paper [4], we found a geometric approach to Conn's linearization theorem (posed by Weinstein about 30 years ago). Furthermore, we gave a geometric proof and clarified the linearization of proper groupoids [1] (conjectured by Weinstein and, in certain cases, proven by Weinstein/Zung).

Our insight will be used in subprojects 1 and 2.

- *Normal forms*: for instance the normal form around symplectic leaves generalizing Conn linearization [3]. Further normal forms are part of the PhD thesis of Marcut- using Nash-Moser techniques (which came after a "Nash-Moser reading seminar" for the team).

Such Nash-Moser techniques will be essential in our subproject 1, while normal forms in subproject 2.

- *Stability/rigidity phenomena*: in our Inventiones paper [10], we discovered a "Reeb-type stability theorem" (new in Poisson Geometry), which provides new approaches and insights even into the classical theorems of this type (foliations, group-actions).

- *Representations theory*: the theory of representations up to homotopy, motivated by stability phenomena and cohomology [2,5,6,9,PhD-Abad]- which initiated further research in the "Higher Structures community".

Similar ideas are expected to be useful in subproject 1 (for Spencer cohomology).

- *Geometry of PDEs*: after 6-7 years of studying the vast literature, first contributions are in preprint form or are part of Salazar's thesis. Recently, the PI went all the way back to Cartan's original writings and noticed the wealth of ideas still unexplored in relation to the results on integrability of the PI.

This research inspired the very last part of subproject 1.

- *PMCT (Poisson Manifolds of compact type)*: a rich and exciting theory of PMCT (start of part 2 of the project) became apparent in discussion with my collaborators Fernandes and Martinz-Torres. In particular, the PI became very much interested in/excited about various other parts of geometry such as Hyperkähler Geometry*,* Affine Geometry, etc.

All these clearly require/deserve more focused investigation and man-power- therefore our subproject 2.

- *Poisson Topology*: A large part of the research of the PI over the last 5 years is part of his long-term "Poisson Topology program" (which started with his Vidi project). More recently, influenced by [Mit], the PI became interested in Contact Topology, noticing the signs of deep interactions between the two- the start of subproject 3.

4g. International activities

The PI was invited to speak in several occasions. Here is a summary:

- *Poisson Conference*: invited to speak in the regular biennial conference in Poisson geometry, “Poisson Geometry in Mathematics and Physics”:

  - 2014 at Urbana-Champaign, USA
  - 2010 at IMPA, Rio de Janeiro
  - 2008 at EPFL Lausanne
  - 2006 at Univ. Tokyo
  - 2004 at Univ. Luxembourg.
  - 2002 at IST Lisbon.

- *Oberwolfach Meetings* (Germany): several participations/talks given at the workshops organized at Mathematisches Forschungsinstitut Oberwolfach where the participation was by invitation only:

  - *Noncommutative Geometry:*
    - September 2011.
    - March 2002
    - October 2001
    - March 2000
    - August 1998

  - *Poisson Geometry and Applications*: April-May 2007

  - *Quantization of Poisson spaces with singularities*: January 2003.

- *Erwin Schrodinger Institute for Mathematical Physics* (Viena): invited to participate to various programs held at ESI and give talks in the related conferences, such as:

  - *Higher structures in Mathematics and Physics*, October 2010
  - *Poisson sigma-models, Lie algebroids, deformations and higher analogues*, 2007
  - *Gerbes, Groupoids and Quantum Field Theory*, May 2006
  - *Moment maps in Poisson Geometry*, August-November 2003
  - *Aspects of Foliations Theory in Geometry, Topology and Physics*, September 2002

- *MSRI*: invited to speak in various conferences organized at the MSRI at Berkeley:
  - *Noncommutative Geometry and quantization,* 2001
  - *Symplectic geometry, noncommutative geometry and physics*, 2010

- *Higher Structures conferences*: invited to speak in this series of conferences:
  - *Higher structures*, Gottingen, November, 2011
  - *Higher structures in Mathematics and Physics*, Viena, October 2010
  - *Higher Structures in Mathematics and Physics*, Zurich, November 2009
  - *Higher structures in Mathematics and Physics*, IHP Paris, 2007

- *Other conferences*: Invited to speak in various other conferences. I mention here:

  - *Geometry and Mechanics*, Moscow, December 2013
  - *Conference in honor of Alan Weinstein, Bernoulli Center, EPFL Lausanne, July 2013*
  - *From Poisson to String Geometry*, September 2012, Erlangen
  - *Poisson Geometry and Applications*, Figueira da Foz, Portugal, June 13-16, 2011.
  - *Quantization of Singular Spaces*, Center for the Topology and Quantization of Moduli Spaces, Aarhus, Dercember 2010
  - *XVIII International Fall Workshop on Geometry and Physics*, CCB Pedro Pascal, 2009
  - *Quantization of Singular Spaces*, Center for the Topology and Quantization of Moduli Spaces, Aarhus, Dercember 2010
  - *XVIII International Fall Workshop on Geometry and Physics*, CCB Pedro Pascal, 2009
  - *FNRS contact group in Differential Geometry*, Han-sur-Lesse, Belgium, 2008.
  - *GESTA 2008: Symplectic geometry with algebraic techniques*, CRM Barcelona, 2008
  - *Séminaire algèbres d'opérateurs,* Paris 6, 2007
  - *Lie Algebroids and Lie Groupoids in Diff. Geometry*, LMS meeting, Sheffield, 2007
  - *Conference on Poisson Geometry*, ICTP, Trieste, July 2005
  - *Poisson Geometry,* Def. Quant. and Group Representations, Brussels, June 2003
  - *Quantization, deformations and new homological and categorical methods In Mathematical Physics*, LMS meeting, Manchester, 2001
  - *Séminaire algèbres d'opérateurs,* Paris 6, 1999.
  - *Symplectic Geometry and Microlocal Analysis*, Penn State University, 1999

- *Minicourses*: invited to give several mini-courses related to my research, such as at:

  - *Workshop on Exterior Differential Systems and Lie Theory*, Fields Institute, 2014, Minicourse on "Lie groupoids and Lie pseudo-groups" (4 lectures).
  - *Winter School on Math. Physics*, Les Diablerets, Switzerland, February 2010 Minicourse on "Stability in Geometry" (4 lectures).
  - *Poisson 2008 summer school,* Bernoulli Center, Lausanne, July 1-7 2008. Minicourse "Introduction to Poisson Geometry" (3 lectures)
  - *Geometric Flows,* Equivariant problems in Symplectic geometry program, CRM Barcelona, 2008. Minicourse "Poisson geometry and Symplectic groupoids"-3 lectures
  - *Groupoids and Stacks,* Physics and Geometry program, IHP Paris, Jan.-April 2007. Minicourse on "Lie pseudogroups and groupoids" (4 lectures).
  - *Aperiodic orders program,* CIRM Luminy, September 2007. Minicourse on cohomology of groupoids and cyclic cohomology (3 lectures)
  - *Summer school on Poisson Geometry,* ICTP Trieste, 4- 15 July 2005. Course on "Lie algebroids and groupoids, and applications" (8 lectures).
  - *Au-dela des algebroides de Lie,* Ecole Polytechnique Paris, 2004. Minicourse on "Momentum maps" (3 lectures).
  - *Journees Mathematique et Physique,* Clermont- Ferrant, France, June 2003. Minicourse on "The integrability problem and applications" (3 lectures).
  - *Seminaire Mediterraneen d'Algebre et Topologie,* Univ. Paul Sabatier, Toulouse,2002. Minicourse on "Integrability of Lie algebroids".

- *Various other talks* during visits abroad, such as University of Geneva (October 2007, invited by A. Alekseev and A. Heafliger, then March 2009, invited by A. Alekseev), Instituto Superior Tecnico Lisbon (2001, 2005, 2006, 2007 invited by R.L. Fernandes), University of California at Los Angeles (UCLA, 2001), Stanford University (2002), Muenster University (2002), Universite Paul Sabatier (Toulouse, 2002) etc.

4h. Other academic activities

- *Director of the NWO-research cluster GQT* (Geometry and Quantum Theory)-since 2013.

- *Involvement in the organization of conferences*:

  - 2013-2014: member of the scientific committee for the conference Poisson 2014.
  - 2012- : the Utrecht organizer of "Higher Geometric Structures along the Lower Rhine"- a series of workshops jointly organized by the Geometry/Topology groups in Utrecht, Bonn and Nijmegen (one meeting organized in Bonn in January 2012, one in Nijmegen in December 2012, the next one will be in Utrecht in September 2013).
  - 2010-2012: main organizer of the conference "Poisson 2012", held in Utrecht during July 30- August 4, 2012 (this activity belongs to a seria of conferences organized every other year). There were more than 200 participants.
  - 2010-2012: main organizer of the summer school "Poisson 2012", held in Utrecht during July 16- July 29, 2012 (this activity belongs to a seria of summer schools organized every other year). There were more than 150 participants.
  - 2010 (June 28- July 2): Organized, with G. Heckman and J. Heinlot, the "GQT conference" marking the end of the NWO cluster Geometry and Quantum Theory.
  - 2009-2010: Member of the Scientific Committee for th confer. "Poisson 2010".
  - 2008 (March 21- 22): Organized the "TopoLogical Workshop", which was a 2-days workshop held in Utrecht, with the support of NWO and MRI.
  - 2007 (April 29- May 5): Co-organized the Oberwolfach meeting "Poisson Geometry and applications".
  - 2003 (March): Organized a one-day workshop in honour of Alan Weinstein, in relation with the honorary doctorate awarded to him by the University of Utrecht.

- *Journal-related activities*:

  - Member of the editorial board of the journal of the Dutch Royal Academy, known as Indagationes (since 2010).
  - Member of the editorial board of the journal Mathematica (since 2006).
  - Referee for various journals such as Inventiones Mathematica, Journal of Differential Geometry, Duke Mathematical Journal, Advances in Mathematics, J.Reine Angew. Math., American Journal of Mathematics, Journal of Symplectic Geometry, Pacific Journal of Mathematics, Annales de l'institut Fourier, K-Theory, etc.

- *Participation in various committees* such as:

  - "Commissie Onderzoek" of PWN (representing Utrecht in this commission).
  - The scientific commission of the romanian education ministry, for awarding the academic titles (prof., etc).
  - "Beoordelingscommissie" of the NWO-Vrije Competitie scheme (2010, 2012).
  - Various selection committees in Utrecht, such as PhD selection committees, or the selection committee for the UD position opened in 2008, supported by the GQT cluster (total number of applications: 105).

- *Master Class coordinator* (2004- 2009): Coordinator of the "Master Class" programme of the Dutch Mathematical Research Institute. This programme aims at attracting bright students from abroad to study in the Netherlands and bringing them to the level of starting PhD students. Indeed, it has been a very important source of taleneted PhD students (the PI has/is already supervising 3 such students). The role of the coordinator is to supervise the ongoing master classes (schedule, various activities, final ceremony, etc), to attract funds, to attract new MC proposals, to advertise the program, to select the students etc. During this period, the Master Class obtained the financial support of the various dutch clusters; most notably, the GQT cluster supported each year about 8 students (for a period of 6 years). The support was announced at the opening ceremony of the GQT cluster by the Dutch Ministry of Education. During this perdio, I coordinated the following programmes:

- Arithmetic Geometry and Noncommutative Geometry (2009-2010)
- Numerical Bifurcation Analysis of Dynamical Systems (2009-2010)
- Calabi-Yau geometry (2008-2009)
- Quantum groups, affine Lie algebras and applications (2007-2008)
- Symplectic geometry and Beyond (2006-2007) (Also Scientific organizer).
- Logic (2006-2007).
- Finite and infinite dimensional dynamical systems (2005-2006).
- Stochastics in Molecular Biology and Genetics (2004-2005).

I was also the scientific organizer of the 2006/2007 Master Class ("Symplectic geometry and Beyond") and I was involved, together with I. Moerdijk and k. Landsmann, in the scientific organization of the 2003/2004 Master Class ("Noncommutative geometry").

➢ SKOz degree (Senior Research Qualification), Utrecht University (2009), SKO degree (Senior Teaching Qualification), Utrecht University (2008).

➢ *Olympiad activities* (since 2009): I am the coordinator, for the region of Utrecht, of the activities related to the Dutch Mathematical Olympiads for high-school students. In particular, I am organizing every year the "Wiskunde Olympiade- Tweede Ronde" for the region of Utrecht. As coordinator, I organize the day of the contest (there are about 100 participants in the contest plus parents for which special activities are organized), organize 5 extra training days for the best 10-15 students who qualify to the final, and rise the necessary funds for these activities. The PI was himself a participant at various national and international olympiads (wining several prizes), but the main motivation for being involved in such activities is rather different: I believe that the Olympiad activities offer a great opportunity for attracting some of the best students to study mathematics (etc, etc).

➢ *Local and national seminars*: organized/co-organized various local and national seminars in Topology, Poisson Geometry, Noncommutative Geometry, Mathematical Physics. In particular, each year I organize a seminar for the PhD students and the PostDocs from our research group; the aim is two-fold: on one hand to provide a proper framework for the PhD students and their "training" and, on the other hand, to stimulate the interaction between all the members of the group. During the perod 2004- 2008 I was also one of the organizers of the general (weekly) Stafcolloquium of the Mathematics Department,Utrecht University.

4i. Grants, scholarships and prizes

Grants: In all the following, my role was that of PI:

Grant Principal Investigator	Amount	Year of award
NWO Vrije Competitie project "Generalised Lie algebra sheaves "	200k Euro (3 years PostDoc)	2013
NWO Vrije Competitie project "Flexibility and Rigidity of Lie brackets"	200k Euro (3 years PostDoc)	2012
ERC Starting Grant project "New-Poetry"	1100k Euro (2 PhD, 6 years PostDoc, 0.4fte PI for 5 years).	2011
NWO Vrije Competitie project "Geometry of PDE's and Poisson structures"	200k Euro (3 years PostDoc).	2009
GQT-cluster PHD position (funds available because of my Vidi)	200k Euro (PhD position)	2009
NWO Vidi project "Poisson Topology"	600k euro (1 PhD, 3 years PostDoc, 1fte PI for 5 years)	2007
NWO Open Competitie project "Symmetries and Deformations in Geometry"	352k euro (1PhD, 3 years PostDoc)	2004
KNAW (Dutch Royal Academy of Arts and Sciences) Fellowship	350k euro (covering the PI)	2002
Miller Research Fellowship, UC Berkeley (interrupted to start the KNAW Fellowship)	? (it generously supported my salary and other research costs)	2001
Subtotal	3202k Euro	

Prize:

2008: The Andre Lichnerowicz Prize for contributions to Poisson Geometry. (see e.g. the Notices of the American Mathematical Society 55/2008 and 56/2009, pp 244, or the Notices of the European Mathematical Society 70/2008 and 71/2009). The prize is honorific and is awarded every other year.

Note about the Miller Fellowship at UC Berkeley: Applications are only by nomination, the fellowship is for all sciences, and at most one/year per field. The position was for 3 years, but was interrupted half way to return to The Netherlands (KNAW Fellowship).

Others:
- Also the one-month Invited Professorships (at Institute Henri Poincare- Paris, and Univ. Blaise Pascal in Clermont-Ferrand- see 4d) are of honorific nature.
- Co-project leader in several other projects.
- Rising funds for the organization of Poisson 2012 (about 100k euro).
- Various other smaller scale grants related to conferences/workshops.

List of publications

5a. Publications (Papers in refereed journals):

1. *On the linearization theorem for proper Lie groupoid*s, joint with I. Struchiner, Annales scientifiques de l'ENS, in press (2013).

2. *Representations up to homotopy and Bott spectral sequence*s, with C.A. Abad, Advances in Mathematics, in press (2013).

3. *A normal form theorem around symplectic leaves*, joint work with I. Marcut, J. Differential Geom. 92 (2012), 417-461.

4. *A geometric approach to Conn's linearization theorem*, joint w.L. Fernandes Annals of Mathematics 173 (2011), 1119-1137.

5. *Representations up to homotopy of Lie algebroids*, joint with C.A. Abad J.Reine Angew. Math. 663 (2012), 91-126.

6. *The Weil algebra and van Est isomorphisms*, joint with C. Arias Abad, Annales de L'Institut Fourier, 61 (2011), 927- 970.

7. *On the existence of symplectic realizations,* joint with I. Marcut, Journal of Symplectic Geometry 9 (2011), 435-444.

8. *Generalized complex structures and Lie brackets,* Proceedings Poisson 2010, Bull. Braz. Math. Soc. 42 (2011), 559-578.

9. *Tensor products of reprentations up to homotopy,* with .A. Abad and B. Dherin, J. Homotopy Relat. Struct. 6 (2011), no. 2, 239–288.

10. *Stability of symplectic leaves*, joint work with R.L. Fernandes, Inventiones Math. 180, no. 3, (2010), 481-533.

11. *Birkhoff Interpolation with Rectangular Sets of Nodes*, joint with N. Crainic, Journal of Numerical Mathematics, 2010.

12. *Polya conditions for multivariate Birkhoff interpolation: from general to rectangular sets of nodes*, joint with N. Crainic, Acta Mathematica Universitatis Comenianae, 2010.

13. *Dirac geometry and quasi-Poisson actions*, joint work with H. Bursztyn, Journal of of Differential Geometry 82 (2009), 501-566.

14. *Normal bivariate Birkhoff interp. schemes and Pell's equation*, joint with N. Crainic, Commentationes Mathematicae Universitatis Carolinae, 50 (2009) 265-272.

15. *Deformations of Lie brackets: cohomological aspects*, joint with Ieke Moerdijk, Journal of European Mathematical Society, 10 no. 4 (2008), pp. 1037-1059.

16. *Birkhoff interpolation with rectangular sets of nodes ...*, joint with N.Crainic,. East Journal on Approximations, 14 no 4 (2008), pp. 423-437.

17. *Integrability of Jacobi and Poisson structures*, joint work with Chenchang Zhu,. Annales de l'institut Fourier, 57 no. 4 (2007), pp. 1181-1216.

18. *Rigidity and flexibility in Poisson geometry*, joint with R.L. Fernandes,
Travaux mathematiques. Fasc. XVI, 53--68, Trav. Math., XVI, Univ. Luxemb. (2005).

19. *Quasi-Poisson structures as Dirac structure*s, joint with H. Bursztyn and P. Severa,
Travaux matheatiques. Fasc. XVI, 41--52, Trav. Math., XVI, Univ. Luxemb. (2005)

20. *Secondary characteristic classes of Lie algebroids*, joint work with R.L. Fernandes,
Quantum field theory and noncom. geom, Lect. Notes in Phys. 232 (2005), 157-176

21. *Dirac structures, moment maps, and quasi-Poisson manifolds*, joint with H. Bursztyn,
The breadth of symplectic and Poisson geom., Progr. Math. 157 (2005), pp. 1—40

22. *Prequantization and Lie brackets*,
J. Symplectic Geom. 2 (2004), pp. 579--602

23. *Integrability of Poisson brackets*, joint work with R.L. Fernandes,
J. Differential Geom. 66 (2004), pp. 71--137

24. *Integration of twisted Dirac brackets*, joint with H. Bursztyn, A. Weinstein, C. Zhu ,
Duke Math. J. 123 (2004), pp. 549--607

25. *Cech-De Rham theory for leaf spaces of foliations*, joint work with I. Moerdijk,
Math. Ann. 328 (2004), pp. 59--85

26. *Differentiable and algebroid cohomology, van Est isomorphisms ...*,
Comment. Math. Helv. 78 (2003), pp. 681--721

27. *Integrability of Lie brackets*, joint work with R.L. Fernandes,
Annals of Mathematics 157 (2003), pp. 575--620

28. *Cyclic cohomology of Hopf algebras*,
J. Pure Appl. Algebra 166 (2002), pp. 29--66

29. *Foliation groupoids and their cyclic homology*, joint work with I. Moerdijk,
Advances in Mathematics 157 (2001), pp. 177--197

30. *A homology theory for etale groupoids*, joint work with I. Moerdijk,
J. Reine Angew. Math. 521 (2000), pp. 25--46

31. *Cyclic cohomology of etale groupoids: the general case*,
K-Theory 17 (1999), pp.319-362

32. *On 2-primary alg. K-theory of quadr. numb. rings with focus on K2*,
joint with P. Ostvaer, Acta Arithmetica 87 (1999), pp. 223--243

33. *A note on the denseness of complete invariant metrics*, joint work with V. Csaba,
Publ. Math. Debrecen 51 (1997), pp. 265—271

Contribution to books (refereed):
34. *Lectures on integrability of Lie brackets*, joint with R.L. Fernandes,
Geometry & Topology Monographs 17 (2010).
Lecture notes for the 2005 Summer School in Poisson Geometry held at ICTP-Trieste.

5b. Median impact factors for your own field: -

5c. Top publications

A normal form theorem around symplectic leaves, joint work with I. Marcut,
J. Differential Geom. 92 (2012), 417-461.

A geometric approach to Conn's linearization theorem, joint with R.L. Fernandes
Annals of Mathematics 173 (2011), 1119-1137.

Stability of symplectic leaves, joint work with R.L. Fernandes,
Inventiones Math. 180, no. 3, (2010), 481-533.

Differentiable and algebroid cohomology, van Est isomorphisms and characteristic classes,
Comment. Math. Helv. 78 (2003), pp. 681—721

Integrability of Lie brackets, joint work with R.L. Fernandes,
Annals of Mathematics 157 (2003), pp. 575—620
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Statements by the applicant

Ethical aspects: not applicable.

☐ **I have completed this form truthfully**

Name: Marius Crainic

Place: Utrecht, The Netherlands

Date: August 27, 2013

There is a possibility to send a list of non-referees (maximum of three names). This is optional for every applicant. The individuals will NOT be asked to assess your application as referees. Please send the list with your application in a separate PDF-file.

Please submit the application to NWO in electronic form (pdf format is required!) using the Iris system, which can be accessed via the NWO website (www.nwo.nl/vi). For any technical questions regarding submission, please contact the Iris helpdesk (iris@nwo.nl).