**Exercises for Group Theory**

Week 3 (January 21)

**Exercise 1** About the puzzle mentioned at the end of the lecture (with the 15 numbers on a $4 \times 4$ table): explain why, if we brake the game, interchange two number, and then glue it back, the resulting game can never be solved.

**Exercise 2** (to prepare for the exam)
Consider the group $D_6$. Recall that

$$D_6 = \{e, r, r^2, r^3, r^4, r^5, s, rs, r^2s, r^3s, r^4s, r^5s\},$$

$r^6 = e$, $s^2 = e$, $rsr = s$.

1. Describe all the cyclic subgroups of $D_6$.
2. For each element of $D_6$, find its order and compute its conjugacy class and its centralizer in $D_6$.
3. Describe all the subgroups of $D_6$. Point out the Sylow subgroups.
4. Which of these subgroups are normal? In each case compute the quotient.
5. Compute the center of $D_6$.
6. Compute the abelianization of $D_6$.
7. Show geometrically that $D_6$ is isomorphic to a subgroup of $S_6$.
8. Can $D_6$ be isomorphic to a subgroup of $S_4$? But of $S_5$?
9. Show that $D_6$ is isomorphic to $F(r, s; r^6, s^2, rsrs)$ (or, in the notation from the book: $\{r, s; r^6, s^2, rsrs\}$).

**Exercise 3** Complete the computation of the counting problem that was started in the lecture.