

Exercises for Group Theory
Week 46 (November 12)

Exercise 1 Exercises 9.7, 9.8 in the book.

Exercise 2 Exercise 13.9.

In the proof of Cauchy's theorem in the class, we started with a group G and introduced

$$X = \{(x_1, x_2, \dots, x_p) : x_1, \dots, x_p \in G, x_1 x_2 \dots x_p = e\}.$$

Also, we have defined an "action" of \mathbb{Z}_p on X as follows: for $k \in \mathbb{Z}_p$,

$$x = (x_1, x_2, \dots, x_p) \in X,$$

we defined $k \bullet x \in X$ as

$$k \bullet x := (x_{k+1}, x_{k+2}, \dots, x_p, x_1, \dots, x_k).$$

Given $x \in X$, the \mathbb{Z}_p -orbit of $x \in X$ was defined as the subset of X :

$$\mathbb{Z}_p \bullet x := \{k \bullet x : k \in \mathbb{Z}_p\}.$$

Exercise 3 Check yourself that

- (i) $0 \bullet x = x$ for all $x \in X$.
- (ii) $k \bullet (k' \bullet x) = (k + k') \bullet x$.
- (iii) For any $x, y \in X$, the orbits $\mathbb{Z}_p \bullet x$ and $\mathbb{Z}_p \bullet y$ either coincide or they are disjoint.

Exercise 4 Show that

- (iv) if $x \in X$ is not of type (a, a, \dots, a) , then $\mathbb{Z}_p \bullet x$ has exactly p elements.

(Hint: consider $H_x := \{k \in \mathbb{Z}_p : k \bullet x = x\}$).

Exercise 5 Next, convince yourself that, the fact that $|X|$ is divisible by p , and (iii) and (iv) above, imply the theorem of Cauchy.

Exercise 6 Exercise 13.2.

Exercise 7 Go back to the older exercises that you did not do yet.