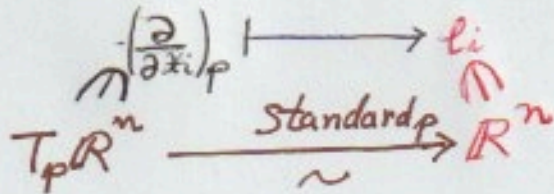


$\mathbb{R}^n$ 

$$T_p \mathbb{R}^n \xrightarrow[\sim]{\text{Standard } \rho} \mathbb{R}^n$$

$\mathbb{R}^n$





$\mathbb{R}^n$

$$\begin{array}{ccc} \frac{ds^a}{dt}(c_0) \in T_p \mathbb{R}^n & \xrightarrow{\text{Standard } p} & \mathbb{R}^n \\ \uparrow \left( \frac{\partial}{\partial x^i} \right)_p & & \uparrow \\ & & e_i \end{array} \Rightarrow \frac{ds^a}{dt}(c) = \left( \frac{ds^1}{dt}(c), \dots \right) = \sum_i \frac{ds^i}{dt}(c) \cdot e_i$$



$\mathbb{R}^n$

$$\sum_i \frac{ds_i}{dt}(c) \left( \frac{\partial}{\partial x_i} \right)_p \xrightarrow{\text{Standard } p} \mathbb{R}^n \ni \frac{ds}{dt}(c) = \left( \frac{ds_i}{dt}(c), \dots \right) = \sum_i \frac{ds_i}{dt}(c) \cdot e_i$$



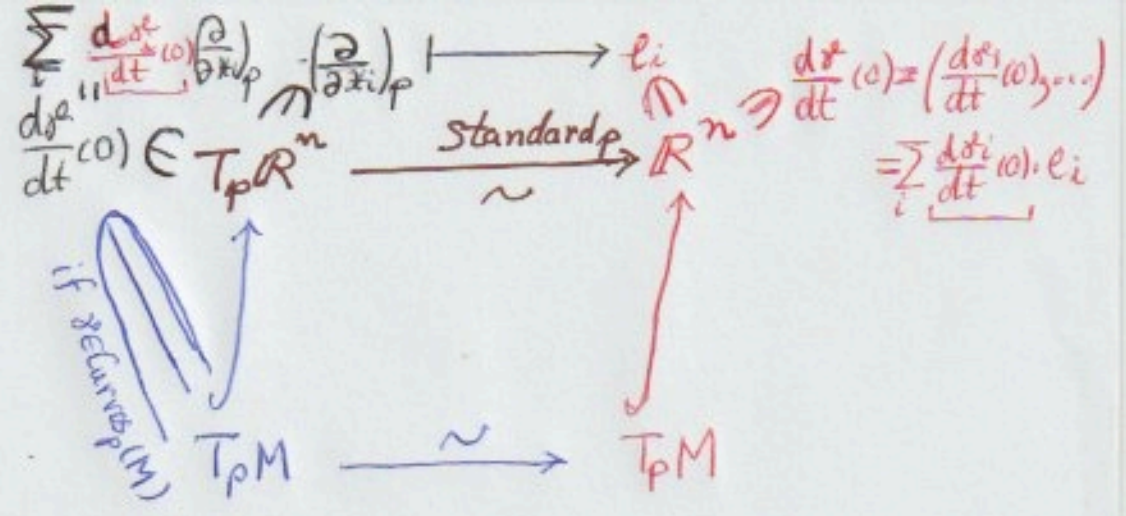
$\mathbb{R}^n$   
 $\uparrow$   
 $M$

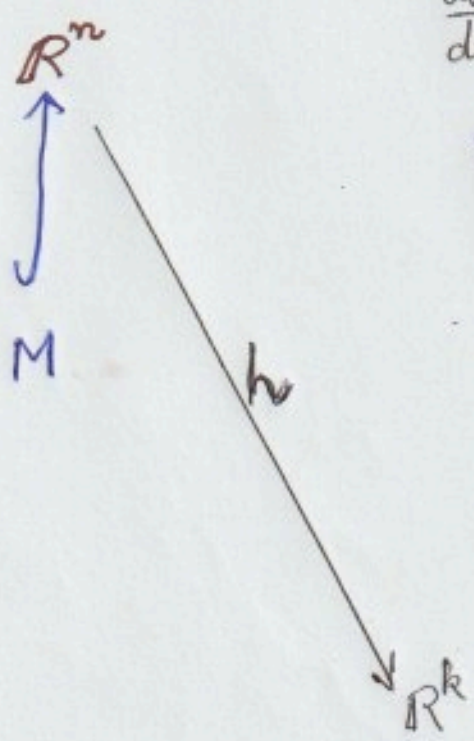
$$\sum_i \frac{ds_i}{dt}(c) \frac{\partial}{\partial x_i} \Big|_p \xrightarrow{\text{Standard } p} \mathbb{R}^n \ni \frac{ds}{dt}(c) = \left( \frac{ds_1}{dt}(c), \dots \right) = \sum_i \frac{ds_i}{dt}(c) \cdot e_i$$





$\mathbb{R}^n$   
 $\uparrow$   
 $M$





$$\sum \frac{d s_i^t(\omega)}{dt} \left( \frac{\partial}{\partial x^i} \right)_p \quad \left( \frac{\partial}{\partial x^i} \right)_p \quad \left| \quad \rightarrow \quad e_i \right.$$

$$\frac{d s^t}{dt}(\omega) \in T_p \mathbb{R}^n \xrightarrow[\sim]{\text{Standard } p} \mathbb{R}^n \quad \left( \frac{d s^t}{dt}(\omega) = \left( \frac{d s_i^t(\omega) \dots}{dt} \right) \right)$$

$$= \sum_i \frac{d s_i^t(\omega)}{dt} \cdot e_i$$

if  $\gamma \in \text{Curves}(M)$

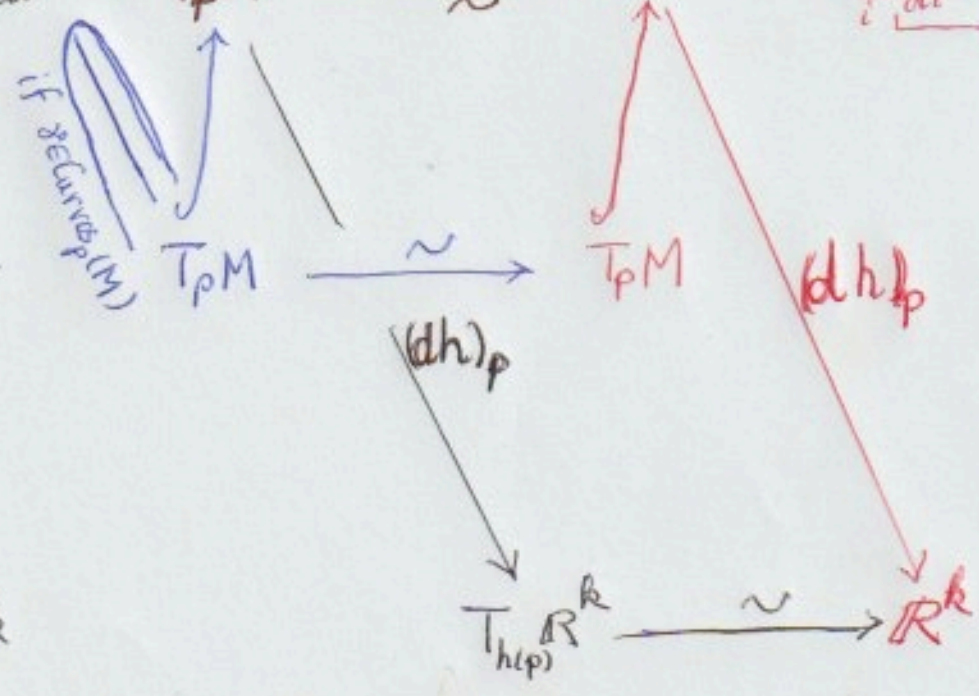
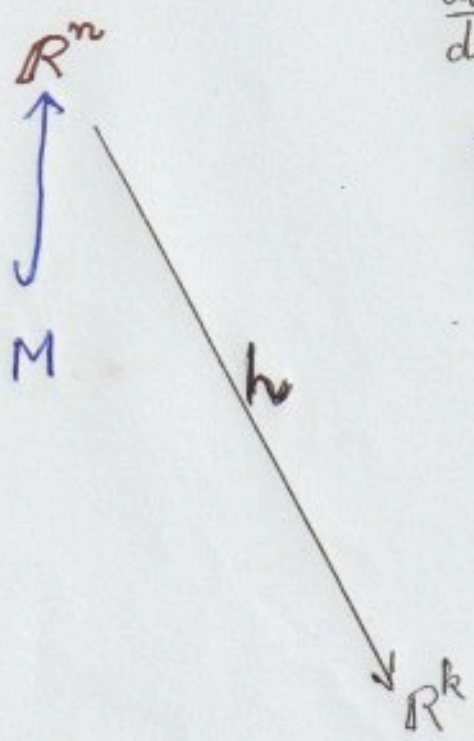
$$T_p M \xrightarrow{\sim} T_p M$$



$$\sum_i \frac{d\sigma_i}{dt}(\sigma) \frac{\partial}{\partial x_i} \Big|_p = \left( \frac{\partial}{\partial x_i} \right)_p \Big|_{\text{Standard } p} \rightarrow e_i$$

$$\frac{d\sigma}{dt}(\sigma) \in T_p \mathbb{R}^n \xrightarrow{\text{Standard } p} \mathbb{R}^n$$

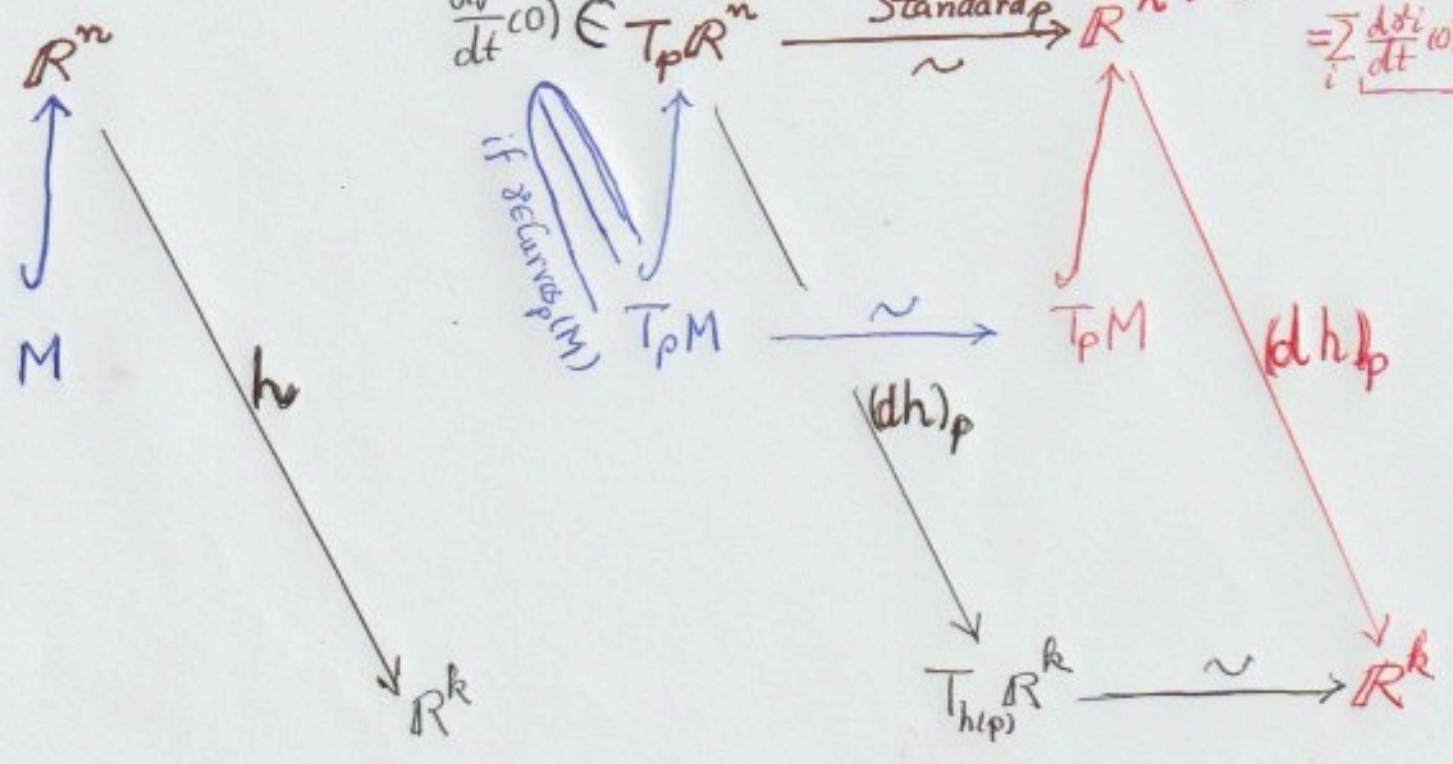
$$\frac{d\sigma}{dt}(\sigma) = \left( \frac{d\sigma_1}{dt}(\sigma), \dots \right) = \sum_i \frac{d\sigma_i}{dt}(\sigma) \cdot e_i$$



$$\sum \frac{dx^i}{dt}(0) \frac{\partial}{\partial x^i} \Big|_p \xrightarrow{\text{Standard}_p} \mathbb{R}^n$$

$$\frac{dx}{dt}(0) \in T_p \mathbb{R}^n \xrightarrow{\text{Standard}_p} \mathbb{R}^n$$

$$\frac{dx}{dt}(0) = \left( \frac{dx^1}{dt}(0), \dots \right) = \sum_i \frac{dx^i}{dt}(0) \cdot e_i$$

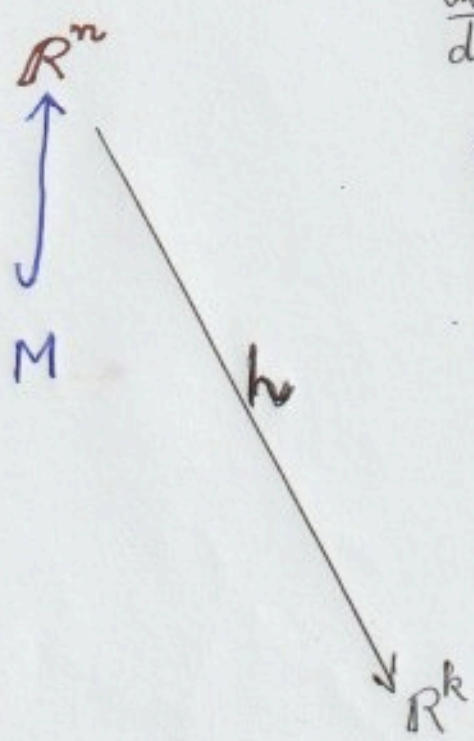


$$(dh)_p(e_i) = \frac{\partial h}{\partial x^i}(p)$$

$$= \left( \frac{\partial h_1}{\partial x^i}(p), \dots, \frac{\partial h_k}{\partial x^i}(p) \right)$$

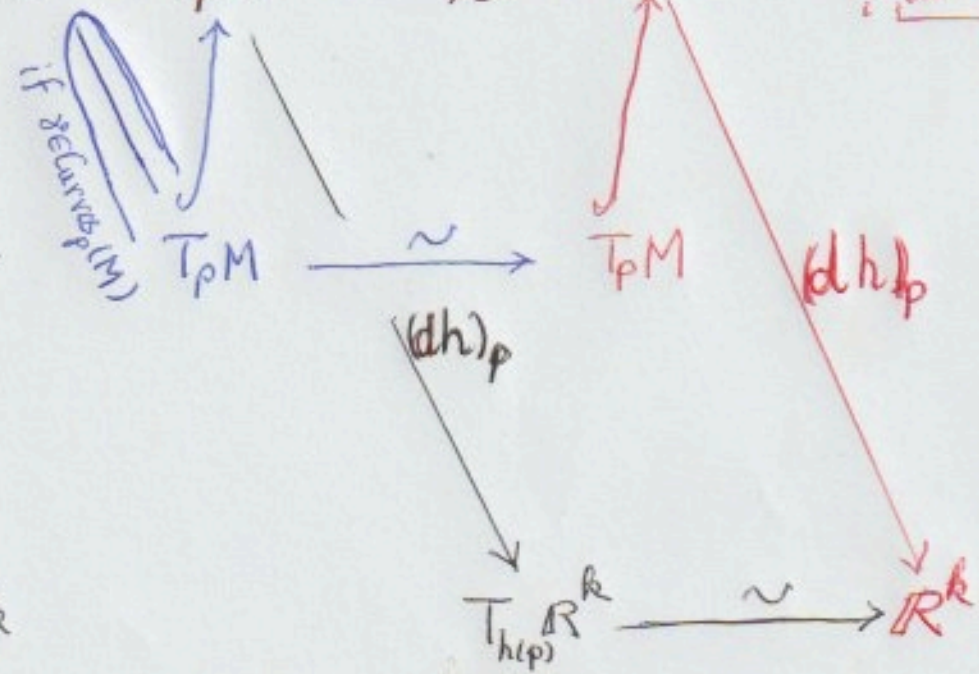
$$= \sum_j \frac{\partial h_j}{\partial x^i}(p) \cdot e_j$$





$$\sum \frac{dx^i}{dt}(c) \left( \frac{\partial}{\partial x^i} \right)_p = \left( \frac{\partial}{\partial x^i} \right)_p \Big|_{\text{Standard } p} \rightarrow e_i$$

$$\frac{dx}{dt}(c) \in T_p \mathbb{R}^n \xrightarrow{\text{Standard } p} \mathbb{R}^n \ni \frac{dx}{dt}(c) = \left( \frac{dx^1}{dt}(c), \dots \right) = \sum_i \frac{dx^i}{dt}(c) \cdot e_i$$

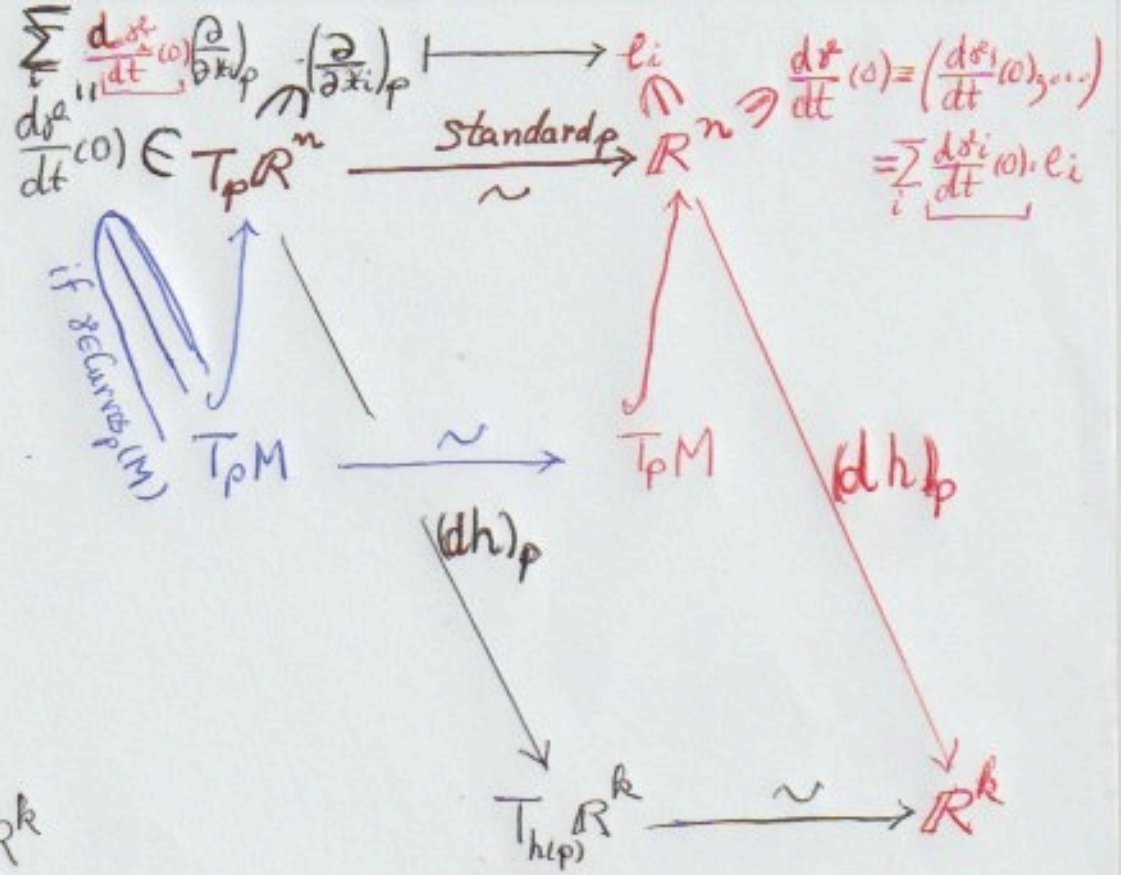
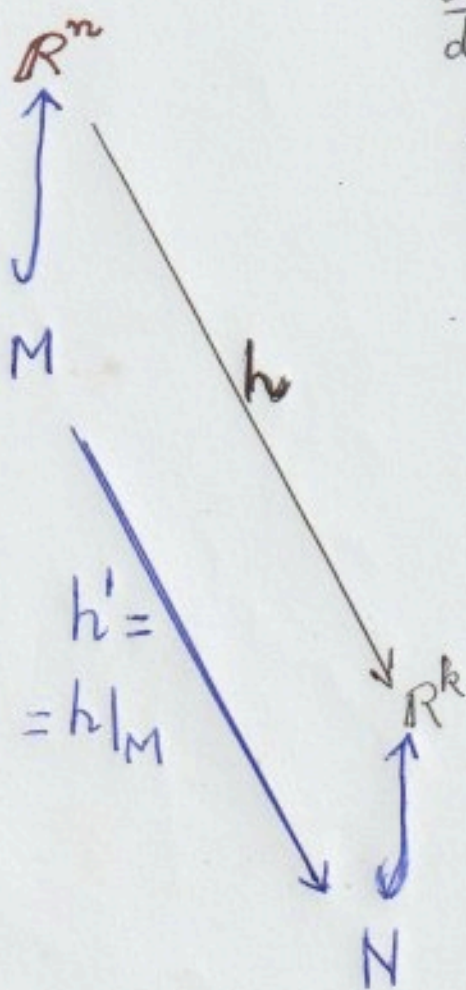


$$(dh)_p(e_i) = \frac{\partial h}{\partial x^i}(p)$$

$$= \left( \frac{\partial h_1}{\partial x^i}(p), \dots, \frac{\partial h_k}{\partial x^i}(p) \right)$$

$$= \sum_j \frac{\partial h_j}{\partial x^i}(p) \cdot e_{ij}$$

$$(dh)_p \left( \left( \frac{\partial}{\partial x^i} \right)_p \right) = \sum_j \frac{\partial h_j}{\partial x^i}(p) \left( \frac{\partial}{\partial x^j} \right)_p$$

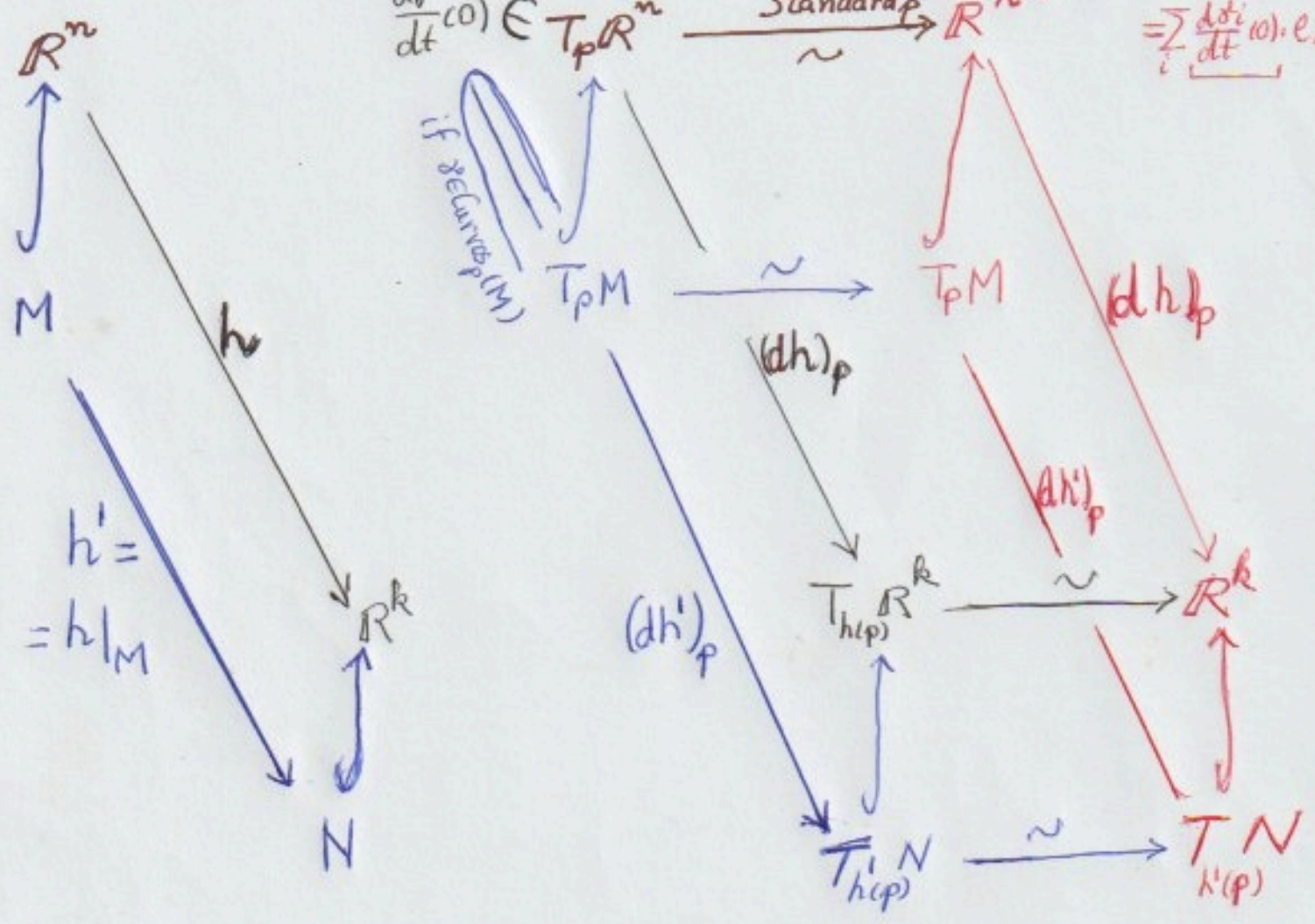


$$\begin{aligned}
 (dh)_p(e_i) &= \frac{\partial h}{\partial x^i}(p) \\
 &= \left( \frac{\partial h_1}{\partial x^i}(p), \dots, \frac{\partial h_k}{\partial x^i}(p) \right) \\
 &= \sum_j \frac{\partial h_j}{\partial x^i}(p) \cdot e_j
 \end{aligned}$$

$$(dh)_p \left( \left( \frac{\partial}{\partial x^i} \right)_p \right) = \sum_j \frac{\partial h_j}{\partial x^i}(p) \left( \frac{\partial}{\partial x^j} \right)_p$$



$$\sum \frac{dx^i}{dt} \left( \frac{\partial}{\partial x^i} \right)_p \quad \left( \frac{\partial}{\partial x^i} \right)_p \quad \xrightarrow{\text{Standard}_p} \quad \mathbb{R}^n \quad \frac{dx}{dt}(c) = \left( \frac{dx^1}{dt}(c), \dots \right) = \sum_i \frac{dx^i}{dt}(c) \cdot e_i$$



$$\begin{aligned} (dh)_p(e_i) &= \frac{\partial h}{\partial x^i}(p) \\ &= \left( \frac{\partial h_1}{\partial x^i}(p), \dots, \frac{\partial h_k}{\partial x^i}(p) \right) \\ &= \sum_j \frac{\partial h_j}{\partial x^i}(p) \cdot e_j \end{aligned}$$

$$(dh)_p \left( \left( \frac{\partial}{\partial x^i} \right)_p \right) = \sum_j \frac{\partial h_j}{\partial x^i}(p) \left( \frac{\partial}{\partial x^j} \right)_p$$