HOMEWORK 6 (OCTOBER 18, 2023)

Exercise 1. Consider the following curve in $\mathbb{R}^3$:

$$\gamma : \mathbb{R} \to \mathbb{R}^3, \quad \gamma(t) = (t^2, t^3, t).$$

a) 1pt Show that the following is a submanifold of $\mathbb{R}^3$ containing γ:

$$M = \{(x, y, z) \in \mathbb{R}^3 : y = xz\}.$$

b) 1.5pt Show that the following defines a vector field on M

$$V := 2z\frac{\partial}{\partial x} + (x + 2z^2)\frac{\partial}{\partial y} + \frac{\partial}{\partial z}.$$

c) 1pt Show that γ is an integral curve of V.

e) 1pt Find vector fields $X, Y \in \mathfrak{X}(M)$ which, at each point in M, give a basis of the tangent space of M.

f) 1.5pt Compute $[X, Y]$.

g) 2pt Compute the flow of V, describing explicitly all the diffeomorphisms ϕ_V^t induced by V.

h) 2pt Find a non-zero 1-form $\theta \in \Omega^1(\mathbb{R}^3)$ such that $\theta|_M = 0$.

i) 2pt Bonus question: find a 1-form on M which is not exact, i.e. cannot be written as df for some $f \in C^\infty(M)$.