Topology and Geometry B (6 July, 2006)

Exercise 1. Prove that, for any space X, the cone $C(X)$ of X is contractible. (9 points)

Exercise 2. Which of the following pairs of spaces are homeomorphic. In each case explain your answer.

(a) $\mathbb{R}^2$ and $\mathbb{R}^3$ (2 points)

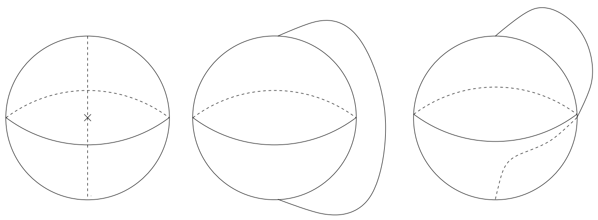
(b) S^2 and $\mathbb{P}^2$ (2 points)

(c) S^1 and $\mathbb{P}^1$ (2 points)

(d) $S^1 \times S^2$ and S^3 (2 points)

(e) $S^1 \times \mathbb{R}$ and $\mathbb{R}^2 \backslash \{(0,0)\}$ (2 points)

Exercise 3. Consider the following spaces



X — (a sphere together with a line joininig the north and the south pole, and which stays inside the ball)

Y — (a sphere together with a line joining the north and the south pole, and which stays outside the ball)

Z — (a sphere together with a line joiniing the north and the south pole, half of which is outside the ball, and half of which is inside the ball)

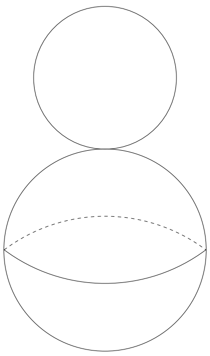
(a) For each of these spaces, describe a cell decomposition (indicate it on the picture).

(b) Which of these spaces are homeomorphic among them? Explain your answer.

(c) Which of these spaces is homotopic equivalent to the space W (explain your answer):

W:

(a sphere together with a circle touching the sphere in the north pole)



Exercise 4. Consider the unit circle

$$S^1 = \{z \in \mathbb{C} : |z| = 1\},$$

$n \geq 1$ integer and define

$$f_n : S^1 \to S^1, f_n(z) = z^n.$$

(a) Show that f_n is a covering map. (2 points)

(b) Consider

$$(f_n)_* : \pi(S^1, 1) \to \pi(S^1, 1)$$

After identifying $\pi(S^1, 1)$ with $(\mathbb{Z}, +)$, show that $(f_n)_*$ is identified with

$$mult_n : \mathbb{Z} \to \mathbb{Z}$$

$$r \mapsto nr$$

(2 points)

(c) Let X_n be the space obtained from S^1 by attaching a two cell with characteristic map f_n. Compute the fundamental group of X_n. (2 points)

(d) Describe X_n as a quotient of D^2. Do you recognize X_2? (2 points)

Note:

- Grade=$1+\frac{\text{Sum of points}}{4}$ (maximum:10).
- You are allowed to use the lecture notes and all the results therein for the exercises.

Important: Please indicate which of the following courses you have already followed, and which of them you have already passed:

- Distributies:
  - attended
  - passed
- Functionaalanalyse:
  - attended
  - passed
- Differentieerbare varieiten:
  - attended
  - passed