**Exercise 1.** (from last year's exam) Let X be the space obtained from a polygon with 7 edges by gluing (some of) its edges according to the labeling in Figure 1. Also, let $A$ be the space obtained from the boundary of the polygon by performing the same gluing.

Let $p$ be the point in $A$ corresponding to the point $P$ in the picture and we consider the fundamental groups of $A$ and $X$ with base-point $p$.

We also consider the space $Y$ which is the connected sum of two Moebius bands (i.e. obtained from two copies of the Moebius band by removing a small open disk from each one of them and gluing them along the boundary circles, as shown in Figure 2).

(a) Make a picture of $A$ and show that its fundamental group is a free group in three generators. Draw the loops at $p$ which represent the generators. (*1p*)

(b) Compute the Euler characteristic of $X$ (explain the computation!!). (*1p*)

(c) Compute the fundamental group of $X$. (*1p*)

(d) Find a description of Y as a quotient obtained from a disk by gluing some of the points on the boundary circle and, using the previous parts of the exercise, compute its fundamental group. (*0.5p*)

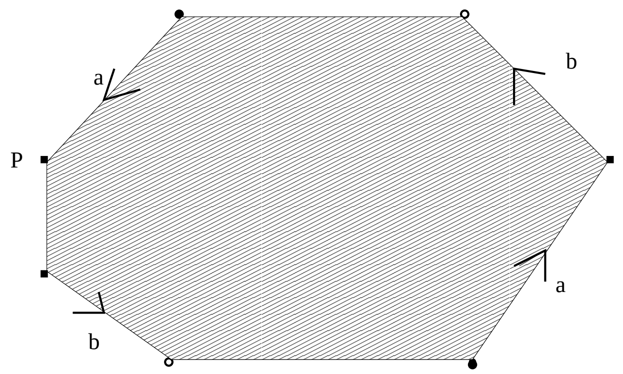


Figure 1:

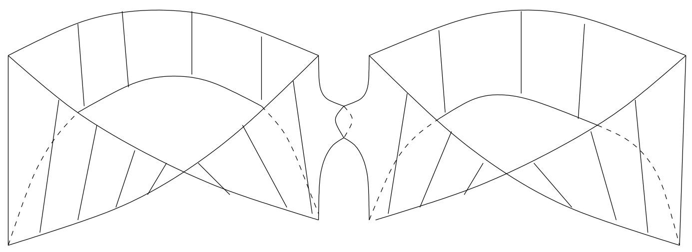

Figure 2: