(a) To show S^1 can not be embedded into $\mathbb{R}$ you want to use the remove a point trick. This says that if X and Y are homeomorphic, then for any $x \in X$ there exists an $y \in Y$ such that $X \setminus \{x\}$ is homeomorphic to $Y \setminus \{y\}$. So if you cand find an $x \in X$ such that $X \setminus \{x\}$ is not homeomorphic to $Y \setminus \{y\}$ for any $y \in Y$, then X and Y are not homeomorphic. You should be careful about what you choose as X and Y. To show $S^1 \times \mathbb{R}$ can be embedded into $\mathbb{R}^2$, it may be easier to embed $S^1 \times (0,1)$ into $\mathbb{R}^2$ as $\mathbb{R} \cong (0,1)$ (where $\cong$ means "homeomorphic to"). It is enough to give the homeomorphism, you don't need to show continuity with epsilon-delta

(b) The elements of $\mathbb{R} \times \mathbb{R}^\infty$ are of the form $(x, (y_1, y_2, \ldots))$ and the elements from $\mathbb{R}^\infty$ are of the form $(y_1, y_2, \ldots)$. There is a natural bijection between them:

$$(x, (y_1, y_2, \ldots)) \mapsto (x, y_1, y_2, \ldots).$$

If $U \subset \mathbb{R} \times \mathbb{R}^\infty$ is open, then for any $p = (x, (y_1, y_2, \ldots)) \in U$, by definition of the product topology there exist opens $V_p \subset \mathbb{R}$, $W_p \subset \mathbb{R}^\infty$ such that

$$x \in V_p, \quad (y_1, y_2, \ldots) \in W_p, \quad V_p \times W_p \subset U.$$

To show $\tilde{U} \subset \mathbb{R}^\infty$ is open, we need to show all the projections of U are open. The key idea to show that each such projection if open is to use the fact, in a general topological space, that anything that is a union of opens is open. For the projection onto the first coordinate, we have a bunch of opens to work with, namely the V_p. So we might hope

$$\tilde{U} = \bigcup_{\tilde{p} \in \tilde{U}} V_{\tilde{p}}$$

This turns out to be the case (this is the first statement you need to prove). For the projections onto the further coordinates, you can do something similar, using that W_p is open, so it's projections are open.
If we start with an open $\tilde{U} \subset \mathbb{R}^\infty$, to show the corresponding $U \subset \mathbb{R} \times \mathbb{R}^\infty$ is open, we need to construct the V_p and W_p. For V_p we can take the projection of $\tilde{p}$ onto the first coordinate, and W_p you will have to find yourself.

(c) Here already a hint was given

(d) Here we have the same V_p and W_p, but now the meaning of W_p being open is different. In a metric space, U is open if for point in U, you can find an epsilon-ball around the point which is still contained in U. So you will have to look what an epsilon-ball is in $\mathbb{R}^\infty$. Then you can do mostly the seem as you did in exercise 3.30 (if you made it, otherwise you might first want to try that one) using Lemma 3.18

General comment: as this is a longer exercise, try to identify which steps/claims are the main ones, and prove those. You don't have to prove every minor subclaim you make (but untrue claims can still result in points deduction), and I won't be grading it with as much scrutiny as Analyse or Bewijzen in de Wiskunde homeworks were graded.