**Diabatic-Dynamical Interaction in the General Circulation (lecture 5)**

**Meridional transport of mass, heat and momentum**

- Hadley-, Ferrel- and Brewer-Dobson circulation
- Wave-zonal mean flow interaction
- Eliassen-Palm flux and planetary wave drag
- Residual circulation = diabatic circulation = Brewer-Dobson circulation

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**General Circulation identified with zonal mean circulation**

Section 12.1:

If both the surface of the planet, in terms of albedo, heat capacity, surface roughness, vegetation and orography, and the composition of the atmosphere are homogeneous in space, the response of this atmosphere to absorption of Solar radiation will be zonally symmetric. In reality this is not quite the case. Nevertheless, in this chapter the "General Circulation" is indeed identified with the **zonally (symmetric) state and its seasonal cycle**. Yet, this "General Circulation" cannot be understood without taking into account the meridional transport of heat (mass) and momentum (vorticity) by zonally asymmetric features of the circulation (chapter 11), which arise, inevitably, because of baroclinic instability of the zonally symmetric state or because of surface inhomogeneities, in particular associated with mountains and land sea-contrasts. In
Eulerian zonal mean flow
Cross section of the subtropical and polar jet streams by latitude.
Hadley cell: driven by diabatic heating.
Ferrel cell: driven by eddy fluxes of heat and momentum!

But, there is more:
Footprint of Lagrangian circulation
Hadley and Ferrel circulations are “Eulerian circulations”. Superposed on these circulations there exist eddies, which also transport mass, heat and momentum. The resulting “Lagrangian circulation” is called the “residual circulation”. This is rather abstract concept. Its manifestation in reality is the zonal mean distribution of ozone and water vapour.

Zonal mean distribution of ozone molecule number density \((10^{18} \text{ m}^{-3})\) at the equinox (22 September), based on measurements taken in the 1960’s
Lagrangian circulation transports mass

Figure 11.14

Ozone, which is formed in the tropical stratosphere, is transported to the poles, where the maximum concentrations are observed.

Brewer’s (1949) original figure, illustrating what came to be known as the Brewer-Dobson circulation. The contours represent isotherms, labeled in units of K. “A supply of dry air is maintained by a slow mean circulation from the equatorial tropopause.”

Residual circulation: transports mass

Figure 11.14

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What is the cause of this residual flow?

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The meridional transport of momentum & heat by planetary waves

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How does this work?
Flux equation

Transport equation for a scalar $Q$ in pressure coordinates is

$$\frac{\partial Q}{\partial t} + u \frac{\partial Q}{\partial x} + v \frac{\partial Q}{\partial y} + \omega \frac{\partial Q}{\partial p} = S$$

Continuity equation multiplied by $Q$ is

$$Q \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} \right) = 0$$

Add these equations yields the “flux-form”:

$$\frac{\partial Q}{\partial t} + u \frac{\partial Q}{\partial x} + v \frac{\partial Q}{\partial y} + \omega \frac{\partial Q}{\partial p} = S$$

=flux-divergence of $Q$

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Zonal average flux-divergence

Repeat:

$$\frac{\partial Q}{\partial t} + u \frac{\partial Q}{\partial x} + v \frac{\partial Q}{\partial y} + \omega \frac{\partial Q}{\partial p} = S$$

Take zonal average of this equation*

$$\frac{\partial [Q]}{\partial t} + \frac{\partial [vQ]}{\partial y} + \frac{\partial [\omega Q]}{\partial p} = [S]$$

The zonal average of the meridional flux of a quantity $Q$ is (sections 1.39 & 11.4)

$$[vQ] = [v][Q] + [v^*Q^*]$$

Therefore, the zonal average flux divergence equation becomes

$$\frac{\partial [Q]}{\partial t} + \frac{\partial [v][Q]}{\partial y} + \frac{\partial [v^*Q^*]}{\partial y} + \frac{\partial [\omega][Q]}{\partial p} + \frac{\partial [\omega^*Q^*]}{\partial p} = [S]$$

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Flux of heat and momentum by waves

Taking account of the flux of a quantity $Q$ due to the waves

* flux-divergence with respect to $Q$
The zonally averaged continuity equation is reads: \( \frac{\partial [v]}{\partial y} + \frac{\partial [\omega]}{\partial p} = 0 \)

Using this, we can write the transport equation as

\[
\frac{\partial [Q]}{\partial t} + [v] \frac{\partial [Q]}{\partial y} + [\omega] \frac{\partial [Q]}{\partial p} + \frac{\partial [v^*Q^*]}{\partial y} + \frac{\partial [\omega^*Q^*]}{\partial p} = [S]
\]

Therefore, **zonal average** of: \( \frac{dQ}{dt} = S \)

"**Apply**" this equation to the following two equations.

**Equation of motion in pressure coordinates**

(x-component):

\[
\frac{du}{dt} = -\frac{\partial \Phi}{\partial x} + fv + F_x \quad \text{Box 9.1}
\]

- Gradient of geopotential
- Friction
- Coriolis

**Energy conservation equation**:

\[
\frac{dT}{dt} = \frac{RT}{c_p p} \omega + \frac{J}{c_p} \quad \text{Box 9.1}
\]

- Adiabatic heating/cooling
- Diabatic heating/cooling

(The metric terms, associated with the curvature of the Earth, are neglected)
We have:
\[
\frac{dQ}{dt} = S \rightarrow \frac{\partial Q}{\partial t} + [v] \frac{\partial Q}{\partial y} + \frac{\partial [v] Q}{\partial p} + \frac{\partial [v^* u^*]}{\partial y} + \frac{\partial [\omega^* u^*]}{\partial p} = [S]
\]

Apply this equation to the following two equations (chapter 9)
\[
\frac{du}{dt} = -\frac{\partial \Phi}{\partial x} + fv + F_x
\]
\[
\frac{dT}{dt} = \frac{RT}{c_p} \frac{\partial T}{\partial p} + \frac{J}{c_p}
\]

For the extra-tropics we make the following approximations:
\[
[v] = [v_g] + [v_a] = \left[1 + \frac{1}{f} \frac{\partial \phi}{\partial x}\right] + [v_a] = [v_a]
\]
\[
[\omega] \frac{\partial u}{\partial p} \ll [v] \frac{\partial u}{\partial y} \quad [\omega] \frac{\partial [v^* u^*]}{\partial p} \ll \frac{\partial [v^* u^*]}{\partial y} \quad [\omega] \frac{\partial u}{\partial y} \ll f = f_0
\]
(Section 11.5)

With these approximations the zonal mean x-momentum equation,
\[
\frac{\partial u}{\partial t} + [v] \frac{\partial u}{\partial y} + \frac{\partial [v^* u^*]}{\partial p} + \frac{\partial [\omega^* u^*]}{\partial p} = \pm f[v] + [F_x]
\]
becomes
\[
\frac{\partial u}{\partial t} = f_0 [v_a] - \frac{\partial [v^* u^*]}{\partial y} + [F_x]
\]
Similarly, we can also simplify the temperature equation to:

\[
\frac{\partial T}{\partial t} = \frac{k T}{p} \frac{\partial T}{\partial p} \left[ \omega \right] - \frac{\partial [v^* T^*]}{\partial y} + \frac{J}{c_p}
\]

or:

\[
\frac{\partial T}{\partial t} = S_p \left[ \omega \right] - \frac{\partial [v^* T^*]}{\partial y} + \frac{J}{c_p}
\]

static stability, \(S_p > 0\)

Wave-Mean Flow Interaction

Simplified (linear) quasi-geostrophic equations, describing the interaction between the zonal mean state and the waves:

\[
\frac{\partial u}{\partial t} = f_0 [v_a] - \frac{\partial [v^* u^*]}{\partial y}
\]

\[
\frac{\partial T}{\partial t} = S_p \left[ \omega \right] - \frac{\partial [v^* T^*]}{\partial y} + \frac{J}{c_p}
\]

divergence of eddy momentum flux

divergence of eddy heat flux
Simplification*: Transformed Eulerian Mean equations

Introduce the following "residual velocities"

\[
[\omega]_r = [\omega] - \frac{\partial}{\partial y} \left[ \frac{v^* T^*}{S_p} \right]
\]

\[
[v_a]_r = [v_a] + \frac{\partial}{\partial p} \left[ \frac{v^* T^*}{S_p} \right]
\]

\([v_a]_r\) and \([\omega]_r\), can be interpreted as "physically consistent" velocity components, because they satisfy the following "continuity equation", which is analogous to the “real” continuity equation in pressure coordinates:

\[
\frac{\partial [v_a]_r}{\partial y} + \frac{\partial [\omega]_r}{\partial p} = 0
\]

Transformed Eulerian Mean (TEM) equations

Substitution of the definitions of the residual velocities, \([v_a]_r\) and \([\omega]_r\), into

\[
\frac{\partial [u]}{\partial t} = f_0 [v_a] - \frac{\partial [v^* u^*]}{\partial y}
\]

\[
\frac{\partial [T]}{\partial t} = S_p [\omega] - \frac{\partial [v^* T^*]}{\partial y} + \frac{[J]}{c_p}
\]

yields the TEM-equations

\[
\frac{\partial [u]}{\partial t} = f_0 [v_a]_r + \nabla \cdot \vec{F}
\]

\[
\frac{\partial [T]}{\partial t} = S_p [\omega]_r + \frac{[J]}{c_p}
\]

\[
\vec{F} \equiv \left( -[u^* v^*], -\frac{f_0}{S_p} [v^* T^*] \right)
\]

\[
\nabla \equiv \left( 0, \frac{\partial}{\partial y}, \frac{\partial}{\partial p} \right)
\]
Eliassen-Palm Flux

Influence of eddies is apparent only in the equation for $[u]!$

Convergence of the Eliassen-Palm flux is associated with a deceleration of the zonal mean wind.

This is referred to as “planetary wave-drag”.

Simulation of an unstable baroclinic wave after 4 days, starting as a small perturbation on a front

**Figure 10.14.** Mature stage of the life-cycle of an adiabatic inviscid baroclinic wave at 865 hPa as simulated by a 36-level primitive equation model. More information is given in the caption of figure 10.13. Note the change in scale of the $Q$-vectors (red arrows) in this figure compared to figure 10.13. The warm front is indicated by “wf”. The cold front is indicated by “cf”. The back-bent front is indicated by “bbf”. On the following website you will find animations of this integration:

**EP-flux in a simulation of an unstable baroclinic wave**

\[
\hat{F} = \left[ -u^* v^* \right] - \frac{\mathbf{f}}{\mathcal{S}_p} \left[ v^* T^* \right]
\]

*Figure 11.12.* Instantaneous Eliassen-Palm flux vector and its divergence at \(t=4\) days in the primitive equation model simulation of a growing baroclinic wave on an \(f\)-plane, which is discussed in detail in section 10.7. The horizontal component of the EP-flux vector is magnified 12 times compared to the vertical component. The contours of EP-flux divergence are labeled in units of \(10^{-2}\) m s\(^{-2}\).

**Steady state**

\[
J_0 \left[ v_a \right]_r = - \nabla \cdot \hat{F} \\
\mathcal{S}_p \left[ \omega \right]_r = - \frac{J}{c_p}
\]

If \(\nabla \cdot \hat{F} < 0\) then \(\left[ v_a \right]_r > 0\) (in the northern hemisphere)

i.e. **poleward residual flow**.

This flow transports heat and hence changes the temperature, such that the temperature is above the radiatively determined temperature over the pole and below the radiatively determined temperature over the equator.

This leads to **diabatic heating** (cooling) over the equator (pole)

*Equator:* \(J > 0 \rightarrow \left[ \omega_r \right] < 0\)  \hspace{1cm} *Pole:* \(J < 0 \rightarrow \left[ \omega_r \right] > 0\)
Residual circulation

Brewer’s (1949) original figure, illustrating what came to be known as the Brewer-Dobson circulation. "A supply of dry air is maintained by a slow mean circulation from the equatorial tropopause". The contours represent isotherms, labeled in units of K.

\[ f_0[v_{a}] = -\nabla \cdot \mathbf{F} \]

Residual circulation

Brewer’s (1949) original figure, illustrating what came to be known as the Brewer-Dobson circulation. "A supply of dry air is maintained by a slow mean circulation from the equatorial tropopause". The contours represent isotherms, labeled in units of K.

\[ S_p(\alpha) = \frac{J}{c_p} \]
Residual circulation

FIGURE 11.11

Monthly average isobars (dashed black lines, labelled in units of hPa) and monthly average tendency of the potential temperature (labelled in units of K day⁻¹; contour interval is 0.5 K day⁻¹) as a function of latitude and potential temperature for March.

Residual circulation: modern view

FIGURE 12.27

Schematic of the residual mean meridional circulation in the atmosphere according to R.A. Plumb. The heavy ellipse indicates the Hadley circulation of the troposphere. The shaded regions (labelled “S”, “P”, and “G”) denote regions of breaking waves (synoptic- and planetary-scale waves, and gravity waves, respectively), responsible for driving branches of the stratospheric and mesospheric circulation. The surf zone is region of mixing of potential vorticity by planetary waves, which acts as a drag force on the mean zonal flow.
Assignment 5

Give a short answer to the following 2 questions

Problem 12.6. Influence of Earth’s rotation rate
How would the Hadley circulation and the zonal mean zonal jets be affected if we would double the value of the Coriolis parameter.

Problem 12.7. Influence of carbon dioxide (CO₂)
How would the Hadley circulation and the zonal mean zonal jets be affected if we would double the carbon dioxide concentration? What will happen to the annual mean global mean temperature at the surface? What will happen to the annual mean global mean temperature at the 10 hPa?

Hand in the answer on Friday, 22 June 2018

Next lecture (6)

Wednesday 20/6, 2018, 15:15-17:00
Room MIN 018

Back to the two-dimensional General Circulation Model:

We introduce the missing element which solves all problems: planetary wave drag.

But, do we really understand the influence of meridional transport of heat and momentum by waves on the zonal mean state?

What is planetary wave drag?

http://www.staff.science.uu.nl/~delde102/BLT&M.htm