Dynamic Meteorology: lecture 10

Sections 1.34 and 1.35

- Quasi-balance of the westerly flow (section 1.34)

- Acceleration and deceleration of the zonal mean zonal wind by eddies (section 1.35)

13/11/2019: tutorial 7
- Problems 1.29, 1.30, 1.32 and 1.33

20/11/2019: tutorial 8
- Problems 1.34 and 1.35
New grading (weights) (in couples)
Project 1: 15% (problem 1.6)
Project 2: 25% (problem 1.24)
Exam: 60% (minimum grade: 5/10)

Dates of deadlines
Project 1 (problem 1.6, page 53): 25 September 2019
Project 2 (problem 1.24, page 185): 22 or 24 January 2020 (oral presentation)
Exam: Wednesday 15 January 2019 (retake in week 5 of 2020)
Quasi-balance of the westerly flow

**Eq. 1.190b (p. 130):** (isentropic coordinates)

\[
\frac{dv}{dt} = -\frac{\partial \Psi}{\partial y} - fu - \frac{u^2 \tan \phi}{a}
\]

**Gradient wind balance:**

\[0 = -\frac{\partial \Psi}{\partial y} - fu - \frac{u^2 \tan \phi}{a}\]

or:

\[0 = -fu_g - fu_{gr} - \frac{u_{gr}^2 \tan \phi}{a}\]

\[u_g = -\frac{1}{f} \frac{\partial \Psi}{\partial y}\]

“geostrophic wind”

“Gradient wind”

or:

\[u_{gr}^2 + Bu_{gr} - Bu_g = 0\]

**Isentropic streamfunction:**

\[\Psi \equiv c_pT + gz\]

**Solution of this equation:**

\[u_{gr} = -\frac{B}{2} \pm \frac{B}{2} \sqrt{1 + \frac{4}{B} u_g}\]

\[B \equiv \frac{fa}{\tan \phi}\]
Quasi-balance of the westerly flow

Geostrophic wind balance equation:

\[
    u_g = -\frac{1}{f} \frac{\partial \Psi}{\partial y}
\]

Gradient wind balance equation:

\[
    u_{gr} = -\frac{B}{2} \pm \frac{B}{2} \sqrt{1 + \frac{4}{B} u_g}
\]

\[
    B \equiv \frac{fa}{\tan \phi}
\]

From gridded reanalysis data of the isentropic streamfunction determine geostrophic wind, \( u_g \), and then the gradient wind, \( u_{gr} \), at each grid point and then calculate the zonal mean of both the geostrophic wind ([\( u_g \)]) and the gradient wind ([\( u_{gr} \)]) at each latitude. Compare these zonal means with the actual zonal mean velocity, [\( u \)]
Figure 1.104: Zonal mean balanced wind speeds (blue: geostrophic; red: gradient wind) in January 2007, daily at 0, 6, 12 and 18 UTC, at $\theta=315$ K and at 50 °N. The straight red line is the best (least squares) linear fit to the gradient wind. The associated correlation coefficient is 0.99. For comparison, the “1 to 1” line is also shown in black. The slope of the best linear fit to the gradient wind is 0.88.
The existence of "super-gradient" zonal mean zonal winds suggests that eddies (zonal asymmetries) accelerate the zonal mean flow.

This is possible (see next slide)!

**Figure 1.104**
Acceleration of the **zonal mean** zonal wind

Eq. 1.190a (p. 130):

**Zonal acceleration:**

\[ \frac{du}{dt} = -\frac{\partial \Psi}{\partial x} + fv + \frac{uv \tan \phi}{a} \]

(isentropic coordinates)

Neglect the last term and assume adiabatic conditions

**Adiabatic conditions:**

\[ \frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - \frac{\partial \Psi}{\partial x} + fv \]

Rewrite this equation:

\[ \frac{\partial u}{\partial t} = -\frac{1}{2} \frac{\partial}{\partial x} \left( u^2 \right) - \frac{1}{2} \frac{\partial}{\partial x} \left( v^2 \right) + v \zeta - \frac{\partial \Psi}{\partial x} + fv \]

With: \[ \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \]
Acceleration of the **zonal mean** zonal wind

Previous slide:

\[
\frac{\partial u}{\partial t} = - \frac{1}{2} \frac{\partial}{\partial x} (u^2) - \frac{1}{2} \frac{\partial}{\partial x} (v^2) + v \zeta - \frac{\partial \Psi}{\partial x} + f v
\]

Take the zonal mean of this equation:

\[
\frac{\partial [u]}{\partial t} = [v \zeta] + f [v]
\]

- Zonal mean meridional flux of relative vorticity
- Zonal mean meridional flux of planetary vorticity
**Acceleration of the zonal mean zonal wind**

Previous slide:

\[
\frac{\partial u}{\partial t} = [\nu \zeta] + f[v]
\]

**Figure 1.105.** Zonal mean eastward acceleration, \(\partial [u]/\partial t\) (l.h.s. of equation), at 50°N, at \(\theta=315\) K, over all 12-hour intervals in January 2007 as a function of the zonal mean net isentropic vorticity flux (r.h.s. of equation), averaged over the corresponding 12-hour intervals. The red straight line represents the best linear fit. The black straight line is the “1-to-1 line”. The large red dot indicates the monthly average value in January 2007 at 50°N, at \(\theta=315\) K.
Contribution of eddies to acceleration of the zonal mean zonal wind

Previous slide:
\[ \frac{\partial [u]}{\partial t} = [v \zeta] + f [v] \]

Substitute: \( v = [v] + v^* \) and \( \zeta = [\zeta] + \zeta^* \)

Result is:
\[ \frac{\partial [u]}{\partial t} = [v]([\zeta] + f) + [v^* \zeta^*] \]

**Eddy vorticity flux** (see section 1.26)

**Vorticity flux** due to the mean meridional circulation
Isentropic Flux of PVS at 45°N

\[ \Delta \text{PVS}(t) = \int_{t_0}^{t} [v \xi_a] dt \quad [\text{m s}^{-1}] \]

**FIGURE 1.55 (alternative version):** Running daily mean poleward isentropic vorticity-flux at 350 K and at 45°N, during the Northern Hemisphere winter of 2009-2010 (red lines). The thick red line represents the contribution from the so-called “Ferrel cell”, while the thin red line represents the contribution from eddies or waves. The black line represents the time integrated net flux of PVS after 1 December 2009 (00 UTC). The blue line represents the zonal mean zonal wind, \([u]\), at 60°N. Based on the ERA-Interim reanalysis.

Data from [http://apps.ecmwf.int/datasets/](http://apps.ecmwf.int/datasets/).
Section 1.35

Contribution of eddies to acceleration of the zonal mean zonal wind

Previous slide:

\[
\frac{\partial[u]}{\partial t} = [v]([\zeta] + f) + [v*\zeta*]
\]

**Figure 1.106.** Monthly average zonal mean isentropic vorticity flux and the monthly average zonal mean eddy isentropic vorticity flux at 315 K and 50°N, for all months of January between 1979 and 2017. Red line: best linear fit to the points (correlation coefficient is -0.97). Slope of this line is 1.07, which means that the absolute value of the eddy flux is on average 7% larger than the absolute value of the mean flux. Red square: ensemble average value for all 39 months of January. Blue dot is for January 2010 (extreme negative NAM phase); Red dot is for January 2007 (extreme positive NAM phase).
Contribution of eddies to acceleration of the zonal mean zonal wind

$\frac{\partial [u]}{\partial t} = [v][\zeta] + f + [v^* \zeta^*]$  

Most of the time eddies accelerate the zonal mean zonal flow at 50°N and $\theta=315$ K.
Zonal mean meridional *eddy* vorticity flux

**Figure 1.107.** Monthly mean, zonal mean *eddy* isentropic vorticity-flux as a function of latitude and potential temperature, in January 2007.

Eddies accelerate the zonal mean zonal wind in middle latitudes!!
Zonal mean meridional mean vorticity flux

**Figure 1.107.** Monthly mean, zonal mean mean isentropic vorticity-flux as a function of latitude and potential temperature, in January 2007.

Equatorward flux: deceleration

Equation: $[v][\zeta + f]$

Mean circulation decelerates the zonal mean zonal wind in middle latitudes.
Meridional circulations (horizontal component)

Average (time and zonal) horizontal velocity components \([u]\) (black contours) and \([v]\) (red: northward; blue: southward) in January 1979-2017 (ERA-Interim). Labels indicate the value in units of m s\(^{-1}\).

Upper level meridional component of Ferrel circulation: response to poleward flux of vorticity due to eddies/waves". 
Dynamic Meteorology: lecture 11

Sections 1.36-1.39

Jet streaks (1.36)
Rossby waves (1.37)
Excitation of Rossby waves due to baroclinic instability (1.38)
Baroclinic life cycle: Norwegian cyclone model (1.39)

13/11/2019: tutorial 7
Problems 1.29, 1.30, 1.32 and 1.33

20/11/2019: tutorial 8
Problems 1.34 and 1.35