Dynamic Meteorology: lecture 2

13/9/2019 (Friday)

Sections 1.3-1.5 and Box 1.5

- Potential temperature
- Radiatively determined temperature (boxes 1.1-1.4)
- Buoyancy (-oscillations) and static instability,
- Brunt-Vaisala frequency
- Introduction to project 1 (problem 1.6) (in couples)
- Convective Available Potential Energy (CAPE)

19/9/2018: tutorial

- Problems 1.1, 1.3, 1.4, 1.6 + extra problems in this lecture
- project 1 in couples
Potential temperature, $\theta$

\[
\theta \equiv T \left( \frac{p_{\text{ref}}}{p} \right)^k
\]

\( \kappa \equiv \frac{R}{c_p} \)

\( R = c_p - c_v \)

\[
J \, dt = c_v \, dT + pd\alpha
\]

\[
d\theta = \frac{J}{\Pi}
\]

\[
\Pi \equiv c_p \left( \frac{p}{p_{\text{ref}}} \right)^k
\]

If \( J=0 \) (adiabatic) \( \theta \) is materially conserved!!
Zonal mean and time mean (over the period 1979-2001) stratification of the atmosphere (potential temperature), as deduced from a state of the art data-assimilation system. Labels are in Kelvin (K): potential increases with height!
In an “ideal atmosphere”
(well mixed, one greenhouse gas, Solar radiation absorbed only by earth’s surface):

Equilibrium temperature, determined by radiation (see box 1.4) is

\[ T = \left( \frac{Q}{2\sigma} \left( \frac{Q_a q_a P}{g} + 1 \right) \right)^{1/4} \] (eq. 1.27a)

Problem 1.5 (page 38)

Show that this is a statically stable temperature profile for earth-like values of the parameters \( Q, g, \sigma_a, q_a \) and \( p \)
Vertical motion due to “buoyancy”

\[ p_b - p_t = -dp = \frac{F_p}{dx\,dy} \]

Vertical component of equation of motion:

\[ m \frac{dw}{dt} = -V \frac{\partial p}{\partial z} - mg \]

http://en.wikipedia.org/wiki/Buoyancy
Vertical motion due to “buoyancy”

Vertical component of equation of motion:

\[ m \frac{dw}{dt} = -V \frac{\partial p}{\partial z} - mg \]

Hydrostatic balance:

\[ 0 = -V_0 \frac{\partial p_0}{\partial z} - mg \]

Perturb this equilibrium state so that:

\[ m \frac{dw}{dt} = -(V_0 + V') \frac{\partial (p_0 + p')}{\partial z} - mg \]
Vertical motion due to “buoyancy”

Vertical component of equation of motion:

\[ m \frac{dw}{dt} = -(V_0 + V')(\frac{\partial p_0}{\partial z} + \frac{\partial p'}{\partial z}) - mg = -V_0 \frac{\partial p_0}{\partial z} - V' \frac{\partial p_0}{\partial z} - V \frac{\partial p'}{\partial z} - mg \]

Hydrostatic balance:

\[ 0 = -V_0 \frac{\partial p_0}{\partial z} - mg \]

Pressure distribution on an immersed cube

http://en.wikipedia.org/wiki/Buoyancy
Vertical accelerations

Previous slide:
\[
m \frac{dw}{dt} = -V' \frac{\partial p_0}{\partial z} - V \frac{\partial p'}{\partial z}
\]

\[
\frac{\partial p_0}{\partial z} = -\rho_0 g
\]
(hydrostatic balance)

Air-parcel accelerates in vertical direction due to gravity and due to the pressure gradient

\[
m \frac{dw}{dt} = \rho_0 g V' - V \frac{dp'}{dz}
\]

Source of a sound wave

Archimedes force ("buoyancy")

Weight of the air displaced

Archimedes (287-212BC): “A body or element immersed or floating in a fluid at rest experiences an upward thrust which is equal to the weight of the fluid displaced”

problem 1.1
Vertical accelerations

Previous slide:

\[
m \frac{dw}{dt} = \rho_0 g V' - V \frac{dp'}{d z} \]

\[p' \approx 0\]

\[m \frac{dw}{dt} \approx \rho_0 g V' \]

(vertical motion due to “buoyancy”)

Equation of state:

\[p_0 (V_0 + V') = m R (T_0 + T')\]

\[p_0 V_0 = m R T_0\]

\[p_0 V' = m R T'\]

\[\frac{dw}{dt} \approx g \frac{T'}{T_0} \approx g \frac{\theta'}{\theta_0}\]

Gravity is dynamically important if there are temperature differences
Acceleration due to buoyancy

Vertical acceleration:
\[ \frac{d^2 z}{dt^2} = g \frac{\theta'}{\theta_0} \]

At \( z = z^* \) an **air parcel** has potential temperature, \( \theta = \theta^* \)

Potential temperature of this air parcel is conserved!

Assume that in the **environment**: \( \theta_0 = \theta^* + \frac{d\theta_0}{dz} \delta z \)
Acceleration due to buoyancy

Vertical acceleration: \[
\frac{d^2 z}{dt^2} = g \frac{\theta'}{\theta_0}
\]

At \(z = z^*\) an **air parcel** has potential temperature, \(\theta = \theta^*\)

Potential temperature of this air parcel is conserved!

Assume that in the **environment**: \(\theta_0 = \theta^* + \frac{d\theta_0}{dz} \delta z\)

**Buoyant force** is proportional to

\[
\frac{\theta'}{\theta_0} = \frac{\theta^* - \theta^* - \frac{d\theta_0}{dz} \delta z}{\theta_0} = -\frac{d\theta_0}{dz} \delta z
\]

Therefore

\[
\frac{d^2 \delta z}{dt^2} = -\frac{g}{\theta_0} \frac{d\theta_0}{dz} \delta z
\]
Stability of hydrostatic balance

Repeat:

\[
\frac{d^2 \delta z}{dt^2} = - g \frac{d \theta_0}{\theta_0} \delta z \equiv -N^2 \delta z
\]

Brunt Väisälä-frequency, \( N \):

\[
N^2 \equiv \frac{g}{\theta_0} \frac{d \theta_0}{dz}
\]
Stability of hydrostatic balance

\[
\frac{d^2 \delta z}{dt^2} = -\frac{g}{\theta_0} \frac{d\theta_0}{dz} \delta z \equiv -N^2 \delta z
\]

Brunt Väisälä-frequency, \( N \):

\[ N^2 \equiv \frac{g}{\theta_0} \frac{d\theta_0}{dz} \]

The solution:

\[ \delta z = \exp(\pm iNt) \]

If \( N^2 = \frac{g}{\theta_0} \frac{d\theta_0}{dz} < 0 \) Exponential growth \( \rightarrow \) instability

If \( N^2 = \frac{g}{\theta_0} \frac{d\theta_0}{dz} > 0 \) oscillation \( \rightarrow \) stability
manifestation of static instability

Formation of cumulus clouds

Figure 1.26: (Espy, 1841) (Leslie Bonnema, 2011)
manifestation of static stability

Mountain induced vertical oscillations
Brunt Väisälä frequency

\[ N^2 \equiv \frac{g}{\theta_0} \frac{d\theta_0}{dz} \]

\( N \) is about 0.01-0.02 s\(^{-1}\)

**Extra problem:**

Demonstrate that the Brunt-Väisälä frequency is constant in an isothermal atmosphere.

What is the typical time-period of a buoyancy oscillation in the isothermal lower stratosphere?
Parcel model of a buoyancy oscillation in a stratified environment

Construct a numerical model (in e.g. Python; see next 2 slides), which calculates the vertical position and vertical velocity of a dry air-parcel, which is initially at rest just above the earth’s surface. Initially this air parcel is warmer than its environment. The governing equations are

\[
\frac{dz}{dt} = w; \quad \frac{dw}{dt} = g \frac{\theta'}{\theta_0}; \quad \frac{d\theta}{dt} = 0.
\]

The potential temperature of the environment of the air parcel is

\[
\theta_0 = \theta_0(0) + \Gamma z.
\]

A more complicated environmental potential temperature is prescribed in problem 1.6.
# Python 3: Fibonacci series up to n
>>> def fib(n):
    a, b = 0, 1
    while a < n:
        print(a, end=' ')
        a, b = b, a+b
    print()
>>> fib(1000)
0 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987

Functions Defined

The core of extensible programming is defining functions. Python allows mandatory and optional arguments, keyword arguments, and even arbitrary argument lists. [More about defining functions in Python 3](https://docs.python.org/3/tutorial/errors.html)

Python is a programming language that lets you work quickly and integrate systems more effectively. [Learn More](https://docs.python.org/3/)

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**Get Started**

Whether you’re new to programming or an experienced developer, it’s easy to learn and use Python.

Start with our Beginner’s Guide

**Download**

Python source code and installers are available for download for all versions! Not sure which version to use? Check here.

Latest: Python 3.5.2 - Python 2.7.12
Learn by example

Solution problem 0.1 in Python

See the Python-script *Problem0.1AD.py* on Blackboard (Assignments)

```python
#Problem 0.1 Atmospheric Dynamics

import numpy as np
import matplotlib.pyplot as plt

a = 3.75
len_t = 40
n_len = 2

Y = np.zeros((len_t, n_len))
t = np.zeros((len_t))

for n in range(n_len):
    i = 0
    t[i] = 0
    Y[i, 0] = 0.5  # initial condition 1
    Y[i, 1] = 0.51  # initial condition 2
    for i in range(len_t):
        if i>0:
            Y[i, n] = (a * Y[i-1, n]) - (Y[i-1, n] ** 2)
            t[i] = t[i-1]+1

plt.plot(t, Y[:, 0], label='Y[0]='+str(Y[0,0]), color='blue')
plt.plot(t, Y[:, 1], label='Y[0]='+str(Y[0,1]), color='red')
plt.ylabel('time')
plt.title('Problem 0.1: solution for two slightly different initial conditions')
plt.legend()
plt.text(1, 3.75, 'a=3.75', fontsize=14, color='black')
plt.show()
```
Convective Available Potential Energy (CAPE)

Given by:
\[ \frac{dw}{dt} = g \frac{\theta'}{\theta_0} \equiv Bg \]

Where:
- \( B = \text{buoyancy} \)
- \( B \equiv \frac{\theta'}{\theta_0} \)

Assuming a stationary state and horizontal homogeneity, we can write:
\[ \frac{dw}{dt} \approx w \frac{dw}{dz} = Bg. \]

Or
\[ wdw = Bgdz. \]
Integrate this equation from a level $z_1$ to a level $z_2$. An air parcel, starting its ascent at a level $z_1$ with vertical velocity $w_1$, will have a velocity $w_2$ at a height $z_2$ given by

$$w_2^2 = w_1^2 + 2 \times CAPE,$$

where

$$CAPE = g \int_{z_1}^{z_2} Bdz.$$

What is CAPE in your model problem of **project 1 (problem 1.6)**? What is the associated maximum value of $w$? Does your model reproduce this value?
The relatively sharp downdraughts at the edge of the cumulus cloud are a typical feature of cumulus clouds. What effect is responsible for these downdraughts?

The vertical motion in a **fair weather cumulus cloud** 1.5 km deep, over a track about 250 m below cloud top
Updraughts in cumulus clouds

Upward velocities of 50 m/s!
What value of CAPE is required to get this upward velocity?

The vertical motion in (b) a cumulo-nimbus cloud more than 10 km deep, over a track 6 km above the ground
Vendredi 13 septembre 2019
7:45 locale
Next: Tutorial

Wednesday, 18/9/2019

Problems 1.1, 1.3, 1.4, 1.6 + extra problems of lecture 2

project 1 in couples
Dynamic Meteorology: lecture 3

Sections 1.9-1.11, 1.14-1.15

Moist (cumulus) convection
Relative humidity, mixing ratio
Clausius Clapeyron equation
Dew point (lapse rate)
Lifted condensation level
Equivalent potential temperature

25/9/2019: tutorial

Problems 1.7, 1.8 + extra problems