Moist (cumulus) convection

- Relative humidity, mixing ratio
- Clausius Clapeyron equation
- Dew point (lapse rate)
- Lifted condensation level
- Equivalent potential temperature
- Thermodynamic diagram

Sections 1.9-1.11, 1.14-1.16

25/9/2019: tutorial 2

Problems 1.7, 1.8 + extra problems + exercises of first tutorial
Lectures
Friday 11:00-12:45 in weeks 37-50 of 2019 (except week 45) (14 lectures)

Practical (exercise) sessions
Wednesday 13:15-17:00 in weeks 37-50 of 2019 (except week 45), 2-4 of 2020
Friday 11:00-12:45 in weeks 2-4 of 2020

Exam
First period week 51 (Wednesday, December 18, 2019) (retake: week 5, 2020)

No lectures or practical sessions
On Wednesday, 23/10/2019 and on Friday, 29/11/2019

Excursion KNMI
Not yet set (January 2020)
Atmospheric Circulation and Climate Changes

09.30 Walk in
10.00 Gert-Jan Steeneveld (NVBM): Opening
10.05 Rein Haarsma (KNMI): Circulation change in high resolution global climate models
10.40 Michiel Baatsen (IMAU): Circulation and heat distribution in an extreme greenhouse climate during the Eocene
11.15 Coffee and tea break
11.45 Karin van der Wiel (KNMI): Dynamics of the atmospheric diagonal Convergence Zone over the South Pacific.
12.20 Eveline van der Linden (Wageningen University): The role of circulation changes in European drought projections
12.55 Lunch
14.00 Paul Williams (Reading University): Changing atmospheric circulation patterns: Impacts on flight routes and turbulence
14.35 Paul de Witte (Dutch Air Force): The challenges of climate change for the Dutch Armed Forces
15.10 Coffee and tea break
15.30 Peter Kerkmans (Vattenfall): Impact of weather changes on energy supplies in Europe, now and in the near future
16.05 Closure and Drinks

Date: Friday 29 November 2019
Venue: Utrecht University, Booth room, Heidelberglaan 3, Utrecht (Science Park)
Free for NVBM members and students, Non-members 50 euro
Please register before 24 November 2019 at www.nvbm.nl or bestuur@nvbm.nl
Moist (cumulus) convection

(Espy, 1841)

An air parcel will cool as it ascends. At some level it will become saturated. This level is the **Lifted Condensation Level (LCL)**. This is the level of the cloud base.
Vertical acceleration of an air parcel

Governed by:
\[ \frac{d^2 \delta z}{dt^2} = - \frac{g}{\theta_0} \frac{d\theta_0}{dz} \delta z \equiv -N^2 \delta z \]

Brunt Väisälä-frequency, \( N \):
\[ N^2 \equiv \frac{g}{\theta_0} \frac{d\theta_0}{dz} \]
\( N \) is about 0.01-0.02 s\(^{-1}\)

The solution:
\[ \delta z = \exp(\pm iNt) \]

If
\[ N^2 = \frac{g}{\theta_0} \frac{d\theta_0}{dz} < 0 \]
Exponential growth \( \rightarrow \) instability

If
\[ N^2 = \frac{g}{\theta_0} \frac{d\theta_0}{dz} > 0 \]
oscillation \( \rightarrow \) stability
Dry adiabatic lapse rate

**Hydrostatic stability if**

\[ \frac{d\theta_0}{dz} > 0 \]

Equivalent to

\[ \frac{dT_0}{dz} > -\frac{g}{c_p} \quad \text{(page 22-23)} \]

Dry-adiabatic lapse rate:

\[ \frac{dT_0}{dz} = -\frac{g}{c_p} \quad \text{(see eq. 1.20)} \]
Measurement of the vertical profile of (potential) temperature by radiosonde

De Bilt, 26 Sep. 2013, 12 UTC

Launching a radiosonde at KNMI on 30 November 2012
Will an air parcel at the surface, with a temperature of 16.0°C and a dewpoint temperature of 8.0°C, accelerate upwards spontaneously?

### Measurement of the vertical profile of (potential) temperature by radiosonde

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Up to what approximate height will it accelerate upwards?
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De Bilt, 26 Sep. 2013, 12 UTC

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Up to a height between 1348 m and 1481 m
Will it reach the “Lifted Condensation Level” (LCL), i.e. will clouds form?
Determining the Lifted condensation level (LCL)

The LCL can be determined with information of the *dewpoint temperature*, $T_d$, and the *temperature*, $T$, at the surface…

$T_d$ is the temperature, which an air parcel would have if it were cooled to saturation at constant pressure.

What determines saturation?
Clausius-Clapeyron equation for the water vapour equilibrium pressure, $p_e$ (in equilibrium with the liquid phase):

\[
\frac{\partial \ln p_e}{\partial T} = \frac{L_v}{R_v T^2}
\]

Notation in meteorology: $e_s \equiv p_e$

$L_v$ : latent heat of evaporation/condensation ($=2.5\times10^6$ J kg$^{-1}$).

$R_v$ : specific gas constant for water vapour ($=461.5$ J K$^{-1}$ kg$^{-1}$).

Water vapour equilibrium pressure, $p_e$, as a function of temperature, according to the Clausius-Clapeyron equation, assuming $L_v$ is constant and $e_s=6.1$ hPa at $T=0^\circ$C.
Relative Humidity and dew point temperature

Relative humidity is defined as the ratio,

\[ RH \equiv \frac{e}{e_s(T)} = \frac{e_s(T_d)}{e_s(T)} \]

Water vapour is an ideal gas:

\[ e = \rho_v R_v T \]  
\( e \) is the water vapour pressure

\( \rho_v = \) mass of water vapour per unit volume (vapour density)

\( T_d = \) dew point temperature
Relative humidity in a convective layer

Simulation with cloud model (labels %): $T_s \approx 20^\circ C; \; RH_s \approx 45\%$

**Figure 1.28.** Simulation of the relative humidity in the atmospheric boundary layer at midday in June in The Netherlands. The hatched regions correspond to clouds (regions where the relative humidity is 100%). The cloud cover in this case is about 20% (see problem 1.11 on page 89 of the lecture notes). Relative humidity is reasonably constant at the surface, but not in the cloud layer!! Why?

Van Delden & Oerlemans, 1982
Mixing ratio

Water vapour mixing ratio (definition): \[ r_v \equiv \frac{\rho_v}{\rho_d} \]

Dry air and water vapour as ideal gases:

Dry air: \[ p_d = \rho_d R_d T \]

Water vapour: \[ e = \rho_v R_v T \]

Water vapour mixing ratio: \[ r_v \equiv \frac{\rho_v}{\rho_d} = \frac{e}{R_v} \frac{R_d}{p_d} \equiv \frac{\varepsilon e}{p_d} \approx \varepsilon e \]

with \( \varepsilon \equiv \frac{R_d}{R_v} \)

Dalton’s law: total pressure is sum of partial pressures, i.e. \[ p = p_d + p_e \]

\( p_d >> p_e \)
Substitute $e_s = e$ and $T = T_d$ in the **Clausius Clapeyron equation**:

$$\frac{de}{dT_d} = \frac{Le}{R_v T_d^2} \quad \Rightarrow \quad \frac{de}{dT_d} \approx \frac{L r_v p}{R_v \epsilon T_d^2}$$

(1)

Previous slide:

$$e \approx \frac{r_v p}{\epsilon}$$

(2)

$r_v$ is constant (see problem 1.8, p. 78)!
Substitute \( e_s = e \) and \( T = T_d \) in the **Clausius Clapeyron equation**:

\[
\frac{de}{dT_d} = \frac{Le}{R_v T_d^2}
\]

**Previous slide:**

\[
e \approx \frac{r_v p}{\varepsilon}
\]

\[
\frac{de}{dT_d} \approx \frac{L r_v p}{R_v \varepsilon T_d^2}
\]

\( r_v \) is constant (see problem 1.8, p. 78)!

\[
\frac{de}{dT_d} \approx \frac{r_v}{\varepsilon} \frac{dp}{dT_d}
\]

Eqs. (1) and (2) -> eq. for **“dew point lapse rate”**: 

\[
\frac{dT_d}{dp} = \frac{R_v T_d^2}{L p}
\]

With the **hydrostatic equation**:

\[
\frac{dT_d}{dz} = - \frac{\rho g R_v T_d^2}{L p} \approx - \frac{g R_v T_d^2}{LR_d T} \approx - \frac{g T_d}{L \varepsilon}
\]
Dew point lapse rate:  \( \Gamma_{\text{dew}} \equiv -\frac{dT_d}{dz} \)

Previous slide:

\[
\frac{dT_d}{dz} \approx -\frac{gT_d}{L\varepsilon} \approx -\frac{10 \times 285}{2.5 \times 10^6 \times 0.62} \approx -0.0018 \text{ K m}^{-1}
\]

Derive this expression for the dry-adiabatic lapse rate:

\[ \Gamma_d \equiv \frac{g}{c_p} \]
Dew point lapse rate

\[ \frac{dT_d}{dz} \equiv -\Gamma_{dew} \approx \frac{gT_d}{L\varepsilon} \]

**Extra problem**

When is

\[ -\frac{dT_d}{dz} \equiv \Gamma_{dew} > \Gamma_d? \]

Investigate the consequences for cloud formation, precipitation and global water cycle if this is indeed the case.

**Figure 1.27**

- **LCL**
- Moist adiabatic Lapse rate (\(\Gamma_d \sim 4-6\) K/km)
- Dry adiabatic Lapse rate (\(\Gamma_d \sim 9.8\) K/km)
- Dew point Lapse rate (\(\Gamma_{dew} \sim 1.8\) K/km)
Determine the lifted condensation level (LCL). Will clouds form?
The lifting condensation level is found by solving:

\[
T_s - \Gamma_d z_{LCL} = T_{ds} - \Gamma_{dew} z_{LCL}
\]

\[
16.0 - \left(\frac{g}{c_p} z_{LCL}\right) = 8.0 - \left(\frac{1.8}{10^3} z_{LCL}\right)
\]

\[
z_{LCL} = 1 \text{ km}
\]

Yes: clouds will form!
What happens above the LCL?

If the air parcel continues its ascent after reaching the LCL, condensation of water vapour will occur, which will be accompanied by release of latent heat.

The rate of heating due to condensation is $mJ$ ($m$ is mass of air parcel):

$[\text{Joules per second}]$

\[ mJ = -L \frac{dm_v}{dt} \quad \Rightarrow \quad J \approx -L \frac{dr_s}{dt} \]

$r_s$ is saturation mixing ratio

$L (=2.5 \times 10^6 \text{ J kg}^{-1})$ is the latent heat of condensation

$m_v$ = the mass of water vapour in the air parcel

$J$ is the heat transfer to the air parcel per unit mass, per unit time
Latent heat release only in the updraught!

The rate of heating due to condensation is $mJ$ ($m$ is mass of air parcel):

$$mJ = -L \frac{dm_v}{dt}$$

$$J = -L \frac{dr_s}{dt}$$

Change in $r_s$ following the motion is primarily due to ascent (stationary state):

$$\frac{dr_s}{dt} \equiv w \frac{dr_s}{dz} \text{ for } w > 0; \quad \longrightarrow \quad J = -Lw \frac{dr_s}{dz}$$

$$\frac{dr_s}{dt} \equiv 0 \text{ for } w \leq 0.$$
Saturated ascent and unsaturated decent

Assume that $\theta = \theta_0(z) + \theta'$, with $\theta' \ll \theta_0$. Then:

$$\frac{d\theta}{dt} = \frac{d\theta'}{dt} + w \frac{d\theta_0}{dz} = \frac{J}{\Pi}$$

$J=0$ if $w<0$: 

$$-N^2 \equiv \frac{g}{\theta_0} \frac{d\theta_0}{dz} \Rightarrow \frac{d\theta'}{dt} = -\frac{\theta_0}{g} N^2 w \text{ if } w \leq 0;$$

$J>0$ if $w>0$ and air is saturated:

$$\frac{d\theta'}{dt} \approx -\frac{\theta_0}{g} N_m^2 w \text{ if } w > 0,$$
Saturated ascent and unsaturated decent

Assume that $\theta = \theta_0(z) + \theta'$, with $\theta' << \theta_0$. Then:

$$\frac{d\theta}{dt} = \frac{d\theta'}{dt} + w \frac{d\theta_0}{dz} = \frac{J}{\Pi}$$

$J = 0$ if $w < 0$:

$$-N^2 \equiv g \frac{d\theta_0}{dz}$$

$\frac{d\theta'}{dt} = -\frac{\theta_0}{g} N^2 w \text{ if } w \leq 0$;

$J > 0$ if $w > 0$ and air is saturated:

$$\frac{d\theta'}{dt} \approx -\frac{\theta_0}{g} N_m^2 w \text{ if } w > 0,$$

$$J = -L \frac{dr_s}{dz} w \text{ if } w > 0$$

$$N_m^2 \equiv N^2 + \frac{gL}{\theta_0 \Pi_0} \frac{dr_s}{dz},$$

$N_m$ is "moist" Brunt Väisälä frequency
Conditional instability

Assume that \( \theta = \theta_0(z) + \theta' \), with \( \theta' \ll \theta_0 \). Then:

\[
\frac{d\theta}{dt} \approx \frac{d\theta'}{dt} + w \frac{d\theta_0}{dz} = \frac{J}{\Pi}.
\]

\[
\begin{align*}
\frac{d\theta'}{dt} &= \frac{-\theta_0}{g} N^2 w \text{ if } w \leq 0; \\
\frac{d\theta'}{dt} &\approx \frac{-\theta_0}{g} N_m^2 w \text{ if } w > 0,
\end{align*}
\]

\[
N_m^2 \equiv N^2 + \frac{gL}{\theta_0 \Pi_0} \frac{dr_s}{dz},
\]

\( N_m \) is the "moist" Brunt Väisälä frequency

Frequently: \( N_m^2 < 0 \) and \( N^2 > 0 \). In these circumstances the atmosphere is statically or buoyantly unstable only with respect to saturated upward motion. This is called conditional instability.
Define a pseudo- or moist adiabatic process in which a so-called “equivalent potential temperature”, $\theta_e$, is constant, following saturated ascent.

By analogy simply define:

$$N_m^2 = g \frac{d\theta_e}{\theta_e} dz$$

then

$$\theta_e \approx \theta \exp \left( \frac{L r_s}{\theta \Pi} \right)$$ (eq. 1.96a)
Equivalent potential temperature

Section 1.14

For an saturated air parcel:

\[ \theta_e \approx \theta \exp \left( \frac{Lr_s}{\theta \Pi} \right) = \theta \exp \left( \frac{Lr_s}{c_p T} \right) \]

For an unsaturated air parcel:

\[ \theta_e \approx \theta \exp \left( \frac{Lr}{c_p T \text{LCL}} \right) \]

(LCL: lifting condensation level)

approximately conserved!
Potential Instability:
\[
\frac{\partial \theta_e}{\partial z} < 0
\]

De Bilt, 26 Sep. 2013, 12 UTC

### Potential Instability (PI) up to 650 hPa (3600 m)

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<th>DWPT °C</th>
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<th>MIXR g/kg</th>
<th>DRCT deg</th>
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constant $\theta$

constant pressure

constant $\theta_e$

$\theta_e \approx \theta \exp\left(\frac{L_r s}{\theta \Pi}\right)$

constant saturation mixing ratio

$r_s = \frac{R_d e_s}{R_v p}$

isotherm

http://www.staff.science.uu.nl/~delde102/tephigram.pdf

“Tephigram”

see Figure 1.29, p.76
Tropical cyclone “Nadine”: warm/moist core
This weekend: high pressure > two beautiful days
Including the influence of Earth’s rotation (Coriolis effect)
Material derivative in a rotating coordinate system on a sphere
The influence of the Coriolis force on vertical circulations
The sea breeze (a second mode of convection)
Inertial frequency: second “eigen-frequency” of the atmosphere

HAND IN Project 1 (problem 1.6) (in couples)
(deadline: end of Wednesday, 25 September 2019)

Sections 1.6, 1.7 and 1.17

25/9/2018: tutorial 2

Problems 1.7, 1.8 + extra problems
+ weather forecast