PV-θ – View* of Diabatic-Dynamical Interaction in the General Circulation

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* Inspired by Hoskins, Tellus, 1991
This talk:

Is about the zonal mean potential vorticity (PV-) distribution, the associated jets and how diabatic processes (radiation and latent heat release) interact with adiabatic dynamical processes (heat and momentum transport) to produce this PV-distribution.

\[ PV \equiv Z \equiv \frac{\zeta_\theta + f}{\sigma} \quad [m^2 K s^{-1} kg^{-1}] \quad 10^{-6} \ m^2 K s^{-1} kg^{-1} \equiv 1 \ PVU \]

isentropic density

relative vorticity on an isentropic surface
Other variables playing a central role

*Potential temperature* \((\theta)\) *is the vertical coordinate*

\[
\theta \equiv T \left( \frac{p_{\text{ref}}}{p} \right)^{\frac{R}{c_p}} \quad [\text{K}]
\]

*Isentropic density:*

\[
\sigma = -\frac{1}{g} \frac{\partial p}{\partial \theta} = \frac{\rho \partial z}{\partial \theta} \quad [\text{kg m}^{-2}\text{K}^{-1}]
\]
This talk

**Observed** hemispheric mean isentropic density profile

Isentropic density and cross-isentropic flow determined by radiation

**Observed** zonal mean isentropic density and cross-isentropic flow

Zonal mean $PV$-anomalies in the upper troposphere and lower stratosphere

Link between zonal mean zonal wind and zonal mean $PV$-anomalies ($PV$-inversion)

**Simulations** revealing the mechanisms that establish the zonal mean $PV$-distribution and associated jets
Observed mean isentropic density profile

Average isentropic density in N. hemisphere poleward of 10° latitude

CIRA (COSPAR International Reference Atmosphere)

Observed mean isentropic density profile

Average (10°N-NP) at each isentropic level

stratosphere

|Z|: large

troposphere

|Z|: small

\( \sigma_{\text{ref}}(\theta) \)

\( \sigma_{\text{ref}} \) [kg m\(^{-2}\) K\(^{-1}\)]

Potential temperature [K]

0 50 100 150 200 250

July

January

σ_{\text{ref}} [kg m\(^{-2}\) K\(^{-1}\)]
Theoretical radiative equilibrium isentropic density

homogeneous “grey” atmosphere

One well-mixed greenhouse gas and no atmospheric absorption of Solar radiation
Theoretical radiative equilibrium isentropic density

homogeneous “grey” atmosphere

One well-mixed greenhouse gas and no atmospheric absorption of Solar radiation

In reasonable agreement with observations!
Potential temperature: January 15

(in a “grey”, homogeneous atmosphere)

Radiative equilibrium

Radiatively “determined”

Unit: K
Observed potential temperature: January

Tropical Middleworld is inflated with mass !!

* Hoskins, Tellus 1991
Cross-isentropic flow, determined by radiation

(in a “grey”, homogeneous atmosphere)

\[ d\theta/dt \quad \text{Unit: K day}^{-1} \]

Cooling in winter hemisphere
Heating in summer hemisphere
Tropics in radiative equilibrium

\[ C=10^6 \text{ J K}^{-1} \text{ m}^2 \]

January 15
Cross-isentropic flow, determined by radiation (in a “grey”, homogeneous atmosphere)

\[ \frac{d\theta}{dt} \]
Unit: K day\(^{-1}\)

Cooling in winter hemisphere
Heating in summer hemisphere
Tropics in radiative equilibrium

In reality (see next slide):

Cooling in winter hemisphere
Cooling in summer hemisphere below 850 K (10 hPa)
Heating in tropics
Cross-isentropic flow, observed

Earth's surface

DIABATIC HEATING
(radiation)

DIABATIC COOLING
(radiation)

Middleworld

> 2.5 K day⁻¹

Thanks to Paul Berrisford
\[ \sigma = -\frac{1}{g} \frac{\partial p}{\partial \theta} \]

Tropical Middleworld is inflated with mass!!

\(\sigma: \text{small}\)  \(\sigma: \text{large}\)

Middleworld

Earth’s surface

\(g\)
Tropical Middleworld is inflated with mass!!

\[ Z = \frac{\xi_\theta + f}{\sigma} \]

\( \sigma \): small

\( Z \): large

\( \sigma \): large

Middleworld

\( Z \): small

\( Z \): large

500 hPa

100 hPa

200 hPa

100 hPa

500 hPa

90°N

60°N

30°N

90°S

January

CIRA
Enhanced slope of dynamical tropopause

Cold point tropical tropopause

Middleworld

"dynamical tropopause"
What mechanisms lead to these meridional gradients in mass & $PV$?

Cold point tropical tropopause

$T=202\, K$

$Z=2\, PVU$

$Z=-2\, PVU$

$|Z|: \text{ larger}$

enhanced meridional slope of the “dynamical tropopause”
"potential vorticity substance" (PVS)

\[ PV \equiv Z \equiv \frac{PVS}{\sigma} \]

is the "mixing ratio" of PVS

Impermeability theorem for PVS *:

**cross-isentropic flux of PVS=0**

i.e. no flux of PVS across isentropic surfaces!

Cross-isentropic mass flux:

\[ I_d \equiv \frac{d\theta}{dt} \sigma \]

\( \partial I_d / \partial \theta > 0 \) leads to concentration of PVS (decrease of \( \sigma \)), therefore increase of PV

(extratropics)

\( \partial I_d / \partial \theta < 0 \) leads to dilution of PVS (increase of \( \sigma \)), therefore decrease of PV

(tropics)
Cross-isentropic mass flux divergence in the Middleworld

\[ I_d \equiv \frac{d\theta}{\sigma} \]

\[ \sigma \approx 10 \text{ kg m}^{-2}\text{K}^{-1} \]

\[ \sigma \approx 100 \text{ kg m}^{-2}\text{K}^{-1} \]

\[ \frac{\partial I_d}{\partial \theta} < 0 \]

Dilution of PVS

Middleworld

Mass in

Mass out

Earth’s surface

Latitude, \( \phi \)
Cross-isentropic mass flux divergence in the Middleworld

Potential temperature, $\Theta [K]$

Latitude, $\phi$

Mass out

Dilution of PVS

Concentration of PVS

Earth's surface

January ERA40 & CIRA

500 hPa

200 hPa

100 hPa

50 hPa

200 hPa

500 hPa
Cross-isentropic mass flux divergence in the Middleworld

potential temperature, $\Theta [K]$

Concentration of PVS
Dilution of PVS
Concentration of PVS

Earth’s surface

latitude, $\phi$
equator
Cross-isentropic mass flux divergence in the Middleworld

\[ \theta = 315 - 370 \text{ K} \]

**ERA40 & CIRA**

- **PV is enhanced**
- **PV is reduced**

Unit: kg m\(^{-2}\) day\(^{-1}\)

** weak diabatic mass-divergence**

** weak diabatic mass-divergence**
Observed zonal mean *PV*-distribution

Define:

**Reference state**

\[ Z = Z_{\text{ref}} \equiv \frac{f}{\sigma_{\text{ref}}} \]

**Reference state + anomaly**

\[ Z = Z_{\text{ref}} + Z' \]

**Non-dimensional *PV*-anomaly**

\[ Z^* = \frac{Z'}{Z_{\text{ref}}} \]
**Analysed [PV] anomaly**

January n.h.

**Analysed positive and negative PV-anomalies**

\[ Z^* = \frac{Z'}{Z_{\text{ref}}} \]

Unit: [PV]: non-dimensional

[u]: m s\(^{-1}\)

January (CIRA)
Analysed $[PV]$ anomaly

**January n.h.**

Analysed *positive* and *negative* $PV$-anomalies

\[
Z^* = \frac{Z'}{Z_{\text{ref}}}
\]

**Unit:** $[PV]$: non-dimensional

$[u]$: m s$^{-1}$

*Stratospheric winter positive PV-anomaly*

*Surf zone*

*Ex-UTLS positive PV-anomaly*
Now I will show, by piecewise PV-inversion, that the Ex-UTLS PV-anomaly induces the subtropical jet and westerly flow in the troposphere and lower stratosphere
Relation $PV ([Z])$ and zonal wind, $[u]$

Zonal mean potential vorticity ($PV$) inversion equation looks like this:

$$\frac{\partial^2 [u]}{\partial y^2} + A \frac{\partial^2 [u]}{\partial \theta^2} = \frac{df}{dy} - \sigma \frac{\partial [Z]}{\partial y}$$

r.h.s. represents the “forcing”. If “forcing”=0 and homogeneous boundary conditions*, then solution of this elliptic equation is $[u]=0$ (rest state).

This is the case if

$$[Z] = Z_{\text{ref}} \equiv \frac{\sigma_{\text{ref}}}{f}$$

Is associated with the state of rest!

$PV$-anomalies induce the full zonal wind!!

*(no thermal wind at lower boundary!)*
Analysed [PV] anomaly

January n.h.

Analysed positive PV-anomalies

\[ Z^* = \frac{Z'}{Z_{\text{ref}}} \]

Unit: [PV]: non-dimensional

Januray (CIRA)
**Analysed** $[u]$

**January n.h.**

**Analysed positive** PV-anomalies and $[u]$ (zonal mean zonal wind)

Unit: $[PV]$: non-dimensional
$[u]$: m s$^{-1}$

$$Z^* = \frac{Z'}{Z_{ref}}$$
Inverted \([u]\)

January n.h.

Inverted zonal mean zonal wind velocity [m/s] and analysed positive PV-anomalies

Unit: \([PV]\): non-dimensional
\([u]\): m s\(^{-1}\)

\[Z^* = \frac{Z'}{Z_{ref}}\]
**Piecewise inverted** \([u]\)

**January n.h.**

**Inverted** zonal mean zonal wind velocity if \(PV\)-anomaly below \(\theta=\Theta_0\) and lower boundary temperature anomaly are removed.

Unit: \([PV]\): non-dimensional 
\([u]\): m s\(^{-1}\)

\(\Theta_0=480\) K

All anomalies removed
**Piecewise inverted \([u]\)**

**January n.h.**

*Inverted* zonal mean zonal wind velocity if *PV*-anomaly above \(\theta=\Theta_0\) is removed.

Unit: \([PV]\): non-dimensional

\([u]\): m s\(^{-1}\)

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**Potential temperature [K]**

**Latitude [°N]**

\(\Theta_0=480\text{ K}\)

All anomalies removed
Ex-UTLS PV-anomaly induces the subtropical jet and the surface westerlies

What processes establish the Ex-UTLS PV-anomaly?

There is more to it than diabatic convergence or divergence of mass in the Middleworld,
Basic question is:

Why is the cross-isentropic flow upward in the tropics and downward in the extratropics?

2D (lat, height) primitive equation model

Contains parametrizations of

1 Radiation
2 Dry convective adjustment
3 “Wave drag”
4 Water cycle (latent heat release)
Permanent equinox steady state: *no drag; no water*

Legend:

\[\theta\]: K

\[u\]: m s\(^{-1}\)

Dynamical tropopause
Permanent equinox steady state: \textit{drag}; no water

Radiation + adiabatic dynamics

Legend

$[\theta]$: K
$[u]$: m s$^{-1}$
$[\text{drag}]$: 10$^{-5}$ m s$^{-2}$

Wave drag between 10 and 25 km a.s.l. and only if $[u]>0$. 
Permanent equinox steady state: *drag + water*

Add: water cycle (latent heat release)

**Legend**

- \[ \theta \]: K
- \[ u \]: m s\(^{-1}\)
- \[ \text{drag} \]: 10\(^{-5}\) m s\(^{-2}\)

Wave drag between 10 and 25 km a.s.l. and only if \[ u \] > 0.

Heating due to latent heat release

**Legend**

- Subtropical jet
- Stratospheric jet
- "wave drag"

**Units**

- \[ \text{day} \] = 1095.0
- \[ \text{run} \] = 1f
Conclusion

All features of the zonal mean general circulation are interconnected!

The zonal mean general circulation, including the subtropical jets, the cold-point tropical tropopause and the strong meridional gradient of the dynamical tropopause, is a product a complex interaction between radiation, latent heat release and heat transport by the circulation.

“Wave drag” (in fact PV-mixing ! ) plays a crucial role
Thank you

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References

A.J. van Delden and Y.B.L. Hinssen, 2012: PV-theta view of the zonal mean state of the atmosphere. *Tellus A* 2012, 64, 18710, [http://dx.doi.org/10.3402/tellusa.v64i0.18710](http://dx.doi.org/10.3402/tellusa.v64i0.18710)

A.J. van Delden, 2014: PV-theta view of Diabatic-Dynamical Interaction in the General Circulation. Accepted for publication in *Tellus A (18 October 2014)*

[http://www.staff.science.uu.nl/~delde102/publications.htm](http://www.staff.science.uu.nl/~delde102/publications.htm)