Aarnout van Delden http://www.staff.science.uu.nl/~delde102/C&HC.htm

Diabatic-Dynamical Interaction in the General Circulation (lecture 5)

Zonal mean meridional circulations (Hadley, Ferrel)

Wave-zonal mean flow interaction

Eliassen-Palm flux

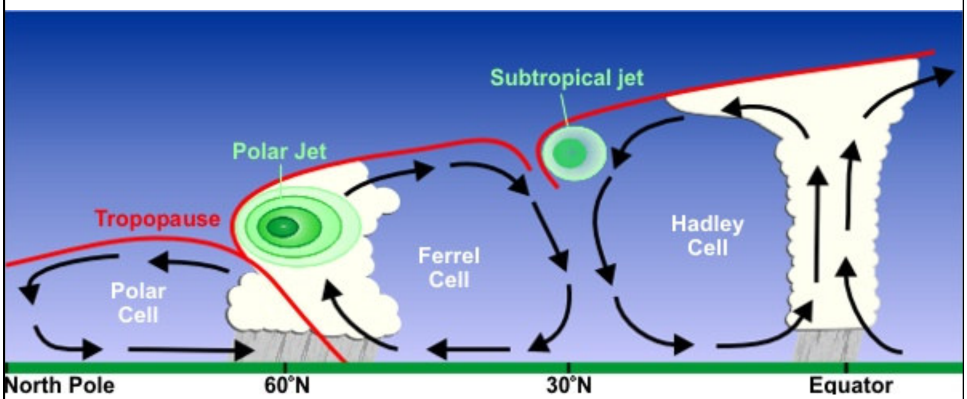
Residual circulation (Brewer-Dobson circulation)

Parametrization of wave drag in the zonal mean pe-model

Results of model simulations including wave drag

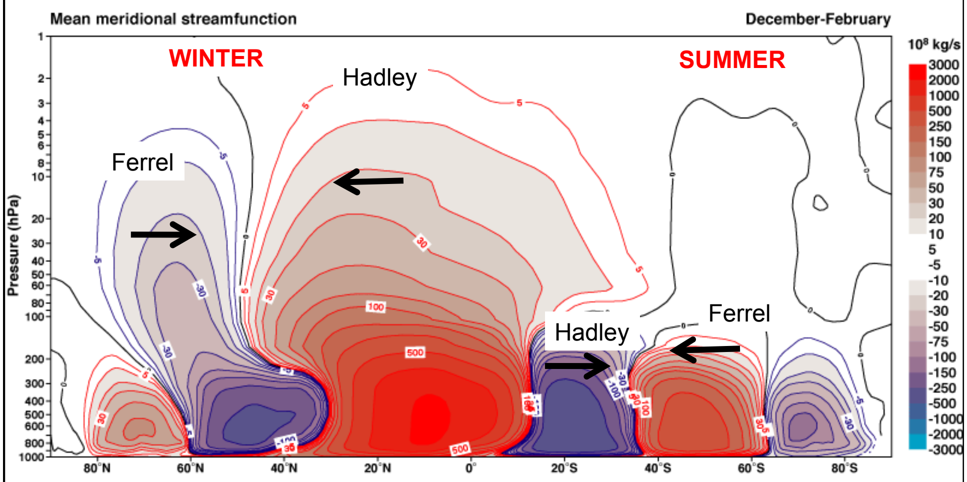
Meridional circulations

Cross section of the subtropical and polar jet streams by latitude.
(http://en.wikipedia.org/wiki/Jet_stream)

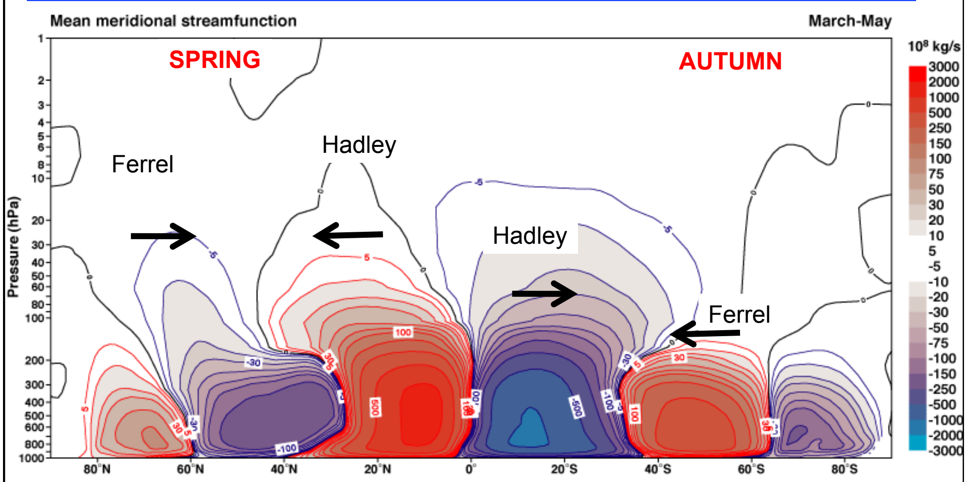


Hadley cell: driven by diabatic heating.
Ferrel cell: driven by eddy fluxes of heat and momentum!

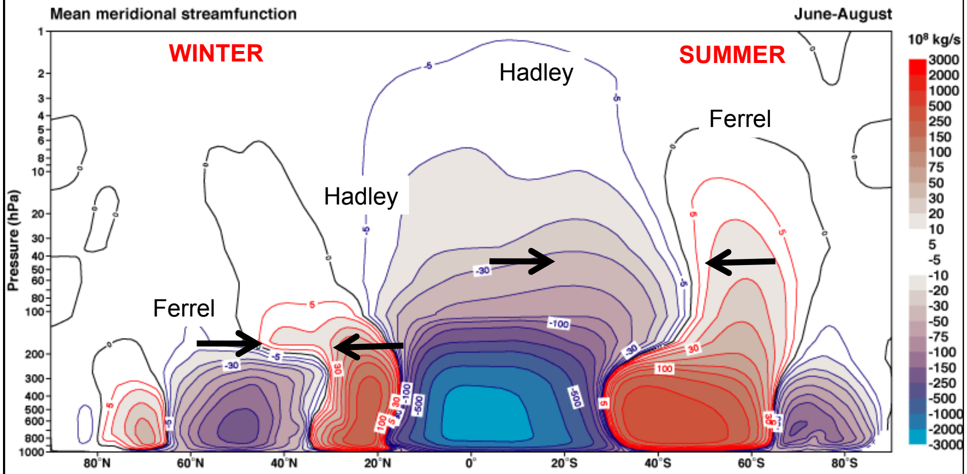
Meridional circulations according to ERA-40



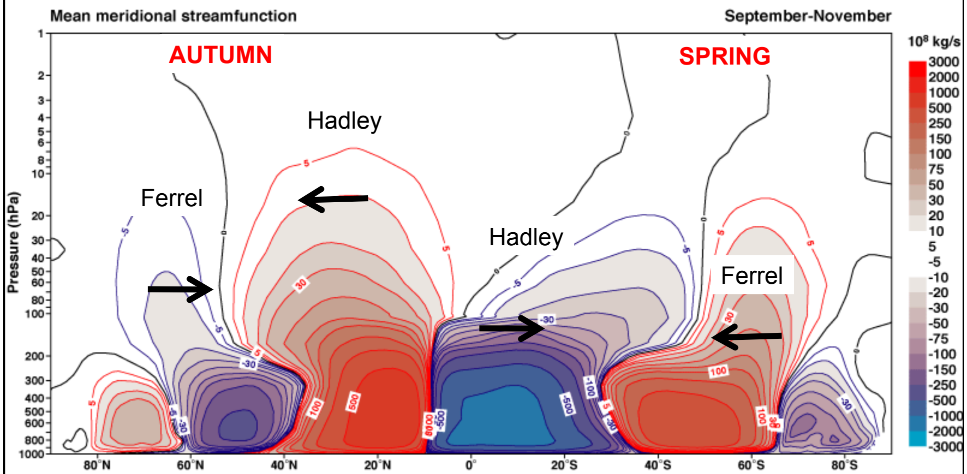
Meridional circulations according to ERA-40



Meridional circulations according to ERA-40



Meridional circulations according to ERA-40

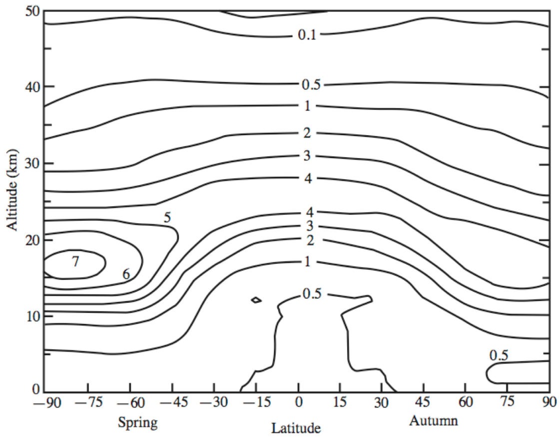


Eulerian/Lagrangian circulations

Hadley and Ferrel circulations are “Eulerian circulations”. Supperposed on these circulations there exist eddies, which also transport mass, heat and momentum. The resulting “Lagrangian circulation” is called the “residual circulation”. This is rather abstract concept. Its manifestation in reality is the zonal mean distribution of ozone and water vapour.

FIGURE 1.23

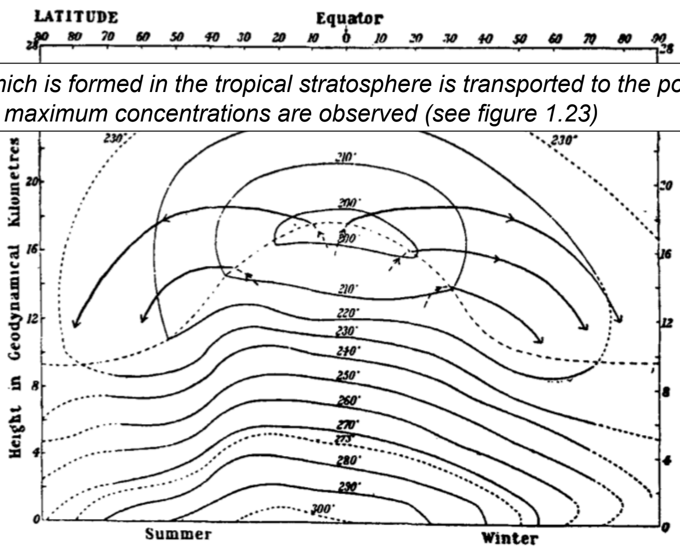
Zonal mean distribution of ozone molecule number density (10^{18} m^{-3}) at the equinox (22 September), based on measurements taken in the 1960’s



Residual circulation: transports mass

FIGURE 11.14

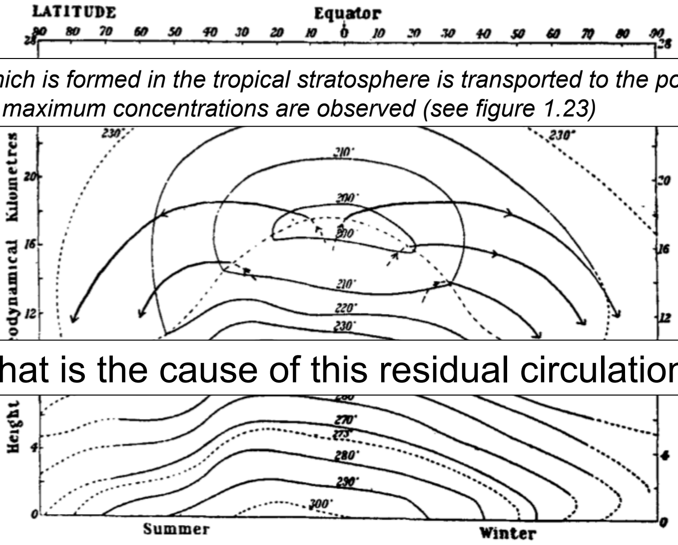
Ozone, which is formed in the tropical stratosphere is transported to the poles, where the maximum concentrations are observed (see figure 1.23)



Brewer’s (1949) original figure, illustrating what came to be known as the Brewer-Dobson circulation. “A supply of dry air is maintained by a slow mean circulation from the equatorial tropopause”. The contours represent isotherms, labeled in units of K.

Residual circulation: transports mass

FIGURE 11.14



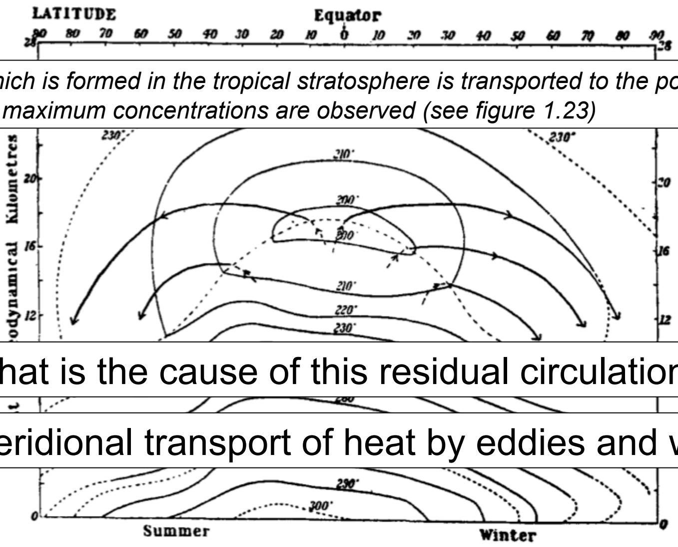
Ozone, which is formed in the tropical stratosphere is transported to the poles, where the maximum concentrations are observed (see figure 1.23)

What is the cause of this residual circulation?

Brewer's (1949) original figure, illustrating what came to be known as the Brewer-Dobson circulation. "A supply of dry air is maintained by a slow mean circulation from the equatorial tropopause". The contours represent isotherms, labeled in units of K.

Residual circulation: transports mass

FIGURE 11.14



Ozone, which is formed in the tropical stratosphere is transported to the poles, where the maximum concentrations are observed (see figure 1.23)

What is the cause of this residual circulation?

This is meridional transport of heat by eddies and waves

Brewer's (1949) original figure, illustrating what came to be known as the Brewer-Dobson circulation. "A supply of dry air is maintained by a slow mean circulation from the equatorial tropopause". The contours represent isotherms, labeled in units of K.

Transport of heat and momentum by eddies

Transport equation

(Section 11.4)

Transport equation for a scalar Q in pressure coordinates is

$$\frac{\partial Q}{\partial t}+u\frac{\partial Q}{\partial x}+v\frac{\partial Q}{\partial y}+\omega\frac{\partial Q}{\partial p}=S$$

$$\omega\equiv\frac{dp}{dt}$$

S=source/sink of Q

Continuity equation multiplied by Q is

$$Q\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \omega}{\partial p}\right)=0$$

Add these equations yields "*flux-form*":

$$\frac{\partial Q}{\partial t}+\frac{\partial uQ}{\partial x}+\frac{\partial vQ}{\partial y}+\frac{\partial \omega Q}{\partial p}=S$$

$$\left(\frac{\partial uQ}{\partial x}+\frac{\partial vQ}{\partial y}+\frac{\partial \omega Q}{\partial p}\right)$$

=flux-divergence of Q

Taking account of the flux of a quantity Q due to the eddies

Zonal average flux-divergence

$$\frac{\partial Q}{\partial t}+\frac{\partial uQ}{\partial x}+\frac{\partial vQ}{\partial y}+\frac{\partial \omega Q}{\partial p}=S$$

Take zonal average of this equation*

$$\frac{\partial [Q]}{\partial t}+\frac{\partial [vQ]}{\partial y}+\frac{\partial [\omega Q]}{\partial p}=[S]$$

* $\left[\frac{\partial Q}{\partial t}\right]=\frac{\partial [Q]}{\partial t}$ etc.

$\left[\frac{\partial Q}{\partial x}\right]=0$

The zonal average of the meridional flux of a quantity Q is (sections 1.39 &11.4)

$$[vQ]=[v][Q]+[v^*Q^*]$$

Therefore, the zonal average flux divergence equation becomes

$$\frac{\partial [Q]}{\partial t}+\frac{\partial [v][Q]}{\partial y}+\frac{\partial [v^*Q^*]}{\partial y}+\frac{\partial [\omega][Q]}{\partial p}+\frac{\partial [\omega^*Q^*]}{\partial p}=[S]$$

$$\frac{\partial[Q]}{\partial t}+\frac{\partial[v][Q]}{\partial y}+\frac{\partial[v^*Q^*]}{\partial y}+\frac{\partial[\omega][Q]}{\partial p}+\frac{\partial[\omega^*Q^*]}{\partial p}=[S]$$

Derivation of the equation for the zonal mean meridional circulation

The zonally averaged continuity equation is reads: $\frac{\partial[v]}{\partial y}+\frac{\partial[\omega]}{\partial p}=0$

Using this, we can write the transport equation as

$$\frac{\partial[Q]}{\partial t}+[v]\frac{\partial[Q]}{\partial y}+[\omega]\frac{\partial[Q]}{\partial p}+\frac{\partial[v^*Q^*]}{\partial y}+\frac{\partial[\omega^*Q^*]}{\partial p}=[S]$$

Advection by mean flow

Eddy-flux divergence

Therefore, **zonal average** of: $\frac{dQ}{dt}=S$

$$\frac{\partial[Q]}{\partial t}+[v]\frac{\partial[Q]}{\partial y}+[\omega]\frac{\partial[Q]}{\partial p}+\frac{\partial[v^*Q^*]}{\partial y}+\frac{\partial[\omega^*Q^*]}{\partial p}=[S]$$

"Apply" this equation to the following two equations.

Equation of motion in pressure coordinates (x-component):

$$\frac{du}{dt}=-\frac{\partial\Phi}{\partial x}+fv+F_x$$

Box 9.1

Gradient of geopotential

Coriolis

Friction

Energy conservation equation:

$$\frac{dT}{dt}=\frac{RT}{c_p p}\omega+\frac{J}{c_p}$$

Box 9.1

Adiabatic heating/cooling

Diabatic heating/cooling

(We have neglected the metric terms, associated with the curvature of the Earth)

We have:

$$\frac{dQ}{dt} = S \quad \longrightarrow \quad \frac{\partial [Q]}{\partial t} + [v]\frac{\partial [Q]}{\partial y} + [\omega]\frac{\partial [Q]}{\partial p} + \frac{\partial [v^* Q^*]}{\partial y} + \frac{\partial [\omega^* Q^*]}{\partial p} = [S]$$

Apply this equation to the following two equations

$$\frac{du}{dt} = -\frac{\partial \Phi}{\partial x} + fv + F_x$$

$$\longrightarrow \frac{\partial [u]}{\partial t} + [v]\frac{\partial [u]}{\partial y} + [\omega]\frac{\partial [u]}{\partial p} + \frac{\partial [v^* u^*]}{\partial y} + \frac{\partial [\omega^* u^*]}{\partial p} = +f[v] + [F_x]$$

$$\frac{dT}{dt} = \frac{RT}{c_p p}\omega + \frac{J}{c_p}$$

$$\longrightarrow \frac{\partial [T]}{\partial t} + [v]\frac{\partial [T]}{\partial y} + [\omega]\frac{\partial [T]}{\partial p} + \frac{\partial [v^* T^*]}{\partial y} + \frac{\partial [\omega^* T^*]}{\partial p} = +\frac{\kappa T}{p}[\omega] + \frac{[J]}{c_p}$$

For the extra-tropics we make the following approximations:

$$[v] = [v_g] + [v_a] = \left[\frac{1}{f}\frac{\partial \phi}{\partial x}\right] + [v_a] = [v_a]$$

Subscript 'g' stands for geostrophic

Subscript 'a' stands for ageostrophic

$$[\omega]\frac{\partial [u]}{\partial p} << [v]\frac{\partial [u]}{\partial y} \qquad \frac{\partial [\omega^* u^*]}{\partial p} << \frac{\partial [v^* u^*]}{\partial y} \qquad \frac{\partial [u]}{\partial y} << f \approx f_0$$

↑ "reference value"

(Section 11.5)

With this the zonal mean x-momentum equation,

$$\frac{\partial [u]}{\partial t} + [v]\frac{\partial [u]}{\partial y} + [\omega]\frac{\partial [u]}{\partial p} + \frac{\partial [v^* u^*]}{\partial y} + \frac{\partial [\omega^* u^*]}{\partial p} = +f[v] + [F_x]$$

becomes

$$\frac{\partial [u]}{\partial t} = f_0[v_a] - \frac{\partial [v^* u^*]}{\partial y} + [F_x]$$

Similarly we can also simplify the temperature equation to:

(Section 11.5)

$$\frac{\partial [T]}{\partial t} = \left(\frac{\kappa T}{p} - \frac{\partial [T]}{\partial p} \right) [\omega] - \frac{\partial [v^* T^*]}{\partial y} + \frac{[J]}{c_p}$$

or:

$$\frac{\partial [T]}{\partial t} = S_p [\omega] - \frac{\partial [v^* T^*]}{\partial y} + \frac{[J]}{c_p}$$

(Section 11.5)

Wave-Mean Flow Interaction

Simplified (linear) quasi-geostrophic equations, describing the interaction between the zonal mean state and the waves:

$$\frac{\partial [u]}{\partial t} = f_0 [v_a] - \frac{\partial [v^* u^*]}{\partial y}$$

$$\frac{\partial [T]}{\partial t} = S_p [\omega] - \frac{\partial [v^* T^*]}{\partial y} + \frac{[J]}{c_p}$$

(Section 11.6)

Simplification: Transformed Eurlerian Mean equations

Introduce the following "**residual velocities**"

$$[\omega]_r \equiv [\omega] - \frac{\partial}{\partial y}\frac{[v^*T^*]}{S_p} \qquad [v_a]_r \equiv [v_a] + \frac{\partial}{\partial p}\frac{[v^*T^*]}{S_p}$$

$[v_a]_r$ and $[\omega]_r$, can be interpreted as "physically consistent" velocity components, because they satisfy the following "continuity equation", which is analogous to the "real" continuity equation in pressure coordinates:

$$\frac{\partial[v_a]_r}{\partial y} + \frac{\partial[\omega]_r}{\partial p} = 0$$

Transformed Eurlerian Mean (TEM) equations

Substitution of the definitions of the residual velocities, $[v_a]_r$ and $[\omega]_r$, into

$$\frac{\partial[u]}{\partial t} = f_0[v_a] - \frac{\partial[v^*u^*]}{\partial y} \qquad \frac{\partial[T]}{\partial t} = S_p[\omega] - \frac{\partial[v^*T^*]}{\partial y} + \frac{[J]}{c_p}$$

Yields the TEM-equations

$$\frac{\partial[u]}{\partial t} = f_0[v_a]_r + \vec{\nabla}\cdot\vec{F} \qquad \frac{\partial[T]}{\partial t} = S_p[\omega]_r + \frac{[J]}{c_p}$$

$$\vec{F} \equiv \left(-[u^*v^*], -\frac{f_0}{S_p}[v^*T^*]\right) \qquad \vec{\nabla} \equiv \left(0, \frac{\partial}{\partial y}, \frac{\partial}{\partial p}\right)$$

(Section 11.6)

Eliassen-Palm Flux

$$\frac{\partial [u]}{\partial t} = f_0 [v_a]_r + \vec{\nabla} \cdot \vec{F}$$

$$\frac{\partial [T]}{\partial t} = S_p [\omega]_r + \frac{[J]}{c_p}$$

Eliassen-Palm Flux:

$$\vec{F} \equiv \left(-[u^* v^*], -\frac{f_0}{S_p}[v^* T^*] \right)$$

<u>Influence of eddies is apparent only in the equation for $[u]$!</u>

<u>Convergence of the Eliassen-Palm flux is associated with a deceleration of the zonal mean wind</u>

Steady state

$$f_0 [v_a]_r = -\vec{\nabla} \cdot \vec{F}$$

$$S_p [\omega]_r = -\frac{[J]}{c_p}$$

If $\vec{\nabla} \cdot \vec{F} < 0$ then $[v_a]_r > 0$ (in the northern hemisphere)

i.e. poleward residual flow.

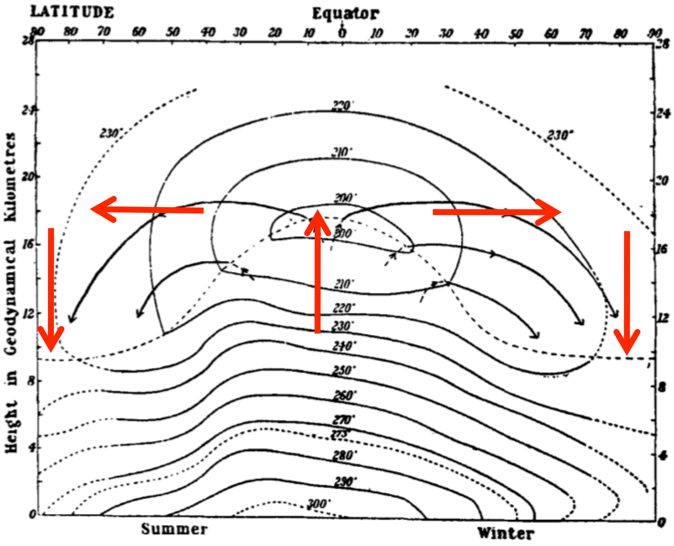
This flow transports heat and hence changes the temperature, such that the temperature is above the radiatively determined state over the pole and below the radiatively determined state over the equator.

This leads to diabatic <u>heating</u> (<u>cooling</u>) over the equator (pole)

Equator : $J > 0 \rightarrow [\omega_r] < 0$ Pole : $J < 0 \rightarrow [\omega_r] > 0$

Residual circulation

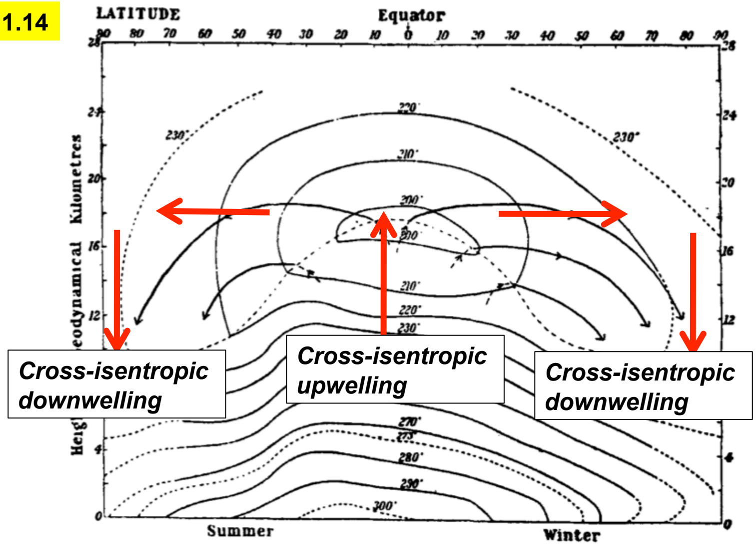
FIGURE 11.14



Brewer's (1949) original figure, illustrating what came to be known as the Brewer-Dobson circulation. "A supply of dry air is maintained by a slow mean circulation from the equatorial tropopause". The contours represent isotherms, labeled in units of K.

Residual circulation

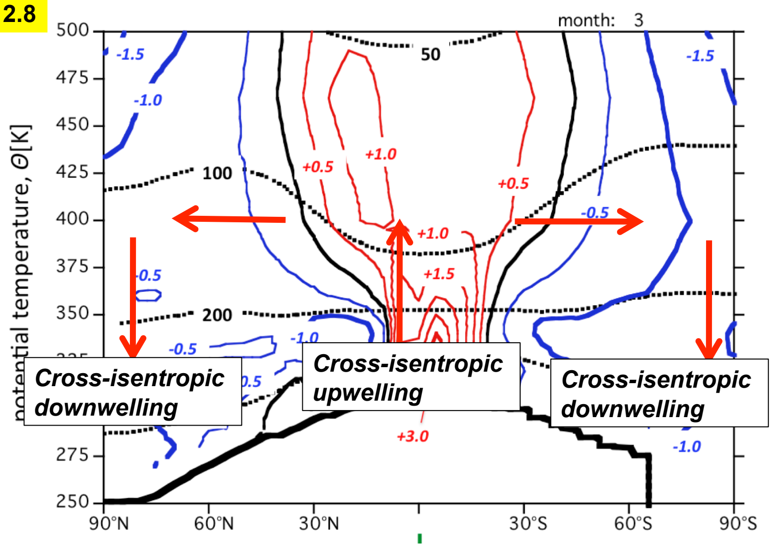
FIGURE 11.14



Brewer's (1949) original figure, illustrating what came to be known as the Brewer-Dobson circulation. "A supply of dry air is maintained by a slow mean circulation from the equatorial tropopause". The contours represent isotherms, labeled in units of K.

Residual circulation

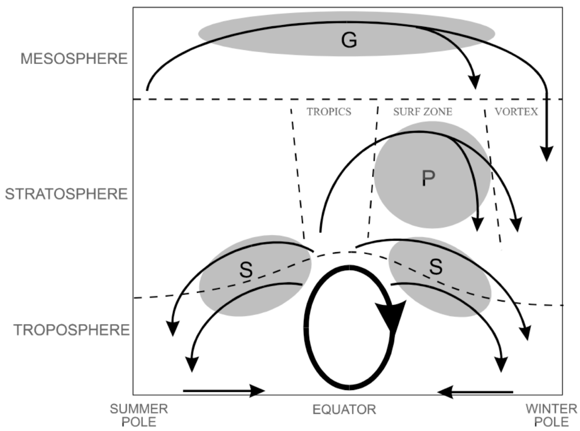
FIGURE 12.8



Monthly average isobars (dashed black lines, labelled in units of hPa) and monthly average tendency of the *potential* temperature (labelled in units of K day^{-1}; contour interval is 0.5 K day^{-1}) as a function of latitude and potential temperature for March

Residual circulation: modern view

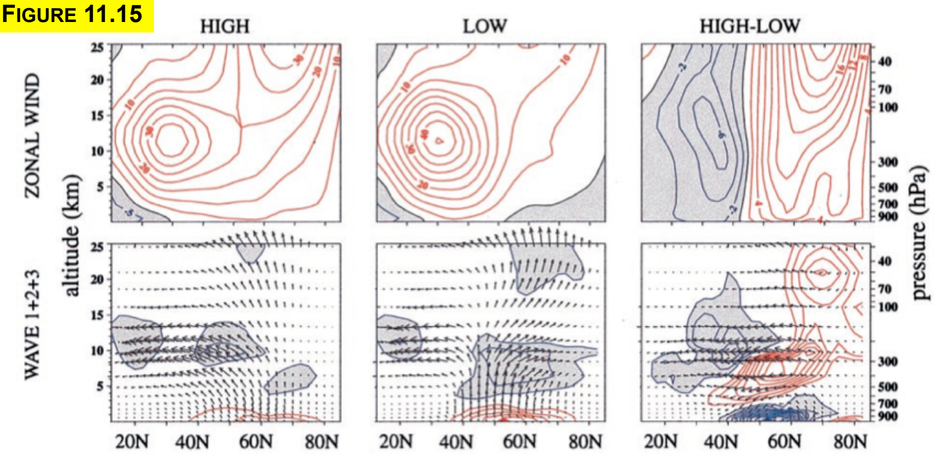
FIGURE 11.14



Schematic of the residual mean meridional circulation in the atmosphere according to R.A. Plumb. The heavy ellipse indicates the **Hadley circulation** of the troposphere. The shaded regions (labelled "S", "P", and "G") denote regions of **breaking waves** (synoptic- and planetary-scale waves, and gravity waves, respectively), responsible for driving branches of the stratospheric and mesospheric circulation. **The surf zone** is region of mixing of potential vorticity by planetary waves, which acts as a drag force on the mean zonal flow.

EP-flux climatology

FIGURE 11.15



Composites for periods of high and low Northern Annular Mode (NAM)-index and their difference (left, centre and right) during the December to March period. Upper panels: zonal wind composites. Red contours correpond to positive values. Lower panels: EP-fluxes and their divergence for the sum of zonal wave numbers 1 to 3. Red contours correspond to negative values of the EP-flux divergence.

The 2D PE-model: how do we incorporate the effects of wave drag?

Wave drag parametrization

(Section 12.6)

Wave drag

$$\frac{\partial p_s u}{\partial t} = -\frac{\partial p_s uv}{\partial y} - \frac{\partial}{\partial \sigma}\left(p_s u \frac{d\sigma}{dt}\right) + \left(f + \frac{u \tan\phi}{a}\right) p_s v + p_s D + p_s F_x$$

$$D = D_0 B(\phi) Z(\Phi)$$

$$B(\phi) = \sin(2|\phi|)$$

$$Z(\Phi) = \sin\left[\pi\left(\frac{z - z_0}{z_1 - z_0}\right)\right] \text{ if } z_0 < z < z_1$$

$$Z(\Phi) = 0 \text{ if } z \le z_0 \text{ or } z \ge z_1$$

$$z_0 = 10 \text{ km and } z_1 = 25 \text{ km}$$

$$D_0 = -0.0001 \text{ m s}^{-2}$$

Temperature, circulation and zonal wind

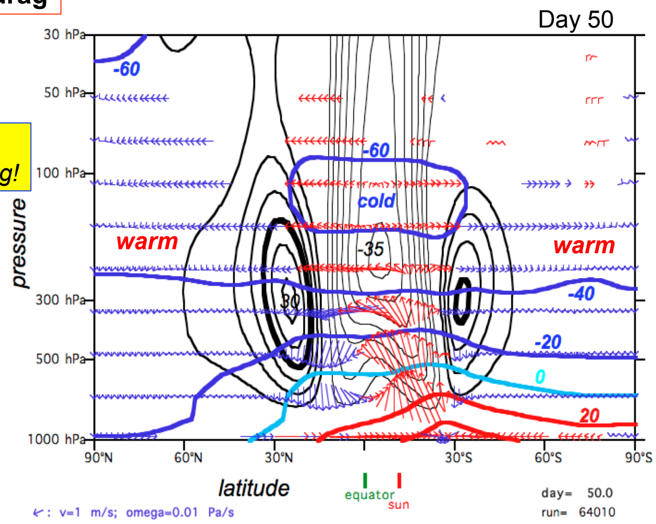
50 days after initialization on 1 January with an isothermal (290 K) atmosphere at rest

With water cycle and drag

$D_0 = -0.0001 \text{ m s}^{-2}$

Cold tropical "cold trap" appears due to wave drag!

Subtropical jets confined to the troposphere



T [°C] and u [m s^{-1}]

Fig 12.15

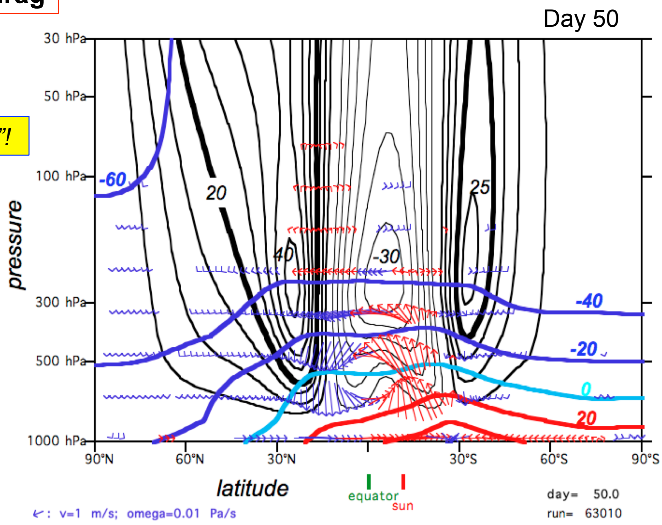
Temperature, circulation and zonal wind

50 days after initialization on 1 January with an isothermal (290 K) atmosphere at rest

With water cycle, no drag

No cold tropical "cold trap"!

Subtropical jets not confined to the troposphere



T [°C] and u [m s^{-1}]

Fig 12.15

Temperature, circulation and zonal wind

50 days after initialization on 1 January with an isothermal (290 K) atmosphere at rest

No water cycle, no drag

No cold tropical "cold trap"!

Upward motion in summer hemisphere; Downward motion in winter hemisphere

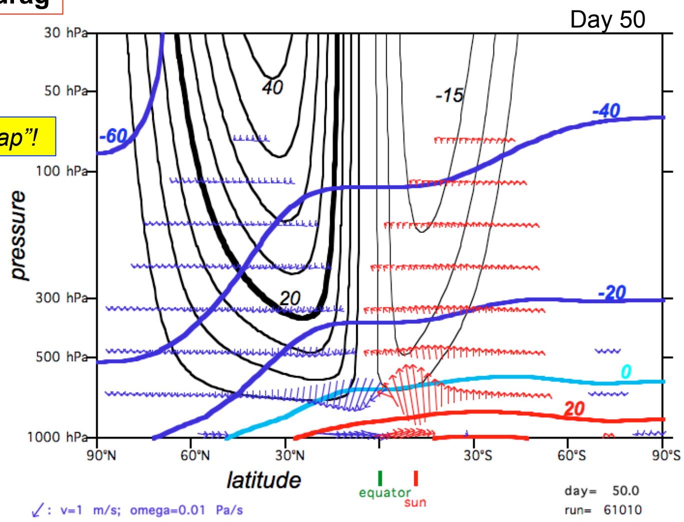


Fig 12.10

T [°C] and *u* [m s^{-1}]

Stratospheric wind reversal due to wave drag!

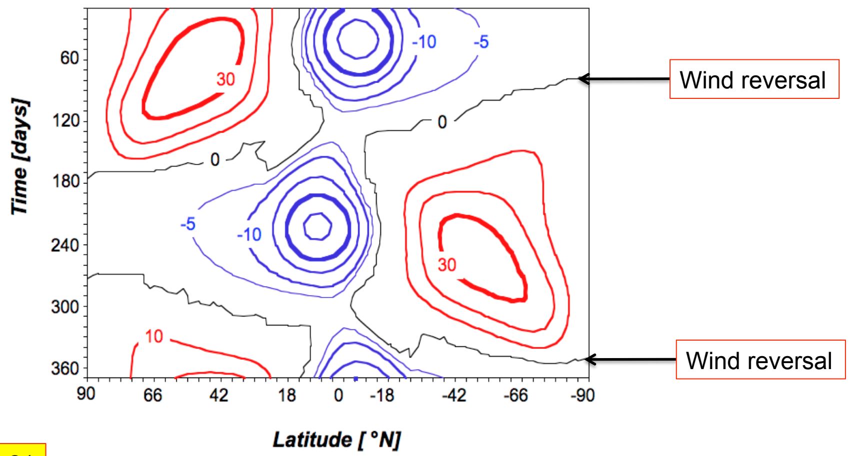


Fig 12.21

Evolution of the zonal wind at **50 hPa** during the second year of the simulation with wave drag and water cycle. Contour interval is 10 m s^{-1}. The black contour corresponds to 0 m s^{-1}.

Last week:

No wave drag: no stratospheric wind reversal!

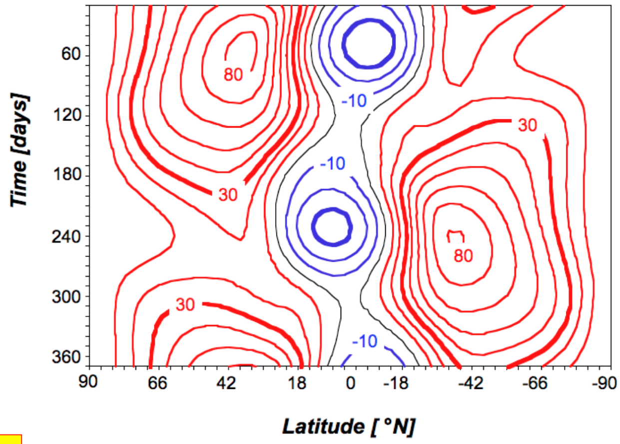


Fig 16

Evolution of the zonal wind at **50 hPa** during the second year of the simulation without wave drag and water cycle. Contour interval is 10 m s^{-1}. The black contour corresponds to 0 m s^{-1}.

Assignment 5

Give a short answer to the following 5 questions

Problem 12.5. Influence of Obliquity angle
How, do you think, does the zonal mean state of the atmosphere (temperature, zonal wind and Hadley circulation) change if both obliquity, δ_{max} (**Box 2.1**) and ϕ_{max} are equal to 55°?

Problem 12.6. Influence of precipitation in the ITCZ
How would the zonal mean state of the atmosphere be affected if we would decrease the fraction, $f_{locprec}$, of the evaporation that is converted to local precipitation from 0.8 to 0.7?

Problem 12.7. Influence of the height of cloud tops in the ITCZ
In the model simulations, which are discussed in this chapter, the pressure of the cloud tops (p_{ct}) in the ITCZ lies is set at 200 hPa (eq. 12.30 and **Table 12.2**). How realistic is this? How would the Hadley circulation, the subtropical jet and the tropopause change if the cloud top pressure in the ITCZ were set at 50 hPa?

See next slide

Assignment 5

Problem 12.8. <u>Influence of Earth's rotation rate</u>
How would the Hadley circulation and the zonal mean zonal jets be affected if we would double the value of the Coriolis parameter.

Problem 12.9. <u>Influence of zonal asymmetry in wave drag</u>
Planetary wave activity is much stronger in the northern hemisphere than in the southern hemisphere, and also probably more variable in the northern hemisphere. What is the influence of this asymmetry on the zonal mean state of the atmosphere in terms of (potential) temperature and zonal mean zonal wind?

Hand in answer on or before 4 June 2014

Assignment 2

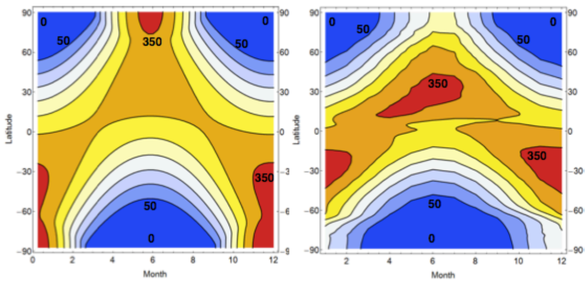


Figure 1. *Absorbed solar radiation (ASR) as a function of latitude and time. (a) Calculated using the equations from box 12.1 of the lecture notes and (b) 5 year-average ASR ('08-'12) from the ERA Interim Reanalysis. In both figures values range from* $< 50\ Wm^{-2}$ *(actually* 0 *over winter poles, but this contour is very jagged and disrupts the figure) shown in blue to* $> 350\ Wm^{-2}$ *shown in red. Contour interval is* $50\ Wm^{-2}$.

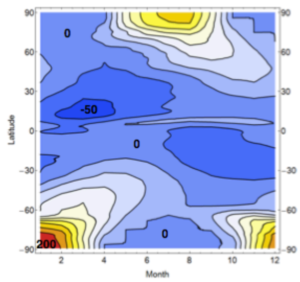


Figure 2. *Difference between the theoretical ASR and reanalysis data. Values range from about* $-50 Wm^{-2}$ *shown by the darkest blue to roughly* $200\ Wm^{-2}$ *shown in red. Contour interval is* $25\ Wm^{-2}$. *Positive values denote an overestimation of the theory w.r.t. reality.*

Assignment 2

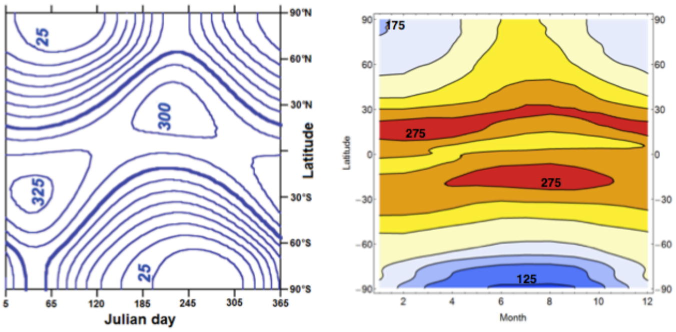


Figure 3. *Outgoing longwave radiation (OLR) as a function of latitude and time calculated from (a) the theoretical radiatively determined state, adapted from figure 12.5 of the lecture notes and (b) 5 year-average ASR ('08-'12) from the ERA Interim Reanalysis. In b, values range from roughly* $125\ Wm^{-2}$ *shown in blue to about* $275\ Wm^{-2}$ *shown in red. Contour interval is* $25\ Wm^{-2}$.

Assignment 2

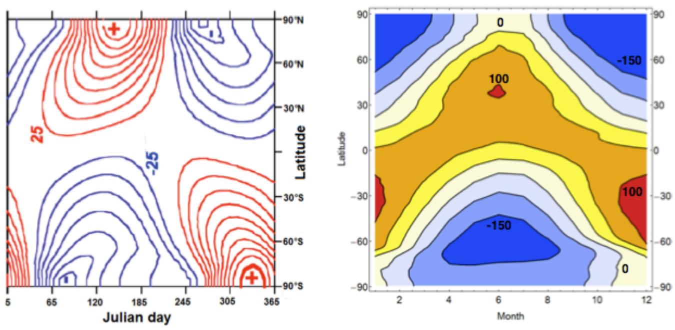


Figure 4. *Difference between ASR and OLR calculated from (a) the theoretical radiatively determined state, adapted from figure 12.6 of the lecture notes and (b) 5 year-average ('08-'12) from the ERA Interim Reanalysis. In b, values range from roughly* $-150\ Wm^{-2}$ *shown in blue to about* $100\ Wm^{-2}$ *shown in red. Contour interval is* $50\ Wm^{-2}$*. Yellow, orange and red colours denote a positive balance: ASR > OLR.*

Schedule of the C&HC-2

23 April: Introduction to radiative transfer; "grey gas"; radiative equilibrium
study sections 2.1-2.4 & boxes 2.1-2.4;
(1)** problem 12.1 (response time)* ***(0.5)
30 April: Radiatively determined state; Reanalyses
(2)** problem 12.2 (radiation at TOA; ERA-Interim)* ***(2.0)
7 May: Radiative-dynamical interaction in a dry atmosphere; GCM's
***(3)** article for review (**yes/no**); Topic of presentation (GCM)*
14 May: Role of water cycle in the general circulation (the ITCZ)
(4)** problem 12.3 (check of model assumptions)* ***(2.5)
21 May: Role of wave drag in the general circulation
(5)** problem 12.5-12.9 (what-if? thought experiments)* ***(1.0)
4 June: The surf zone: further physical interpretation of wave drag
11 June: Hadley circulation, distribution of isentropic density
(6)** problem 12.12 (Hadley-circulation theory)* ***(1.5)
***(7)** hand in review*
18 June: Potential vorticity (inversion)
***(8)** presentations on GCM's (**2.5**)*
25 June: Zonal mean mass- and potential vorticity budget
***(8)** presentations on GCM's (**2.5**)*
No exam

Next lecture

Wednesday 4/6, 2013, 13:15-15:00

No lecture next week

Discussion of fourth assignment (problem 12.3)

The surf zone:
The meaning (physical interpretation) of wave drag

http://www.staff.science.uu.nl/~delde102/C&HC.htm