Aarnout van Delden http://www.staff.science.uu.nl/~delde102/C&HC.htm

Diabatic-Dynamical Interaction in the General Circulation (lecture 8)

***Recapitulation**: Relation PV and wind (PV-inversion)*

Vorticity equation in isentropic coordinates

Potential vorticity substance (PVS)

Impermeability theorem for PVS

Can we understand the formation of PV-anomalies?
(in particular the ex-UTLS PV anomaly)

Potential vorticity inversion

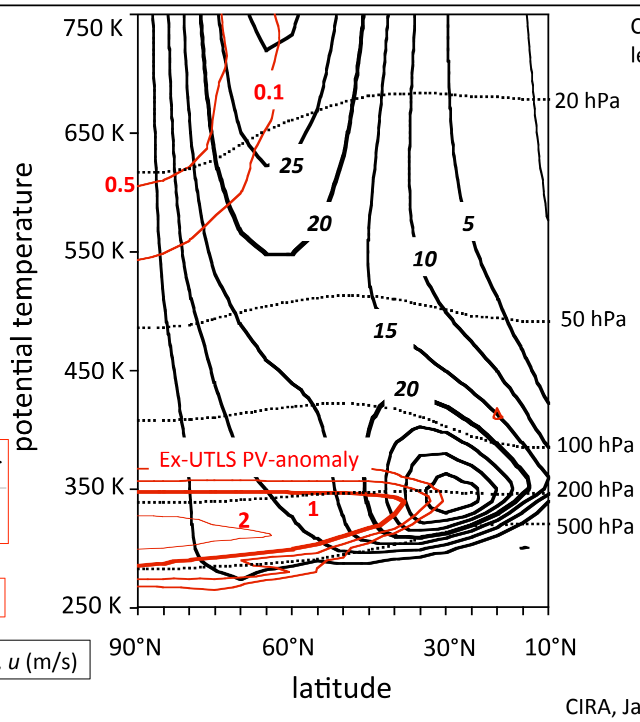
Zonal wind determined from total PV

PV-anomaly:

$$Z' \equiv \frac{Z - Z_{ref}}{Z_{ref}}$$

Red: PV-anomaly

Black: zonal wind, u (m/s)



Chapter 7 lecture notes

CIRA, January mean

Potential vorticity inversion

Zonal wind determined from total PV

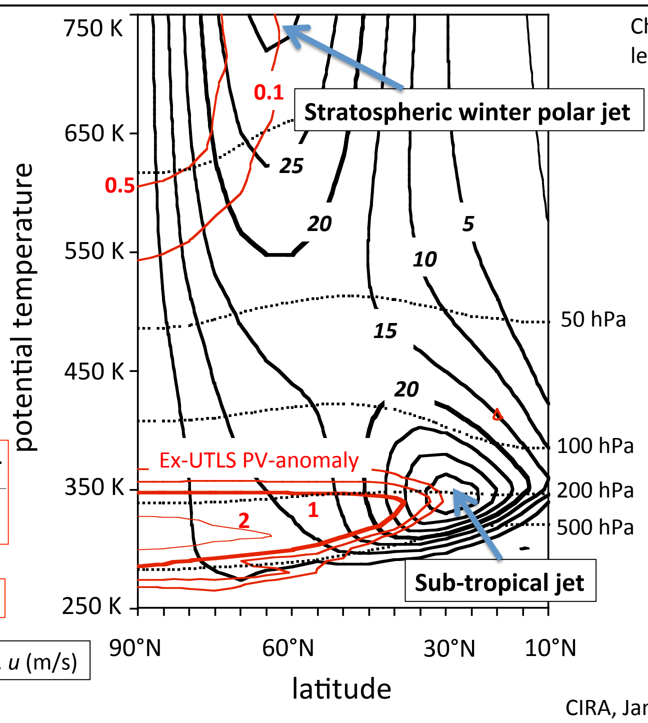
PV-anomaly:

$$Z' \equiv \frac{Z - Z_{ref}}{Z_{ref}}$$

Red: PV-anomaly

Black: zonal wind, *u* (m/s)

Chapter 7 lecture notes



CIRA, January mean

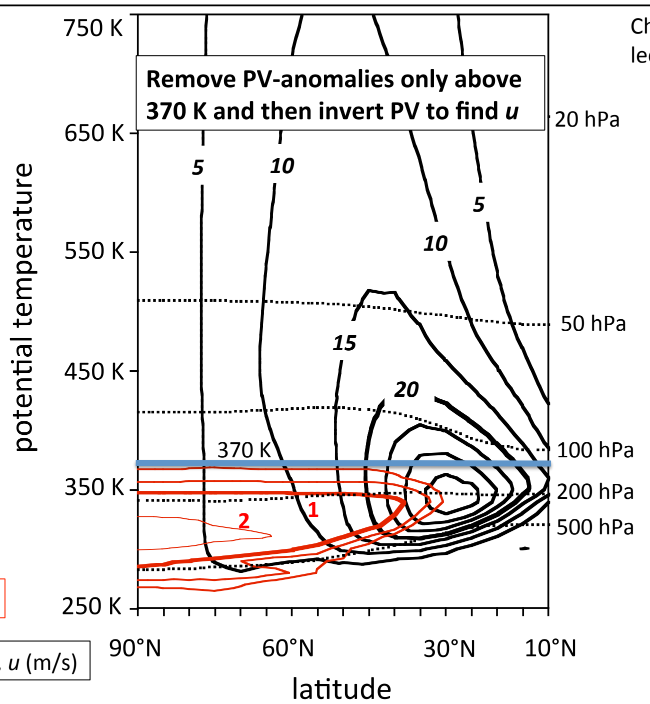
Piecewise Potential vorticity inversion

Zonal wind determined from PV in UTLS

Red: PV-anomaly

Black: zonal wind, *u* (m/s)

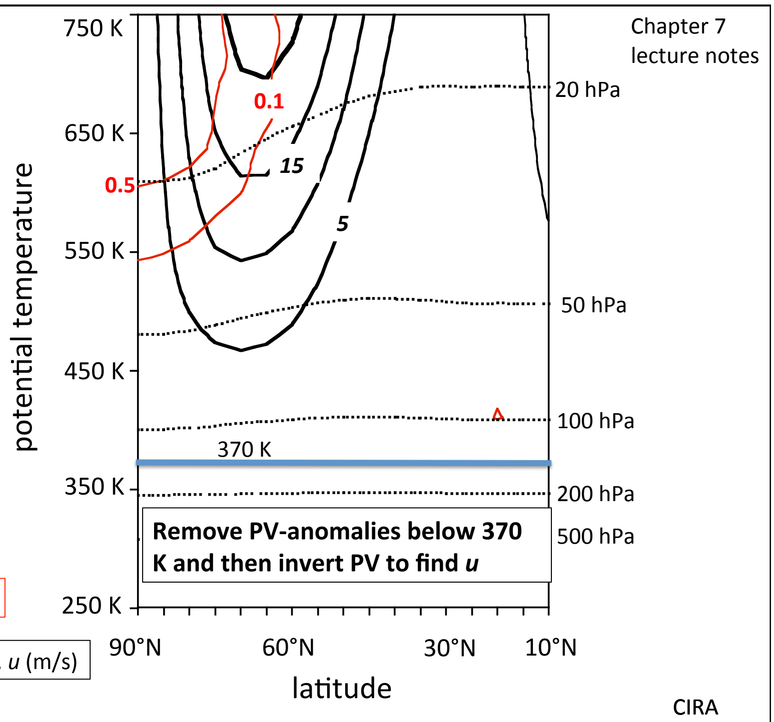
Chapter 7 lecture notes



CIRA

Piecewise Potential vorticity inversion

Zonal wind determined from PV in stratosphere above 370 K



Red: PV-anomaly

Black: zonal wind, *u* (m/s)

Section 7.3

Equations with potential temperature as a vertical coordinate

Mass conservation equation:

$$\frac{1}{\sigma}\frac{d\sigma}{dt} + \left(\frac{\partial u}{\partial x}\right)_\theta + \left(\frac{\partial v}{\partial y}\right)_\theta + \frac{\partial}{\partial \theta}\left(\frac{d\theta}{dt}\right) = 0$$

"density": $\sigma \equiv -\frac{1}{g}\frac{\partial p}{\partial \theta}$

Hydrostatic balance:

$$\frac{\partial \Psi}{\partial \theta} = \Pi$$ (problem 5.4)

"vertical motion": $\frac{d\theta}{dt} = \frac{J}{\Pi}$

Equation of motion:

$$\frac{\partial u}{\partial t} + u\left(\frac{\partial u}{\partial x}\right)_\theta + v\left(\frac{\partial u}{\partial y}\right)_\theta + \frac{d\theta}{dt}\frac{\partial u}{\partial \theta} = -\left(\frac{\partial \Psi}{\partial x}\right)_\theta + fv + F_x$$

$$\frac{\partial v}{\partial t} + u\left(\frac{\partial v}{\partial x}\right)_\theta + v\left(\frac{\partial v}{\partial y}\right)_\theta + \frac{d\theta}{dt}\frac{\partial v}{\partial \theta} = -\left(\frac{\partial \Psi}{\partial y}\right)_\theta - fu + F_y$$

If dθ/dt=0: motion is "two-dimensional"!

Section 7.4

Vorticity equation

Isentropic relative vorticity:

$$\zeta_\theta = \left[\frac{\partial v}{\partial x}\right]_\theta - \left[\frac{\partial u}{\partial y}\right]_\theta$$

$$\frac{\partial u}{\partial t} + u\left(\frac{\partial u}{\partial x}\right)_\theta + v\left(\frac{\partial u}{\partial y}\right)_\theta + \frac{d\theta}{dt}\frac{\partial u}{\partial \theta} = -\left(\frac{\partial \Psi}{\partial x}\right)_\theta + fv + F_x$$

$$\frac{\partial v}{\partial t} + u\left(\frac{\partial v}{\partial x}\right)_\theta + v\left(\frac{\partial v}{\partial y}\right)_\theta + \frac{d\theta}{dt}\frac{\partial v}{\partial \theta} = -\left(\frac{\partial \Psi}{\partial y}\right)_\theta - fu + F_y$$

Vorticity equation:

$$\frac{d}{dt}(\zeta_\theta + f) = -(\zeta_\theta + f)\delta_\theta + \frac{\partial u}{\partial \theta}\frac{\partial}{\partial y}\left(\frac{d\theta}{dt}\right) - \frac{\partial v}{\partial \theta}\frac{\partial}{\partial x}\left(\frac{d\theta}{dt}\right) + \frac{\partial F_x}{\partial y} - \frac{\partial F_y}{\partial x}$$

Stretching term

divergence: $\delta_\theta = \left[\frac{\partial u}{\partial x}\right]_\theta + \left[\frac{\partial v}{\partial y}\right]_\theta$

Vorticity equation

Drop subscript, θ:

$$\frac{d}{dt}(\zeta + f) = -(\zeta + f)\delta + \frac{\partial u}{\partial \theta}\frac{\partial \dot{\theta}}{\partial y} - \frac{\partial v}{\partial \theta}\frac{\partial \dot{\theta}}{\partial x} + \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}$$

$$\zeta_{abs} \equiv \zeta + f = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f, \quad \delta \equiv \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \quad \text{and} \quad \dot{\theta} \equiv \frac{d\theta}{dt}$$

$$\frac{\partial \zeta_{abs}}{\partial t} = -u\frac{\partial \zeta_{abs}}{\partial x} - v\frac{\partial \zeta_{abs}}{\partial y} - \dot{\theta}\frac{\partial \zeta_{abs}}{\partial \theta} - \zeta_{abs}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{\partial u}{\partial \theta}\frac{\partial \dot{\theta}}{\partial y} - \frac{\partial v}{\partial \theta}\frac{\partial \dot{\theta}}{\partial x} + \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}$$

$$\frac{\partial \zeta_{abs}}{\partial t} = -\frac{\partial u\zeta_{abs}}{\partial x} - \frac{\partial v\zeta_{abs}}{\partial y} - \dot{\theta}\frac{\partial}{\partial \theta}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \frac{\partial u}{\partial \theta}\frac{\partial \dot{\theta}}{\partial y} - \frac{\partial v}{\partial \theta}\frac{\partial \dot{\theta}}{\partial x} + \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}$$

Vorticity equation

$$\frac{\partial \zeta_{abs}}{\partial t} = -\frac{\partial u\zeta_{abs}}{\partial x} - \frac{\partial v\zeta_{abs}}{\partial y} - \dot{\theta}\frac{\partial}{\partial \theta}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \frac{\partial u}{\partial \theta}\frac{\partial \dot{\theta}}{\partial y} - \frac{\partial v}{\partial \theta}\frac{\partial \dot{\theta}}{\partial x} + \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}$$

$$\frac{\partial \zeta_{abs}}{\partial t} = -\vec{\nabla} \cdot \vec{J}$$

$$\vec{\nabla} \equiv \left\{\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial \theta}\right\}$$

$$\vec{J} \equiv \left(u\zeta_{abs} + \dot{\theta}\frac{\partial v}{\partial \theta} - F_y,\; v\zeta_{abs} - \dot{\theta}\frac{\partial u}{\partial \theta} + F_x,\; 0\right)$$

Vorticity equation

$$\frac{\partial \zeta_{abs}}{\partial t} = -\frac{\partial u\zeta_{abs}}{\partial x} - \frac{\partial v\zeta_{abs}}{\partial y} - \dot{\theta}\frac{\partial}{\partial \theta}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \frac{\partial u}{\partial \theta}\frac{\partial \dot{\theta}}{\partial y} - \frac{\partial v}{\partial \theta}\frac{\partial \dot{\theta}}{\partial x} + \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}$$

$$\frac{\partial \zeta_{abs}}{\partial t} = -\vec{\nabla} \cdot \vec{J}$$

$$\vec{\nabla} \equiv \left\{\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial \theta}\right\}$$

$$\vec{J} \equiv \left(u\zeta_{abs} + \dot{\theta}\frac{\partial v}{\partial \theta} - F_y,\; v\zeta_{abs} - \dot{\theta}\frac{\partial u}{\partial \theta} + F_x,\; 0\right)$$

vertical component = 0 !

Vorticity equation

$$\frac{\partial \zeta_{abs}}{\partial t} = -\vec{\nabla} \cdot \vec{J}$$

$$\vec{\nabla} \equiv \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial \theta} \right\}$$

$$\vec{J} \equiv \left(u\zeta_{abs} + \dot{\theta}\frac{\partial v}{\partial \theta} - F_y,\ v\zeta_{abs} - \dot{\theta}\frac{\partial u}{\partial \theta} + F_x,\ 0 \right)$$

Divergence theorem: $\int_A \vec{J} \cdot \hat{n} dA = \int_V \vec{\nabla} \cdot \vec{J} dV$

Area, *A*, encloses volume, *V* in (*x*, *y*, *θ*) space

$$\frac{\partial}{\partial t}\left\{ \int_V \zeta_{abs} dV \right\} = -\int_A \vec{J} \cdot \hat{n} dA$$

Potential Vorticity Substance (*PVS*)

Potential vorticity substance in volume *V*:

$$PVS(V) \equiv \left\{ \int_V \zeta_{abs} dV \right\} = \left\{ \int_V \sigma Z dV \right\}$$

Impermeability theorem for *PVS*

Potential vorticity substance in volume *V*:

$$PVS(V) \equiv \left\{ \int_V \zeta_{abs} dV \right\} = \left\{ \int_V \sigma Z dV \right\}$$

$$\frac{dPVS(V)}{dt} = -\int_A \vec{J} \cdot \hat{n} dA$$

$$\vec{J} \equiv \left(u\zeta_{abs} + \dot{\theta}\frac{\partial v}{\partial \theta} - F_y,\ v\zeta_{abs} - \dot{\theta}\frac{\partial u}{\partial \theta} + F_x,\ 0 \right)$$

PVS <u>cannot</u> cross isentropic surfaces!!

Last week:

Mass conservation equation

$$\frac{\partial \sigma}{\partial t} = -\left(\frac{\partial \sigma u}{\partial x}\right)_\theta - \left(\frac{\partial \sigma v}{\partial y}\right)_\theta - \frac{\partial}{\partial \theta}\left(\sigma \frac{d\theta}{dt}\right) = -\vec{\nabla} \cdot \vec{I}$$

<u>Mass flux vector:</u> $\vec{I} \equiv (\sigma u, \sigma v, \sigma \dot{\theta})$

Vertical diabatic component due to cross-isentropic flow

Equation in short: $\frac{\partial \sigma}{\partial t} = -\vec{\nabla} \cdot \vec{I}$

<u>Mass in volume *V*:</u> $M(V) \equiv \left\{ \int_V \sigma dV \right\}$

With the divergence theorem: $\frac{dM(V)}{dt} = -\int_A \vec{I} \cdot \hat{n} dA$

Overview

$$\frac{dPVS(V)}{dt} = -\int_A \vec{J} \cdot \hat{n} dA$$

$$\vec{J} \equiv \left(u\zeta_{abs} + \dot{\theta}\frac{\partial v}{\partial \theta} - F_y,\ v\zeta_{abs} - \dot{\theta}\frac{\partial u}{\partial \theta} + F_x,\ 0 \right)$$

PVS <u>cannot</u> cross isentropic surfaces!!

$$\frac{dM(V)}{dt} = -\int_A \vec{I} \cdot \hat{n} dA$$

$$\vec{I} \equiv \left(u,\ v,\ \dot{\theta} \right)\sigma$$

Mass <u>can</u> cross isentropic surfaces!!

Overview of consequences

PVS <u>cannot</u> cross isentropic surfaces!!

Mass <u>can</u> cross isentropic surfaces!!

Diabatic mass convergence into an isentropic layer dilutes Potential Vorticity Substance (*PVS*), i.e. the *"mixing ratio of potential vorticity substance"* (=*PV*)* decreases

This effect may create a ***potential vorticity anomaly***

This happens in the Middleworld*, or upper troposphere and lower stratosphere (UTLS)*

* $Z \equiv PV = \frac{PVS}{\sigma}$

Diabatic mass flux convergence per unit horizontal area into the Middleworld layer between θ=315 K and θ=370 K

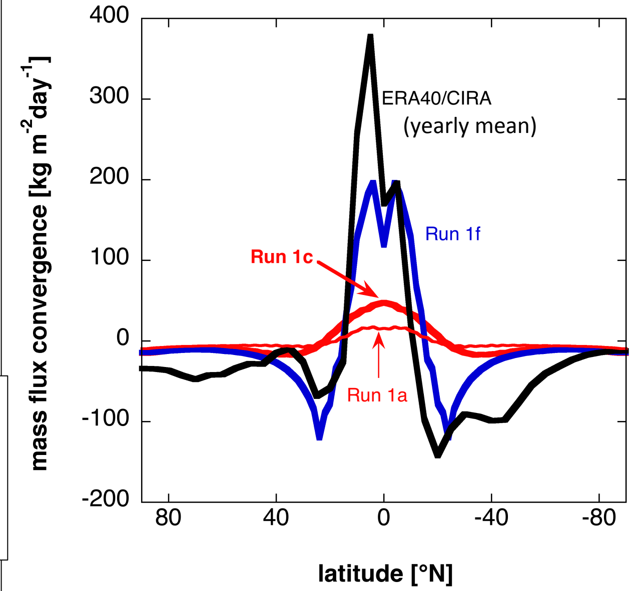
In the *tropics* PVS is *diluted*,
while in the *extratropics* PVS is *concentrated*

Legend:

Permanent equinox runs

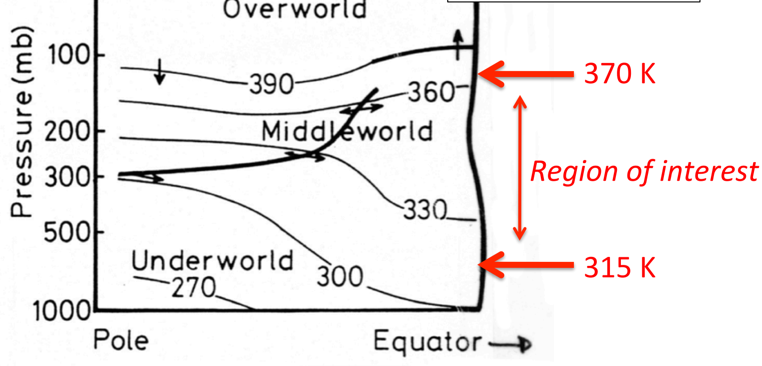
Run 1a:
water: no; wave drag: no
Run 1c:
water: no; wave drag: yes
Run 1f:
water: yes; wave drag: yes

FIGURE 13



Schematic view of the atmosphere

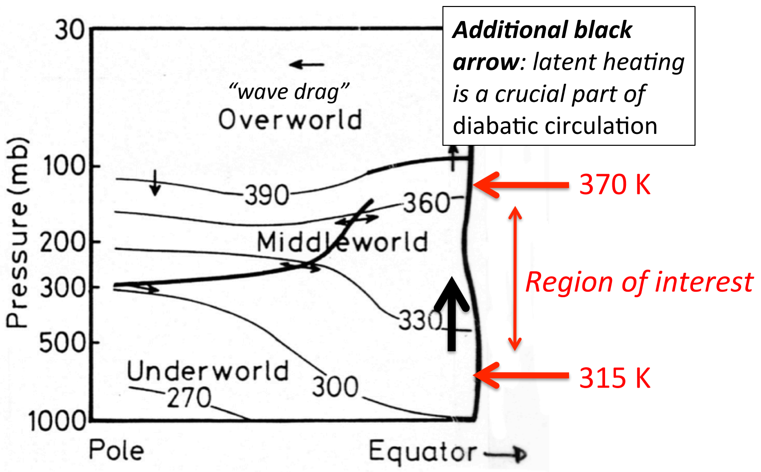
Black arrow: diabatic circulation according to Hoskins



Schematic view of the atmosphere, as a function of latitude and pressure. The tropopause is marked by a thick line, and isentropes every 30 K from 270-390 K by thin lines. The arrows indicate some transports. (from Hoskins, B.J., 1991, **Tellus, 43AB**, 27-35).

Schematic view of the atmosphere

Additional black arrow: *latent heating is a crucial part of* diabatic circulation

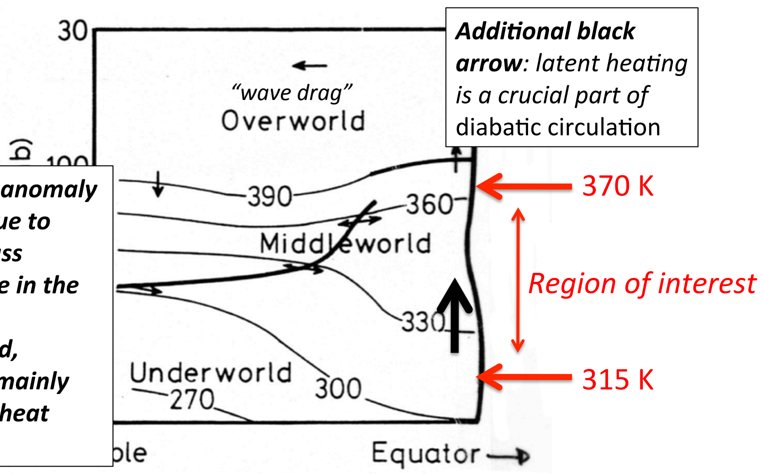


Schematic view of the atmosphere, as a function of latitude and pressure. The tropopause is marked by a thick line, and isentropes every 30 K from 270-390 K by thin lines. The arrows indicate some transports. (from Hoskins, B.J., 1991, **Tellus, 43AB**, 27-35).

Schematic view of the atmosphere

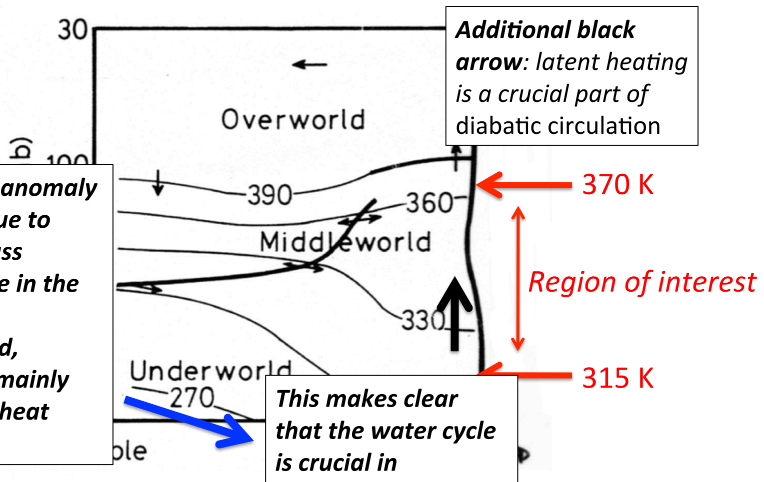
Additional black arrow: *latent heating is a crucial part of* diabatic circulation

Ex-UTLS PV anomaly is formed due to diabatic mass convergence in the tropical Middleworld, associated mainly with latent heat release!



Schematic view of the atmosphere, as a function of latitude and pressure. The tropopause is marked by a thick line, and isentropes every 30 K from 270-390 K by thin lines. The arrows indicate some transports. (from Hoskins, B.J., 1991, **Tellus, 43AB**, 27-35).

Schematic view of the atmosphere



***Additional black arrow**: latent heating is a crucial part of* diabatic circulation

Ex-UTLS PV anomaly is formed due to diabatic mass convergence in the tropical Middleworld, associated mainly with latent heat release!

This makes clear that the water cycle is crucial in determining the structure of the General Circulation!!

Schematic view of the atmosphere, as a f[...]ssure. The tropopause is marked by a thick line, and isentropes [...] by thin lines. The arrows indicate some transports. (from H[...]3AB, 27-35).

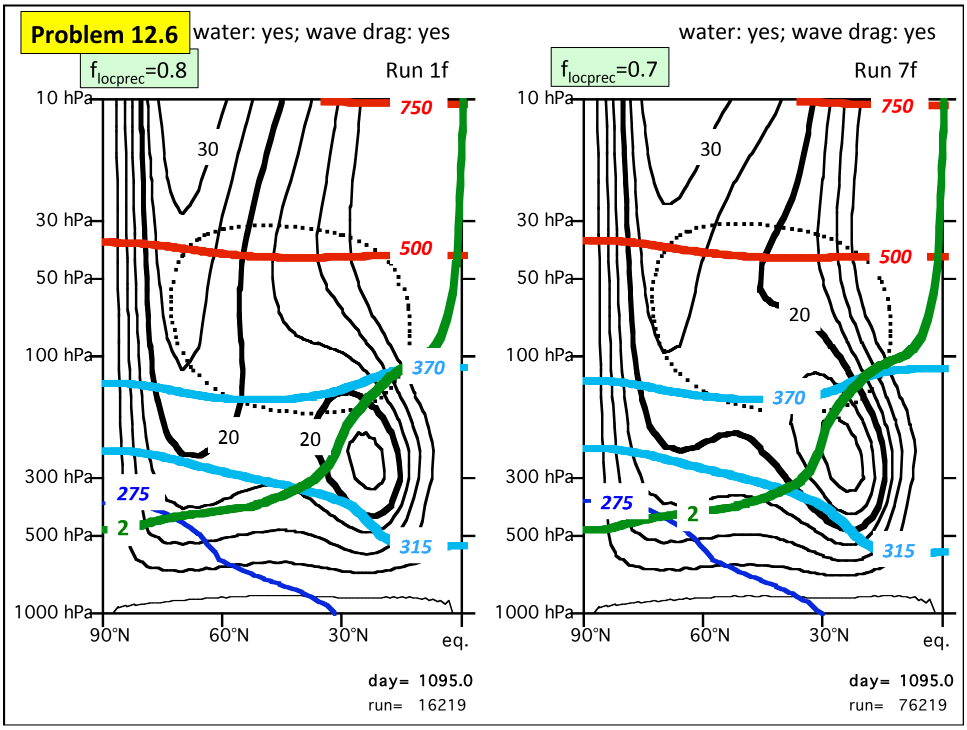
Problems 12.6-12.8

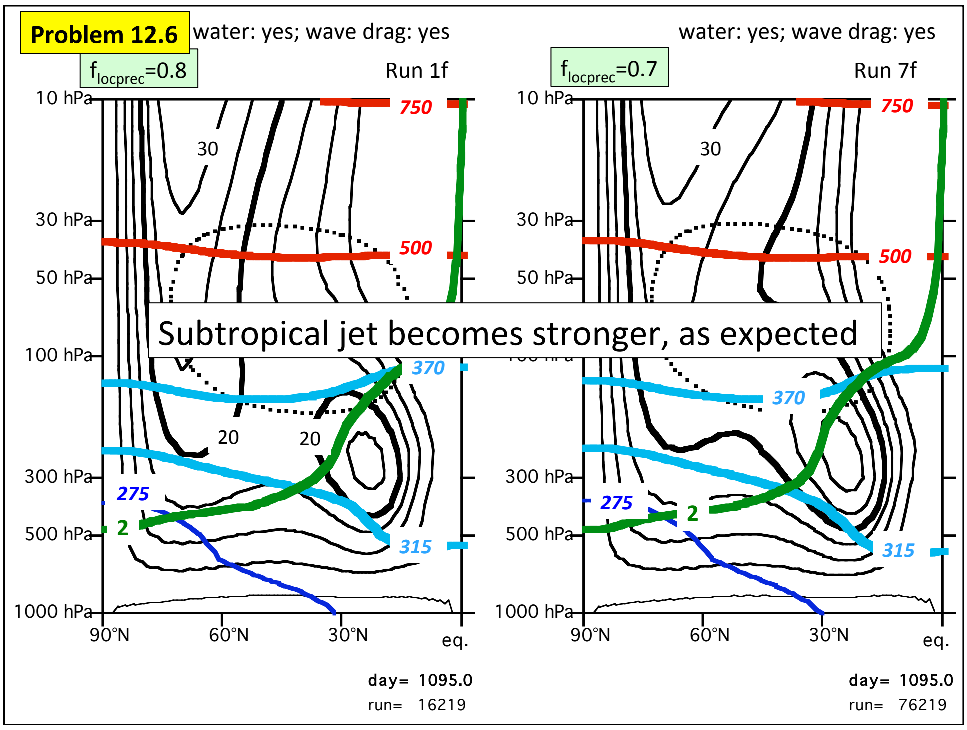
What if questions

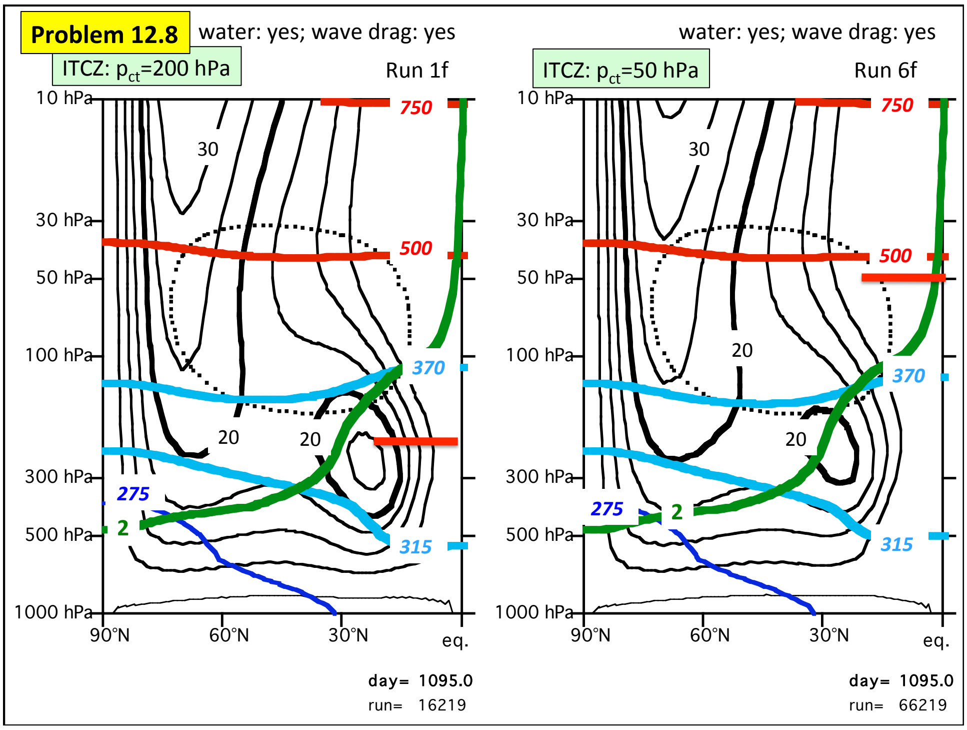
12.6: Changing $f_{locprec}$ from 0.8 to 0.7.

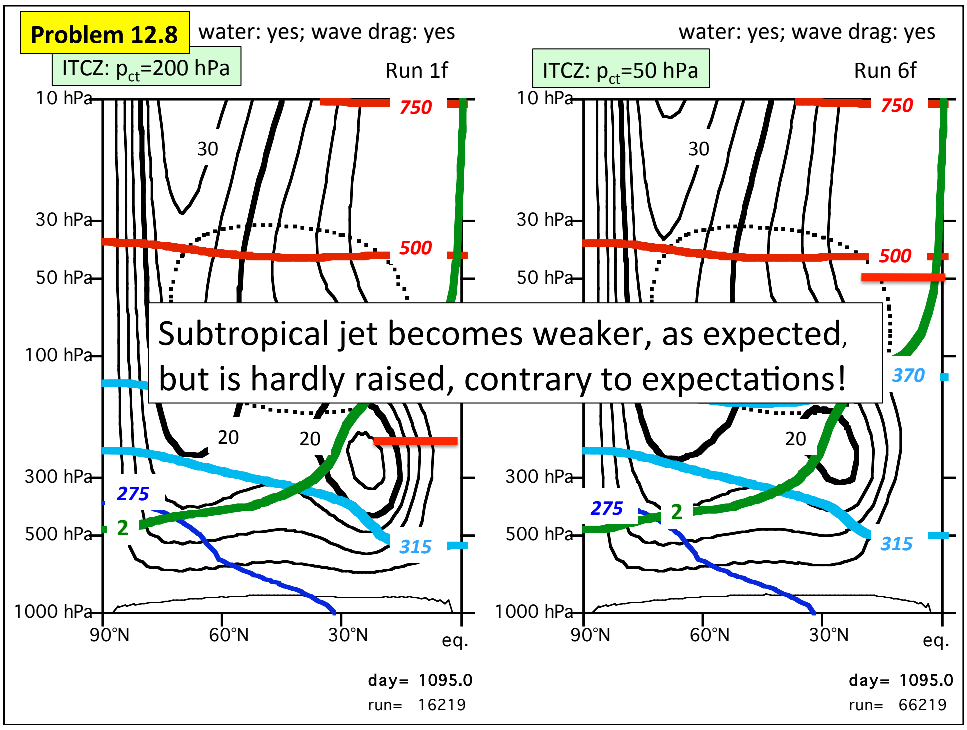
12.7: Changing p_{ct} in the ITCZ form 200 hPa to 50 hPa

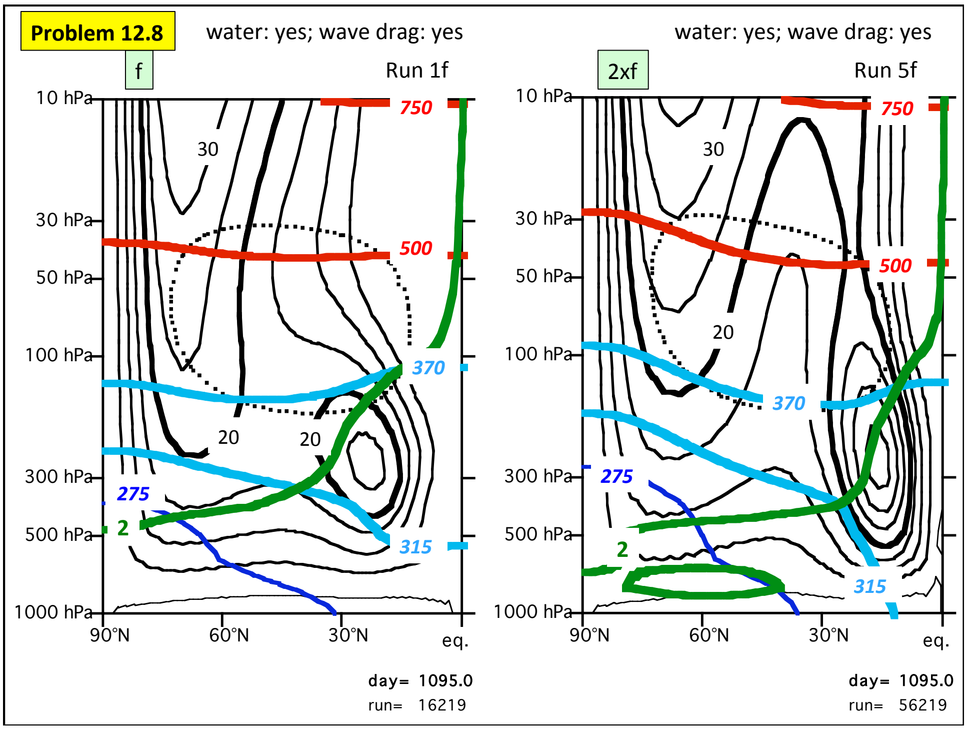
12.8: Doubling the earth's rotation rate

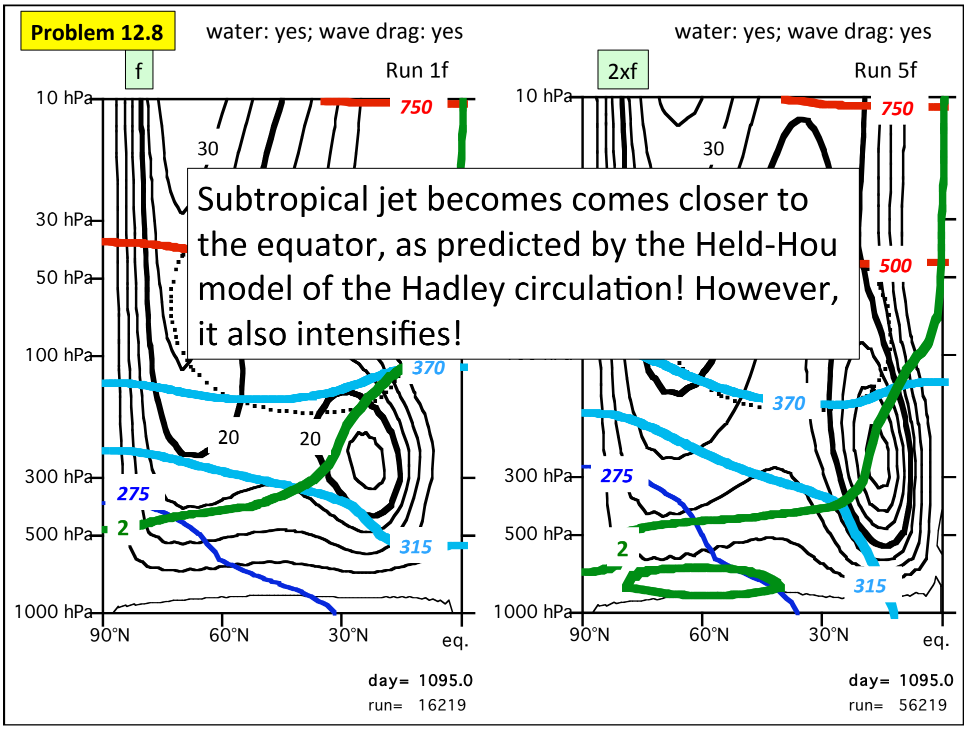

Problem 12.6
water: yes; wave drag: yes
f_locprec=0.8
Run 1f
water: yes; wave drag: yes
f_locprec=0.7
Run 7f
10 hPa
30 hPa
50 hPa
100 hPa
300 hPa
500 hPa
1000 hPa
90°N
60°N
30°N
eq.
750
500
370
315
275
2
30
20
day= 1095.0
run= 16219
day= 1095.0
run= 76219


Problem 12.6
water: yes; wave drag: yes
f_locprec=0.8
Run 1f
water: yes; wave drag: yes
f_locprec=0.7
Run 7f
Subtropical jet becomes stronger, as expected
10 hPa
30 hPa
50 hPa
100 hPa
300 hPa
500 hPa
1000 hPa
90°N
60°N
30°N
eq.
750
500
370
315
275
2
30
20
day= 1095.0
run= 16219
day= 1095.0
run= 76219


Problem 12.8
water: yes; wave drag: yes
ITCZ: p_{ct}=200 hPa
Run 1f
water: yes; wave drag: yes
ITCZ: p_{ct}=50 hPa
Run 6f
10 hPa
30 hPa
50 hPa
100 hPa
300 hPa
500 hPa
1000 hPa
90°N
60°N
30°N
eq.
750
500
370
315
275
2
30
20
day= 1095.0
run= 16219
run= 66219


Problem 12.8
water: yes; wave drag: yes
ITCZ: p_{ct}=200 hPa
Run 1f
water: yes; wave drag: yes
ITCZ: p_{ct}=50 hPa
Run 6f
Subtropical jet becomes weaker, as expected,
but is hardly raised, contrary to expectations!
10 hPa
30 hPa
50 hPa
100 hPa
300 hPa
500 hPa
1000 hPa
90°N
60°N
30°N
eq.
750
500
370
315
275
2
30
20
day= 1095.0
run= 16219
run= 66219


Problem 12.8
water: yes; wave drag: yes
f
Run 1f
water: yes; wave drag: yes
2xf
Run 5f
10 hPa
30 hPa
50 hPa
100 hPa
300 hPa
500 hPa
1000 hPa
90°N
60°N
30°N
eq.
750
500
370
315
275
2
30
20
day= 1095.0
run= 16219
day= 1095.0
run= 56219


Problem 12.8
water: yes; wave drag: yes
f
Run 1f
water: yes; wave drag: yes
2xf
Run 5f
Subtropical jet becomes comes closer to the equator, as predicted by the Held-Hou model of the Hadley circulation! However, it also intensifies!
10 hPa
30 hPa
50 hPa
100 hPa
300 hPa
500 hPa
1000 hPa
90°N
60°N
30°N
eq.
750
500
370
315
275
2
30
20
day= 1095.0
run= 16219
day= 1095.0
run= 56219

Problem 12.12: Held-Hou model

Theory of the permanent equinox Hadley circulation based on

1. Two layer model
2. Conservation of angular momentum in the upper "outflow layer" → $U_M = \frac{\Omega y^2}{a\cos\phi}$
3. Thermal wind balance
4. Lower layer is dominated by friction so that *u*=0
5. Diabatic heating/cooling is determined only by radiation
6. Latent heat release is neglected
7. Radiation is parametrized as "Newtonian cooling", i.e. as a relaxation process to a prescribed radiative equilibrium temperature (constant in time)
8. The radiative equilibrium temperature is prescribed (rather ad-hoc)
9. Assumes that heating in the updraught is balanced by net cooling in the downdraught
10. Predicted circulation intensity is too weak
11. The main result is the interesting expression for the horizontal scale, *Y*, of the Hadley circulation and associated position of the subtropical jet:

$$Y = \left(\frac{5gH\Delta\theta}{3\Omega^2\theta_0}\right)^{1/2} \approx 2200 \text{ km} \text{ (section 8.2)}$$

so that the maximum strength of the subtropical jet is: $U_M = \frac{\Omega y^2}{a\cos\phi} \approx 50-60 \text{ m/s}$

End of the course

I hope that you have found the course interesting and useful!

You will receive your grade tomorrow, together with my response to your review

Thank for your reviews and your attention!