Questions & Answers 28 Sept.
Functional Programming 2017/18
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Format of the exam

Similar to the previous exams in the web

- The exam is on **paper**
  - Some places (wrongly) state that it is digital
- Two types of questions
  - Open (80%): explain something or write some code
  - Multiple choice (20%): choose one answer
- There is a maximum amount of space per question
  - Only the things you write there count
  - Don’t worry! There’s way more space that you need!
- You cannot bring any notes to the exam
- You have to answer in English
  - You can bring a (mother tongue) - English dictionary
Contents of the exam

Everything until today

- Basic types: lists, tuples, numbers
- User-defined data types
- Define functions by pattern matching
- Recursion on lists and other data types
- Define and use higher-order functions
- Write classes and instances
- Infer and check the type of an expression
- **No**: write functions using accumulators
- **No**: use functions to represent data
Contents of the exam

Do I need to know all the types by heart?

Writing and understanding type signatures is something you need to learn, and we need to test

- You have to know or deduce the type of simple functions such as `(++)`, `max`, `==`, and so forth
- Some higher-order functions are very important
  - `map :: (a -> b) -> [a] -> [b]`
  - `filter :: (a -> Bool) -> [a] -> [a]`
  - `foldr :: (a -> b -> b) -> b -> [a] -> b`

In most cases you can deduce their type if you know what they do
Outcome of the exam

- You should expect your grades in about two weeks
- What happens if I fail the exam?
  - *Nothing*, your grade is the average with the final one
  - Remember, $T = 0.3 \times \text{midterm} + 0.7 \times \text{final}$
  - *Reflect* on your mistakes and *act* to fix them
Q&A session
Interactive Q&A session

1. I pose a question that somebody mailed me
Interactive Q&A session

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2. Give you some time to think in groups
Interactive Q&A session

1. I pose a question that somebody mailed me

2. Give you some time to think in groups

3. And then I explain the answer in full
The most repeated question

More examples of type inference

Let me answer some smaller questions before
In the previous lecture...

Monoids provide sane defaults:

\[
\text{lookup}' = \text{findWithDefault} \ \text{mempty} \\
\text{merge}' = \text{mergeWith} \ \text{mappend}
\]

What does “monoids provide sane defaults” mean?
Monoids provides defaults

Whenever you have to:

- Combine elements of a type into one with the same type $\implies$ use `mappend`
- Have a default value for a type $\implies$ use `mempty`
Monoids provides defaults

Whenever you have to:

- Combine elements of a type into one with the same type $\Rightarrow$ use `mappend`
- Have a default value for a type $\Rightarrow$ use `mempty`

Monoids provide *sane* defaults

This is about how people normally use `Monoid`

- The `Monoid` instance for a class is only written when that type has a “natural” way to be combined
  - How to combine lists? Append them
  - How to combine Booleans? Conjunction or disjunction
- In some cases you want to use this “natural” implementation in other places
Difference and/or relation between left- vs. right-bias and foldr vs. foldl
Difference and/or relation between left- vs. right-bias and foldr vs. foldl

Not much, just “right” or “left” in their names

- foldr and foldl are about parenthesis and nesting
  - foldr (+) 0 [1,2,3] = 1 + (2 + (3 + 0))
  - foldl (+) 0 [1,2,3] = ((0 + 1) + 2) + 3

- Bias is about choosing a value when there is a conflict
  - E.g., when merging two values with the same key
What can we do with anonymous function / (lambda) abstractions? Is there something deep?
What can we do with anonymous function / (lambda) abstractions? Is there something deep?

They are just a handier way to write (small) functions

\[
g = \text{map } f \quad \Rightarrow \quad g = \text{map } (\lambda x \rightarrow \text{foo where } f \ x = \text{foo}
\]

They have a limitation: you cannot do case distinction

\[
g = \text{map } f \quad \Rightarrow \quad g = \text{map } (\lambda x \rightarrow ??) \text{ where } f \ True = \text{foo} \quad f \ False = \text{bar}
\]
What is the difference between `name@(...)`, `\z -> ...` and `let x = ... in ...`?
What is the difference between `name@(...)`, `\z \to \ldots` and `let x = \ldots in \ldots`?

**In common:** they introduce a new *name* into scope
- You can use that name in the \ldots part

The difference lies in what they refer to
- `let x = \ldots in \ldots` gives a name to an expression which is part of a bigger expression

```plaintext
unit (x, y) = let norm = \sqrt(x*x + y*y) in (x/norm, y/norm)
```
- The others refer to the *argument* of a function
  - `name@(...)` always appear at the left of the = symbol
What is the difference between `name@(...)`, \( \texttt{\( z \to \ldots \)} \) and `let x = \ldots in \ldots`?

With pattern matching we choose a branch in a function and access the components of a value:

- We can match also in an anonymous function!
- But we can only match one pattern

\[
\text{norm} \ (x,y) = \ldots \quad \text{norm} = \texttt{\( (x,y) \to \ldots \)}
\]

\[
\text{length} \ \texttt{[]} = \ldots \quad \text{length} = \texttt{\( ?? \to \ldots \)}
\]

\[
\text{length} \ (x:xs) = \ldots
\]

With `name@(...)` we give a name to the whole value and then we pattern match in its components

\[
f \text{lst}@\ (x:xs) = \ldots \quad \text{-- 'lst' is the whole list}
\]

\[
\text{-- 'x' is the head of 'lst' and 'xs' is its tail}
\]
Enough waiting! We want to infer some types!

What is the type of `map . foldr`?

**General rule:** If \( f : : A \rightarrow B \) and \( e : : A \), then \( f \ e : : B \)

Infix operator syntax comes to play here

- `map . foldr = (.) map foldr`
- The function `(.)` takes two arguments, `map` and `foldr`

\[
(\cdot) : : (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow a \rightarrow c \\
\text{map} : : (a \rightarrow b) \rightarrow [a] \rightarrow [b] \\
\text{foldr} : : (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b
\]
1. Introduce new names to disambiguate
   - I tend to use \(?n\) to make it clear
   - Other people use Greek letters
   - I don’t care, but make it clear in the exam

\[ a, b \text{ and } c \text{ in each type are unrelated} \]

\[
(\cdot) :: (?b \to ?c) \to (?a \to ?b) \to ?a \to ?c
\]

\[
\text{map} :: (?d \to ?e) \to [?d] \to [?e]
\]

\[
\text{foldr} :: (?u \to ?v \to ?v) \to ?v \to [?u] \to ?v
\]
1. Introduce new names to disambiguate
   - I tend to use $\text{(?n)}$ to make it clear
   - Other people use Greek letters
   - I don’t care, but make it clear in the exam

   -- $a$, $b$ and $c$ in each type are unrelated

   \[
   (.) \to (?b \to ?c) -> (?a \to ?b) \to ?a \to ?c
   \]

   \[
   \text{map} \to (?d \to ?e) -> [?d] -> [?e]
   \]

   \[
   \text{foldr} \to (?u \to ?v \to ?v) \to ?v \to [?u] \to ?v
   \]

2. Write equations to fit the type in the function with the types of the arguments

   \[
   ?b \to ?c = (?d \to ?e) -> [?d] -> [?e]
   \]

   \[
   ?a \to ?b = (?u \to ?v \to ?v) \to ?v \to [?u] \to ?v
   \]
3. Solve the equations

- Remember about the implicit parenthesis for $\rightarrow$

\[
\begin{align*}
?b \rightarrow ?c &= (?d \rightarrow ?e) \rightarrow [?d] \rightarrow [?e] \\
&= (?d \rightarrow ?e) \rightarrow ([?d] \rightarrow [?e])
\end{align*}
\]

$\implies \quad ?b = ?d \rightarrow ?e$

$\implies ?c = [?d] \rightarrow [?e]$

\[
\begin{align*}
?a \rightarrow ?b &= (?u \rightarrow ?v) \rightarrow (?v \rightarrow [?u] \rightarrow ?v) \\
\implies ?a &= ?u \rightarrow ?v \rightarrow ?v \\
&\implies ?b = ?v \rightarrow [?u] \rightarrow ?v
\end{align*}
\]

\[-- \text{ We have to equations for } ?b\]

\[
\begin{align*}
?b &= ?d \rightarrow ?e = ?v \rightarrow ([?u] \rightarrow ?v) \\
\implies ?d &= ?v \\
?e &= [?u] \rightarrow ?v
\end{align*}
\]
4. Obtain the result type

- The remainder of the fn. without the given arguments

\[(.) :: (?b -> ?c) -> (?a -> ?b) -> ?a -> ?c\]

\[==> \text{map} \cdot \text{foldr} :: ?a -> ?c\]

- Substitute unknowns for their values

\[?a -> ?c\]

\[= -- \text{Parentheses!}\]

\[(?u -> ?v -> ?v) -> [?d] -> [?e]\]

\[= -- \text{We can keep substituting}\]


- This type works for any ?u and ?v

\[\text{map} \cdot \text{foldr} :: (u -> v -> v) -> [v] -> [[u] -> v]\]
How do I check that I am right?

Use the interpreter to ask for the type

```
> :t map . foldr
map . foldr
  :: Foldable t => (a1 -> a2 -> a2)
        -> [a2] -> [t a1 -> a2]
> :set -XTYPEApplications
> :t map . foldr @[]
map . foldr @[] :: (a1 -> a2 -> a2)
        -> [a2] -> [[a1] -> a2]
```

The names a1 and a2 do not matter

- But the *shape* of the type must be the same
Rinse and repeat

What is the type of map (map map)?

The result of the inner map map is the arg. to the outer map.
Rinse and repeat

What is the type of map (map map)?

The result of the inner map map is the arg. to the outer map

1. Introduce new names to disambiguate

   ▶ Each map gets different names

   ```
   map :: (?a -> ?b) -> [?a] -> [?b]   -- #1
   map :: (?c -> ?d) -> [?c] -> [?d]   -- #2
   map :: (?e -> ?f) -> [?e] -> [?f]   -- #3
   ```
2. Obtain the type of the inner map :map:
   ▶ Pose and solve the equations

\[
\begin{align*}
?c \to ?d &= (?e \to ?f) \to [?e] \to [?f] \\
\implies ?c &= (?e \to ?f) \\
?d &= [?e] \to [?f]
\end{align*}
\]

▶ Obtain the result type

\[
\text{map : map} :: [?c] \to [?d] \\
&= [?e \to ?f] \to [[?e] \to [?f]]
\]
3. Obtain the type of the whole expression
   ▶ Pose and solve the equations

\[
\begin{align*}
?a \rightarrow ?b &= [?e \rightarrow ?f] \rightarrow [[?e] \rightarrow [?f]] \\
\Rightarrow ?a &= [?e \rightarrow ?f] \\
?b &= [[?e] \rightarrow [?f]]
\end{align*}
\]

▶ Obtain the result type

\[
\text{map (map map) :: } [[?a] \rightarrow [?b]] = [[?e \rightarrow ?f]] \rightarrow [[[?e] \rightarrow [?f]]]
\]

4. The type works for any ?e and ?f

\[
\text{map (map map) :: } [[[a \rightarrow b]]] \rightarrow [[[a] \rightarrow [b]]]
\]