Moduli Spaces
Spring 2014
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[FAC] = Fulton's Algebraic Curves

**Homework 1.**

Let $k$ be an algebraically closed field.

1. Assume that the characteristic of the ground field $k$ differs from 2 **and** 5. Let $X$ be the projective nonsingular model of the affine curve $C$ with equation $y^2 = x^5 + 1$. Determine the divisors (on $X$) of the functions $x$ and $y$. Determine the (canonical) divisors of the differentials $dx$ and $dy$. Determine a basis for the vector space of regular differentials on $X$.

2. Assume that the characteristic of the ground field $k$ equals 2. Let $X$ be the projective nonsingular model of the affine curve $C$ with equation $y^2 + y = x^5$. Determine the divisors (on $X$) of the functions $x$ and $y$. Determine the (canonical) divisors of the differentials $dx$ and $dy$. Determine a basis for the vector space of regular differentials on $X$. (Remark: if you have never worked in characteristic 2 before, this exercise may at first be rather confusing to you.)

These exercises to be handed in before Friday, February 28, 2014. Note (February 20): in the first problem, the characteristic should also differ from 5.

**Homework 2.**

Let $k$ be an algebraically closed field of characteristic 0.

3. Let $C$ be the plane quintic with equation $x^2y^3 + x^2z^3 + y^3z^2 - z^5 = 0$. In parts (a) through (d), you may assume that $C$ is irreducible.

(a) Prove that $C$ has exactly 2 singular points and that these points are ordinary multiple points. Determine the genus of the nonsingular model $X$ of $C$ (with map $f\colon X \to C$).

(b) Determine for $1 \leq m \leq 6$ the spaces $V_m$ of adjoint forms of degree $m$.

(c) Let $L$ be the equation of a line intersecting $C$ in 5 distinct points $P_1, \dots, P_5$. Let $E = \sum_{P \in X}(m_{f(P)} - 1)P$ and $E_m = m\sum_{i=1}^5 P_i - E$ be the divisors discussed in class and [FAC]. Verify in this concrete case that the map $V_m \to L(E_m)$, $G \mapsto G/L^m$ is surjective with kernel the space of forms divisible by the form $F$ defining $C$.

(d) Describe a $g_2^1$ (a linear system of dimension 1 and degree 2) on $X$. (In fact, it is unique.)

(e)* Prove that $C$ is irreducible.

This exercise to be handed in before Friday, March 7, 2014.

Do not hesitate to contact me when you have questions: `C.F.Faber@uu.nl`