

WISB134

Modellen & Simulatie

*Lecture 11 - Dynamica van lineaire
differentiaalvergelijkingen in twee dimensies*



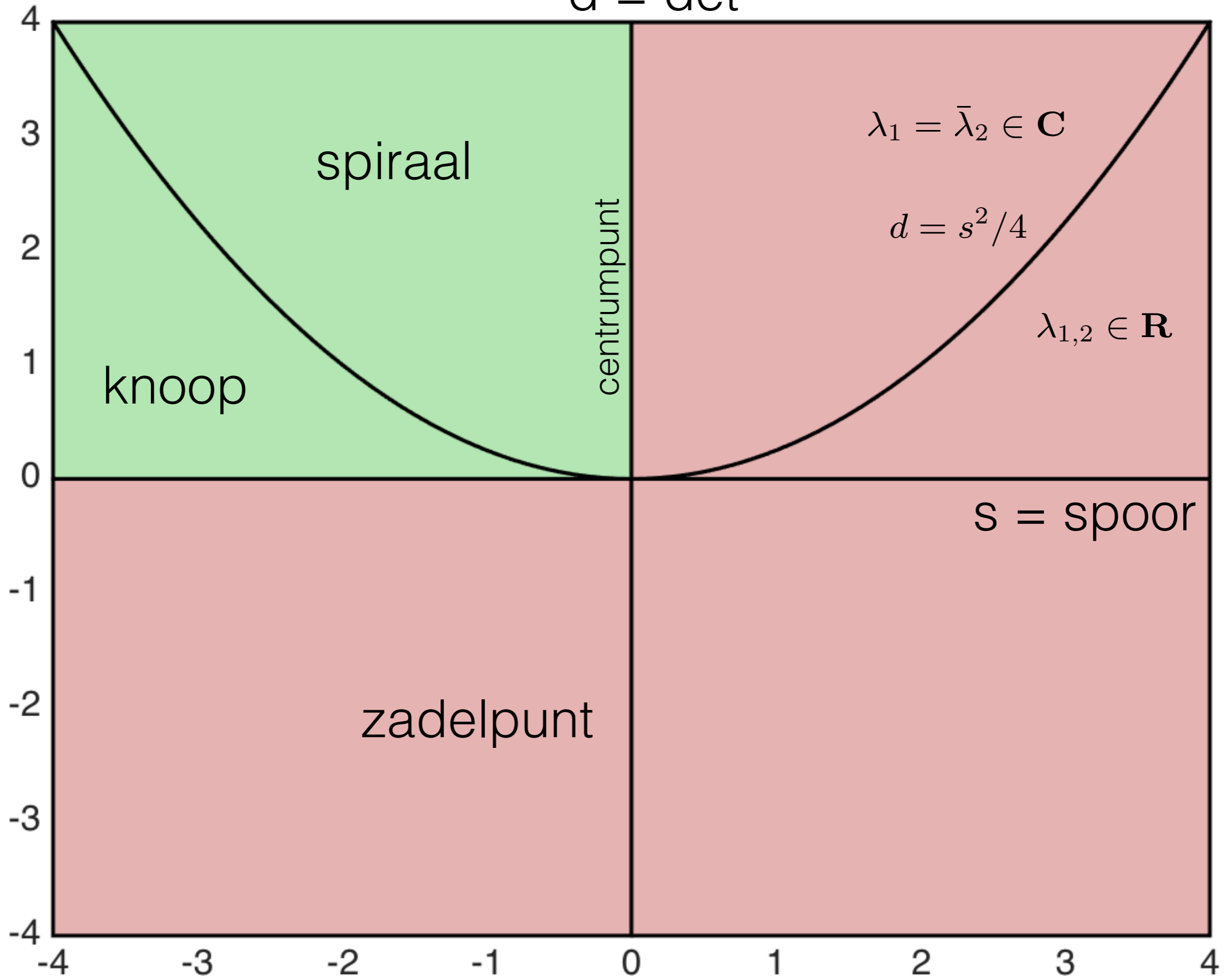
Universiteit Utrecht

Overzicht van ModSim

- Basisbegrippen dynamische modellen
 - Definities recursies, DVs, numerieke methoden
 - Oplossingen DVs
 - Convergentie numerieke methoden
- Dynamica
 - Scalaire dynamica
 - Dynamica op \mathbf{R}^d
 - ➔ Lineaire dynamica op \mathbf{R}^2
- Bijzondere gevallen
 - Lineaire kansmodellen (Markovketens)
 - Niet-autonome systemen (Resonantie)
 - Hogere orde numerieke methoden

Meeste
aandacht
(t/m 1 apr.)

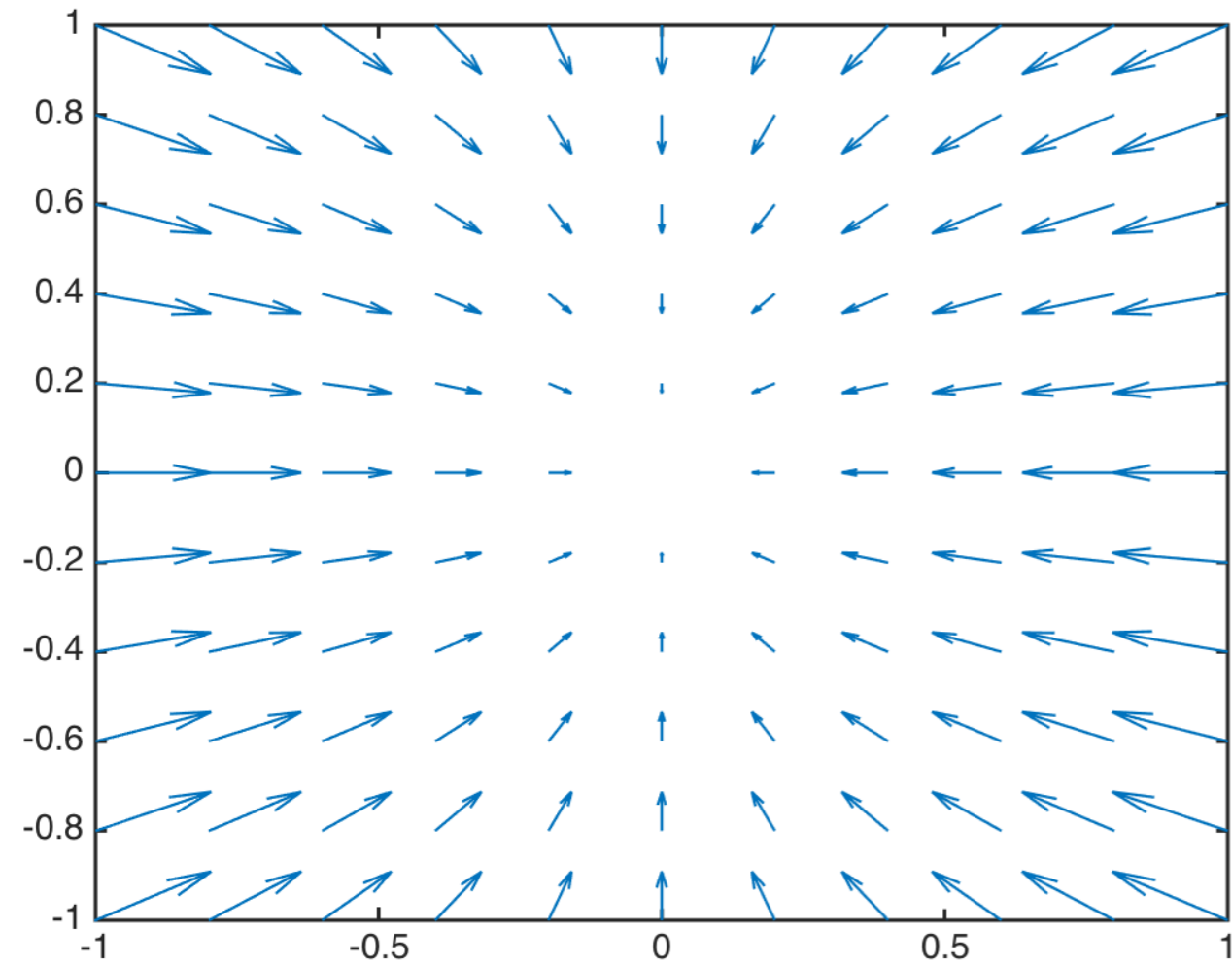
$d = \det$



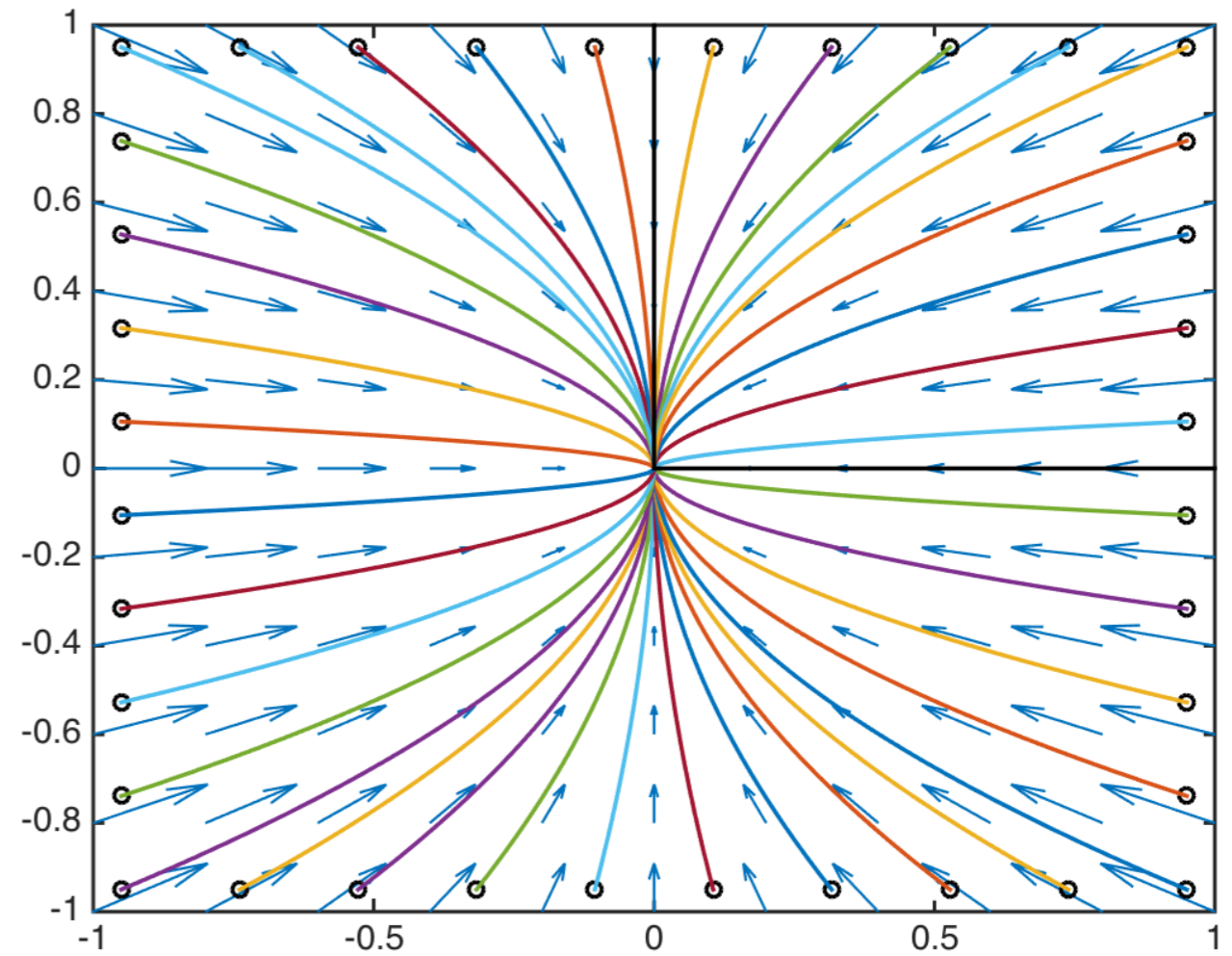
$$\lambda_1 = -1.5, \lambda_2 = -0.8$$

$$v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Vector Field



Phase portrait

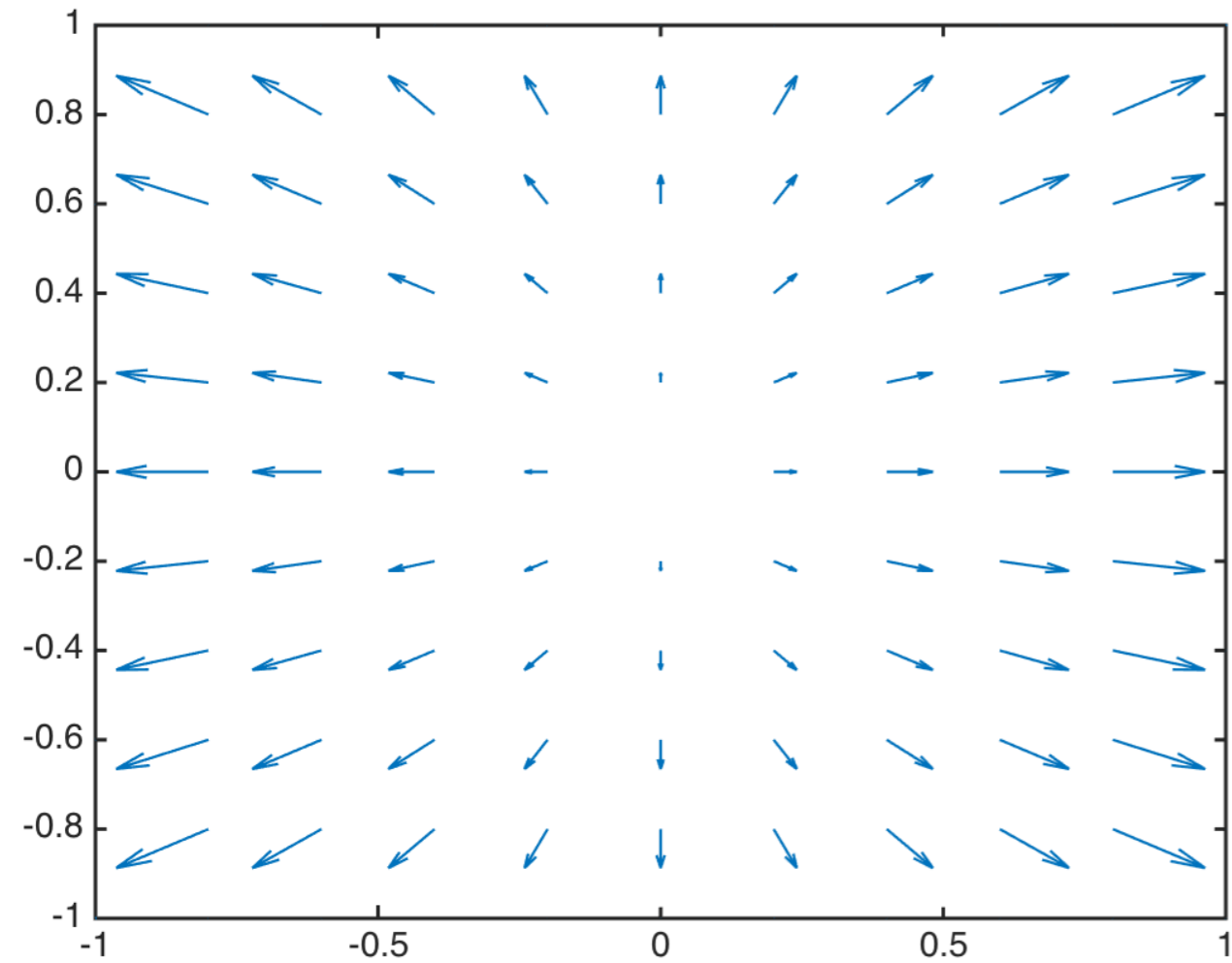


Stable node

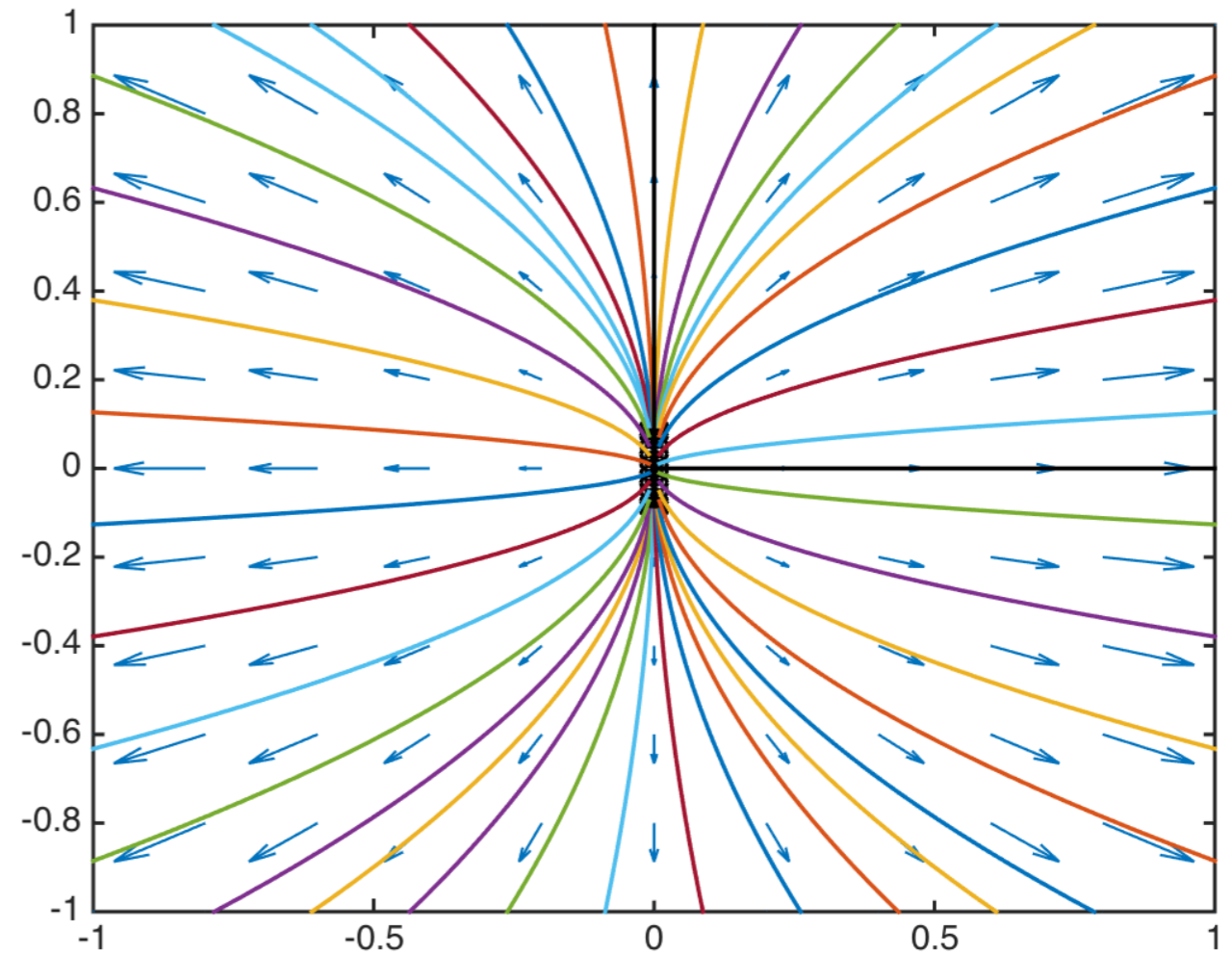
$$\lambda_1 = 1.5, \lambda_2 = 0.8$$

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Vector Field



Phase portrait

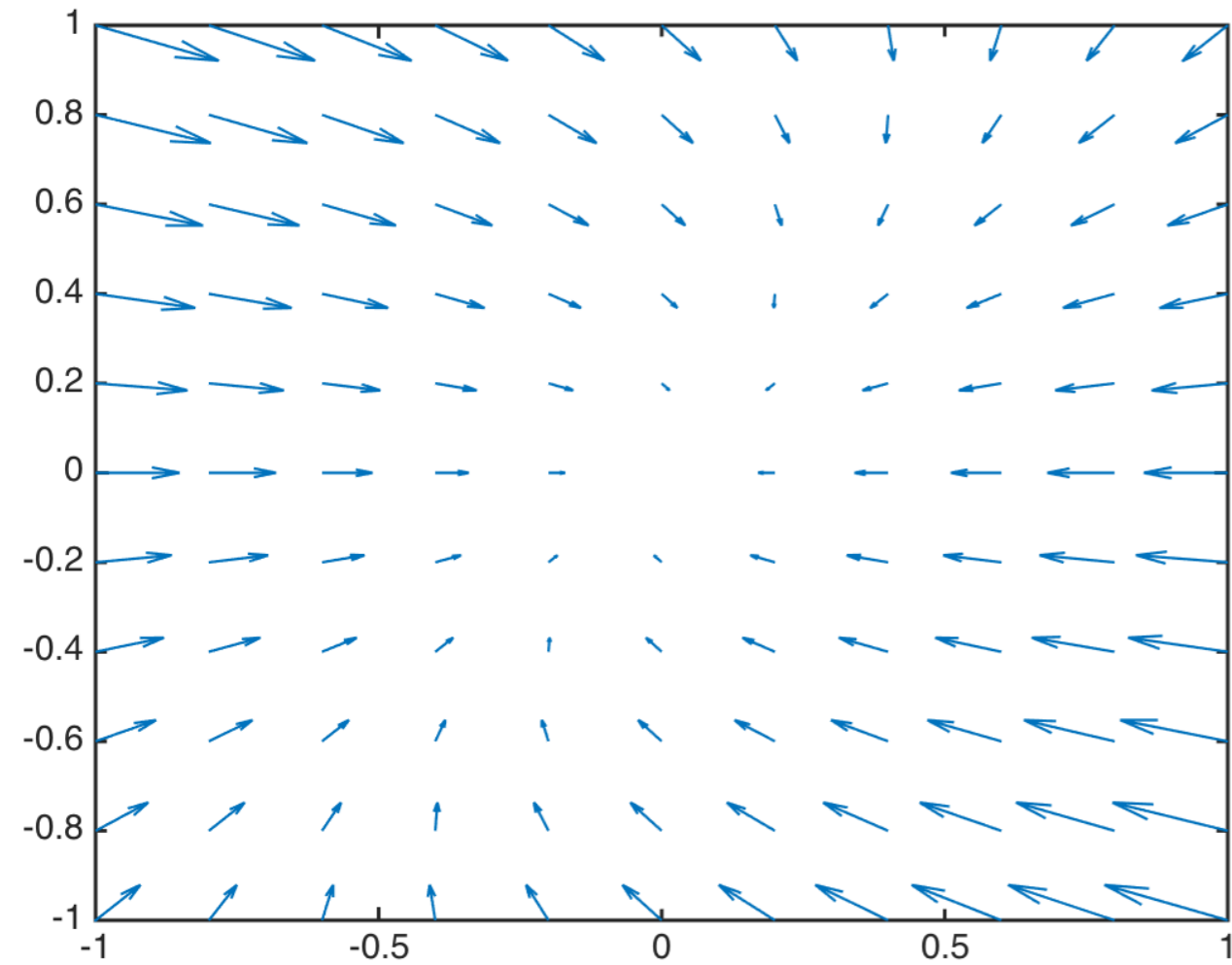


Unstable node

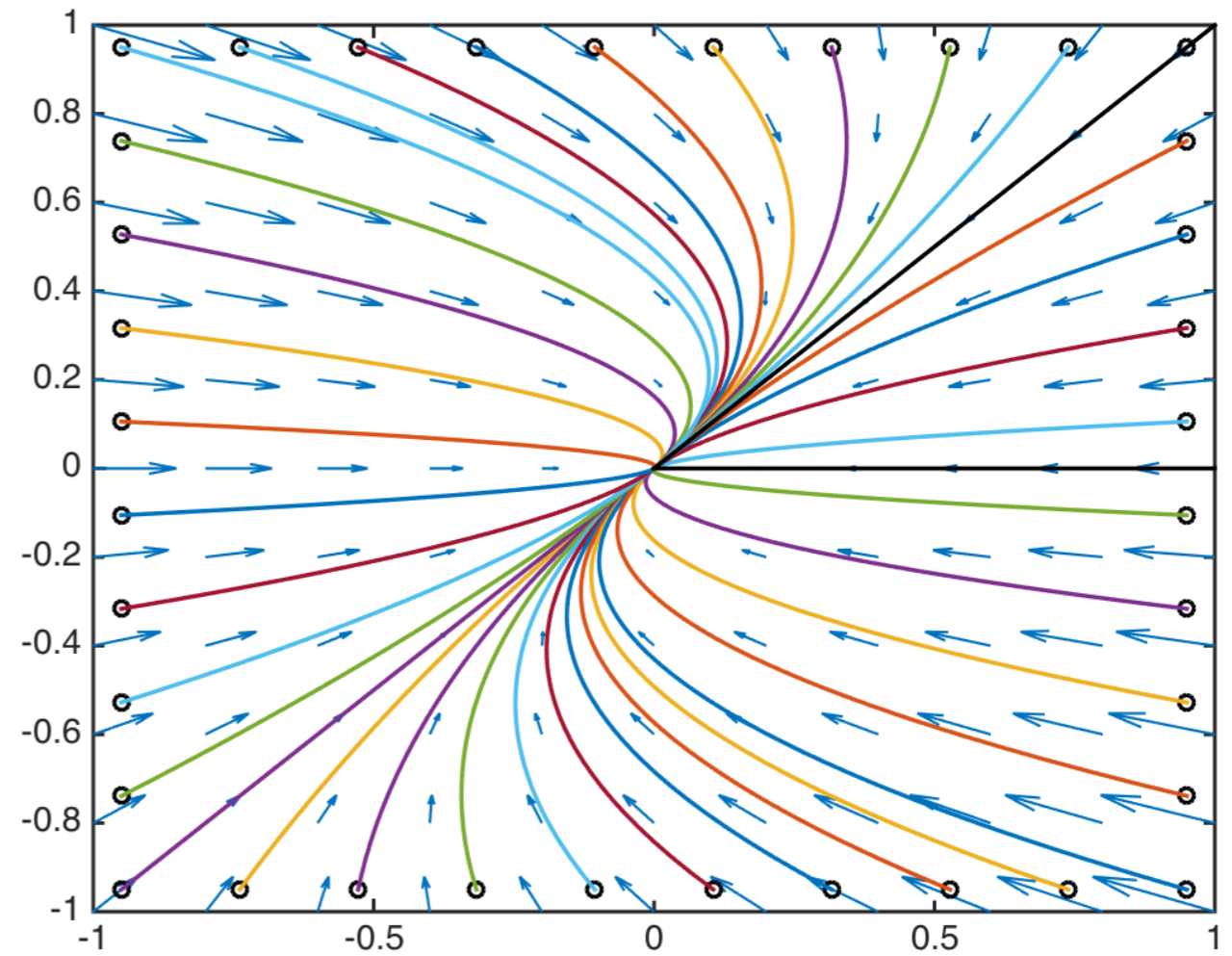
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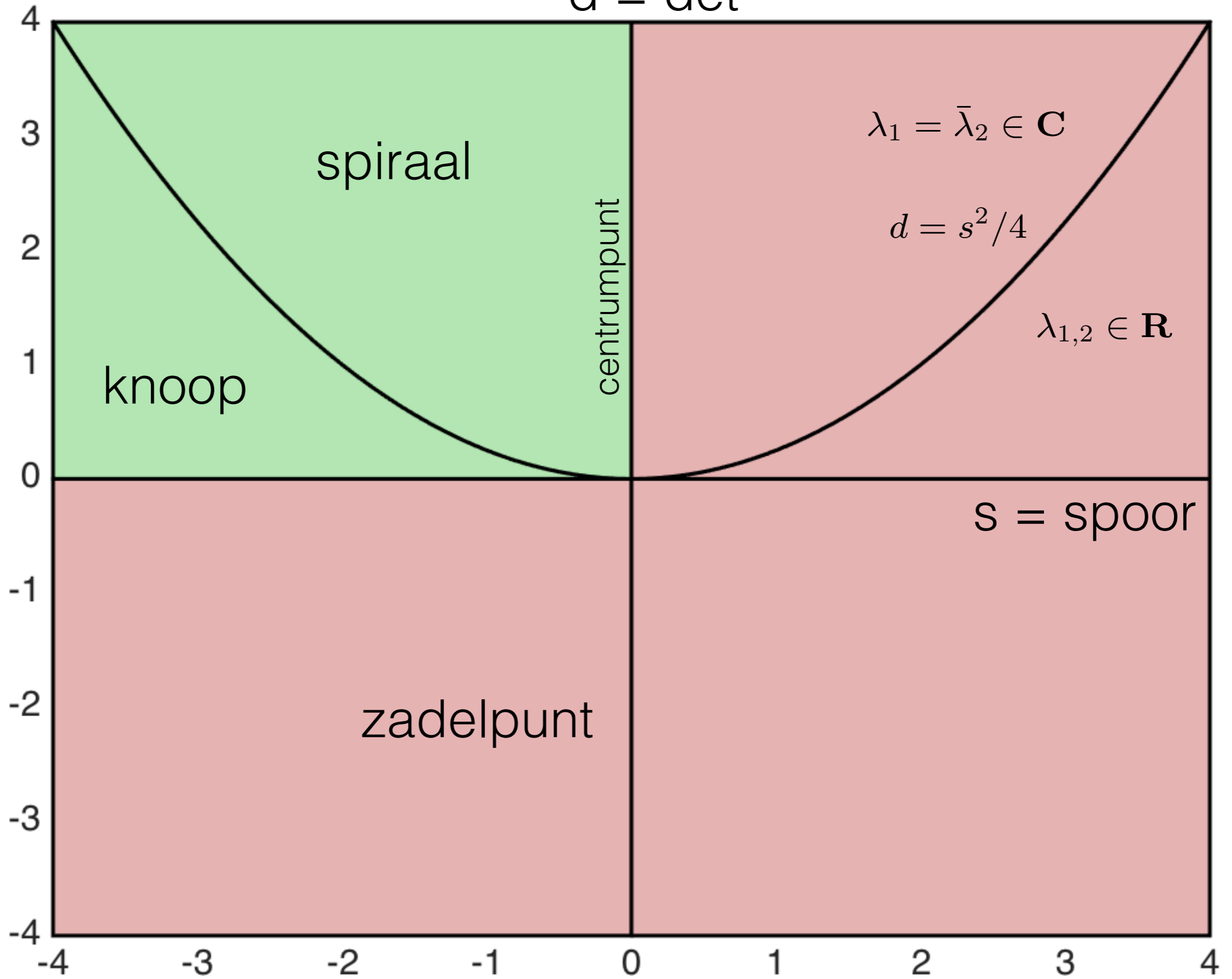


Phase portrait



Stable node

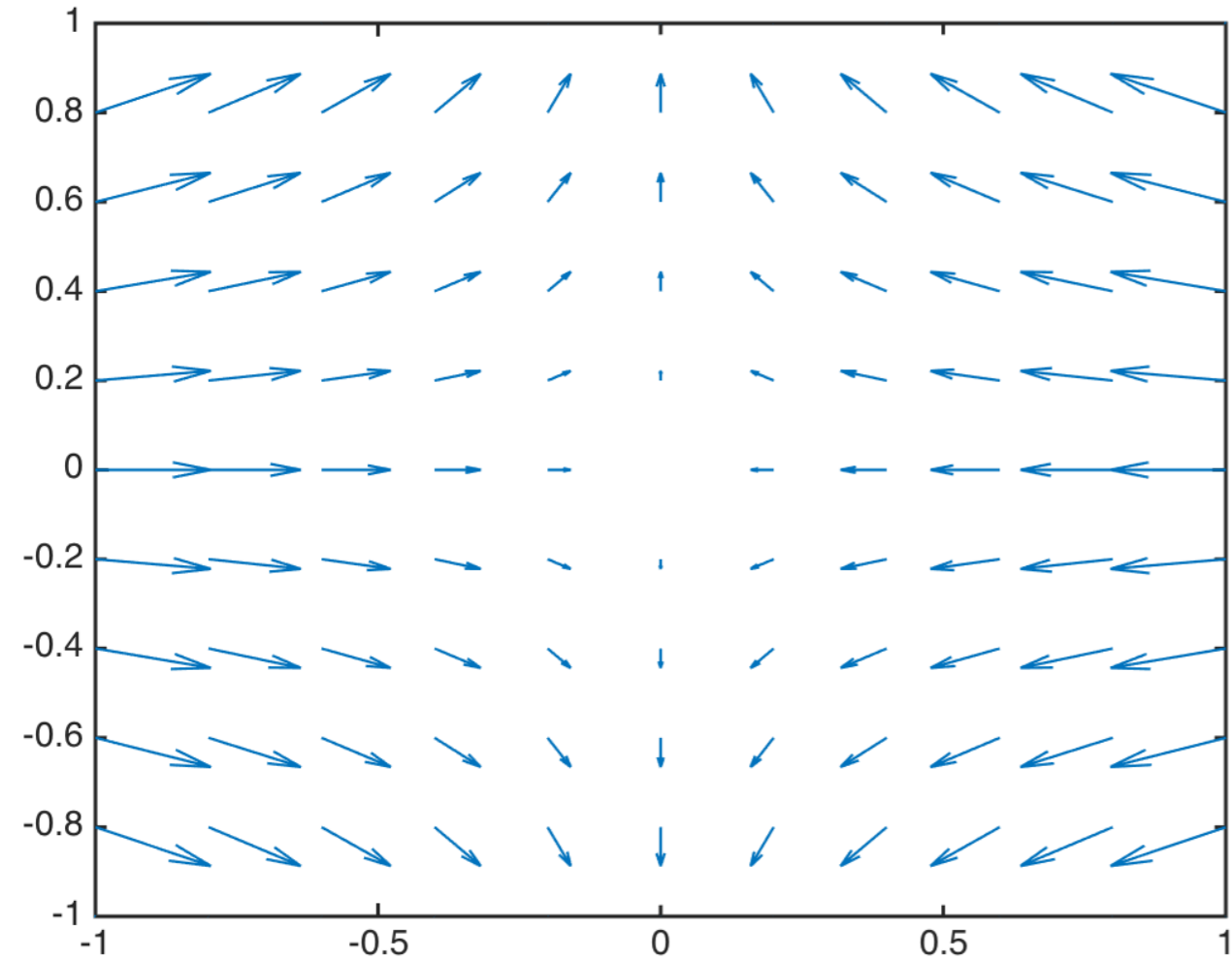
$d = \det$



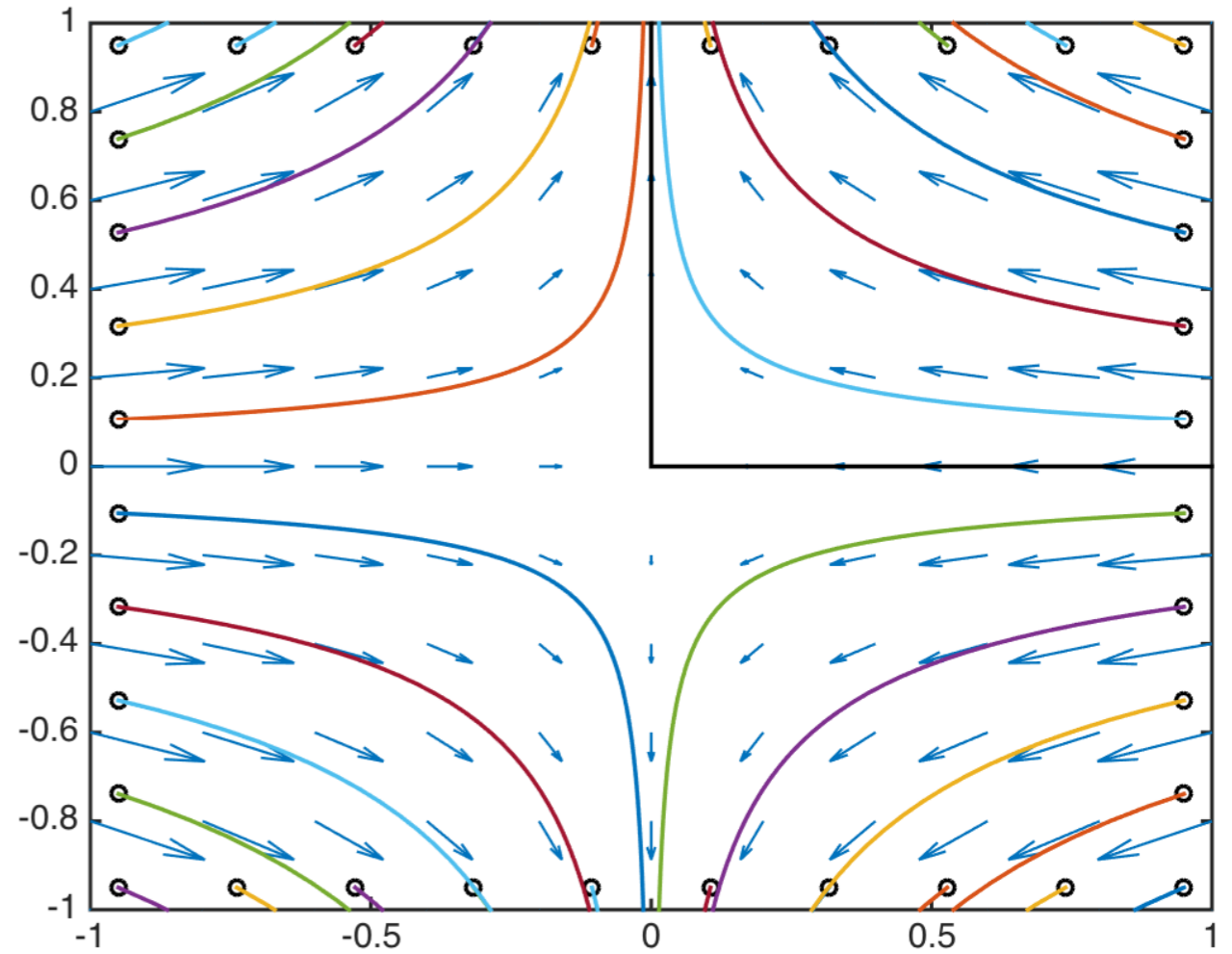
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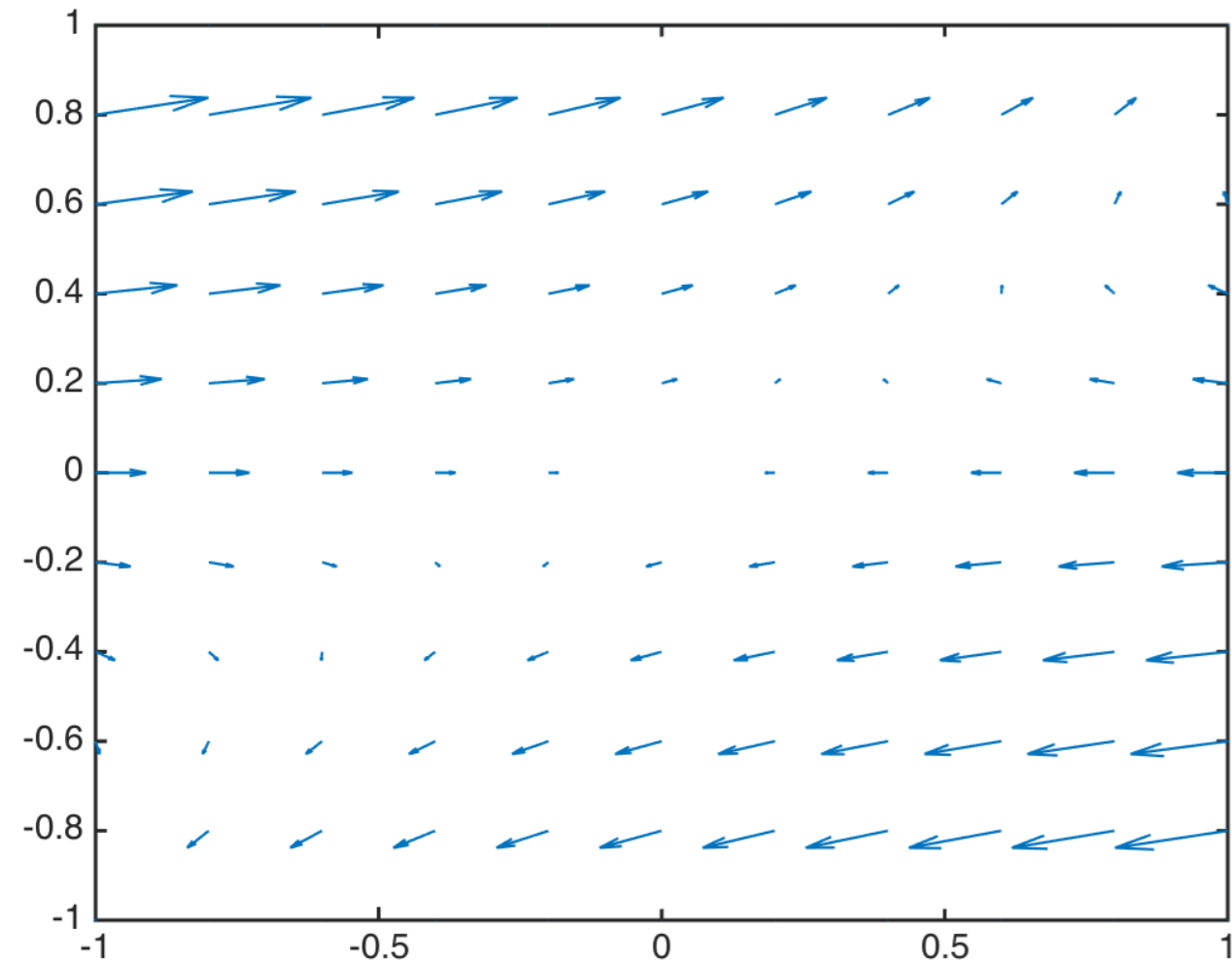


Saddle point

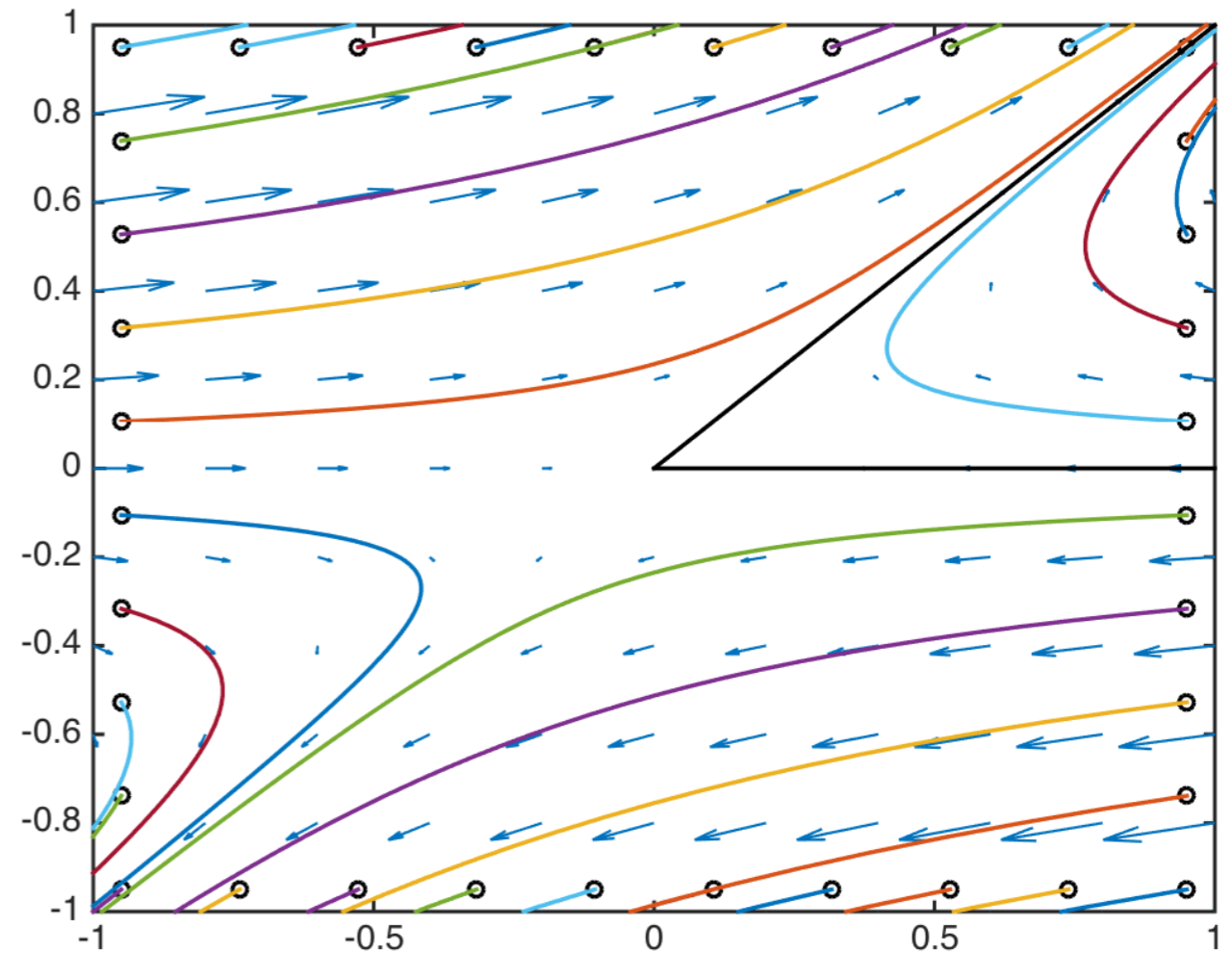
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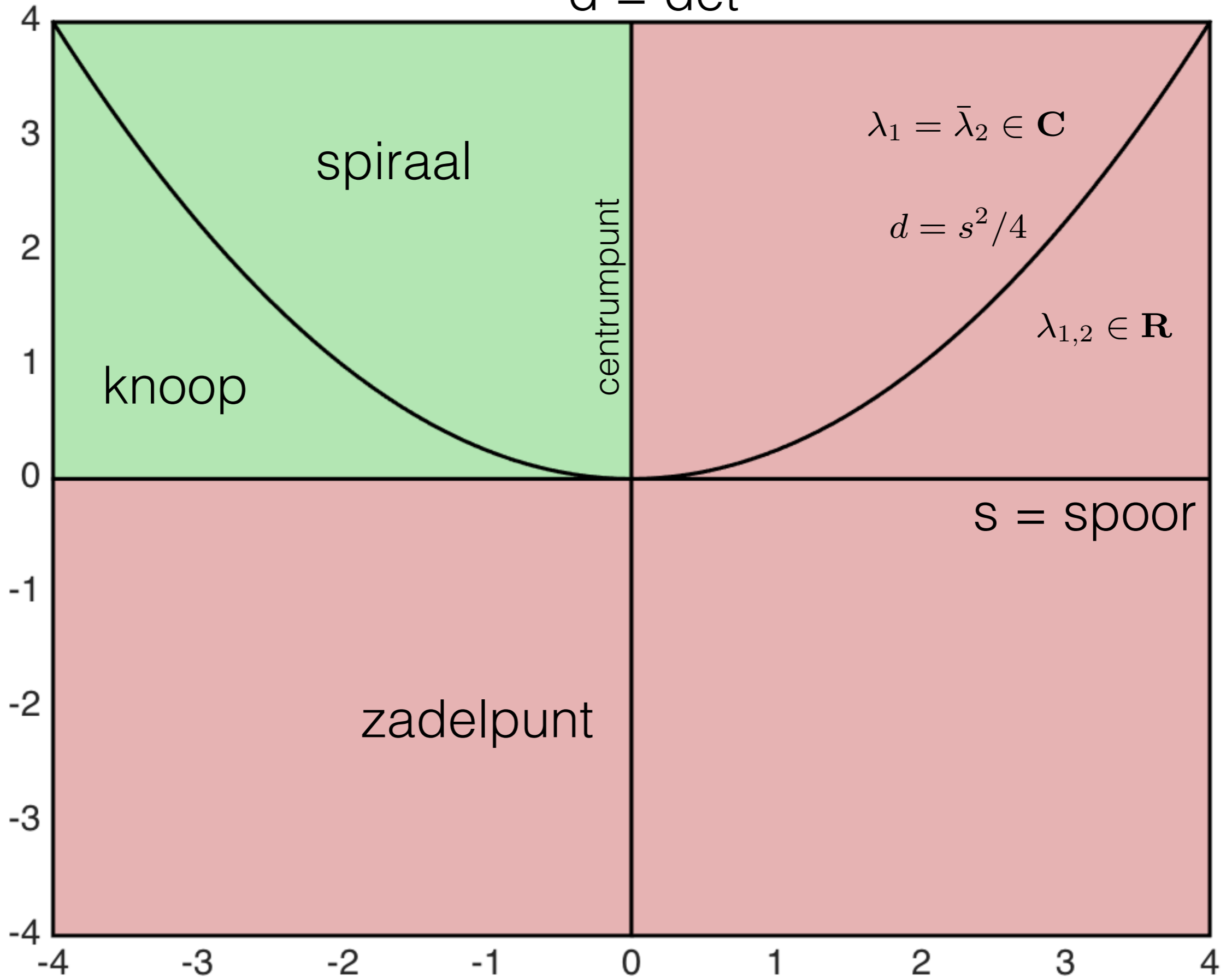


Phase portrait



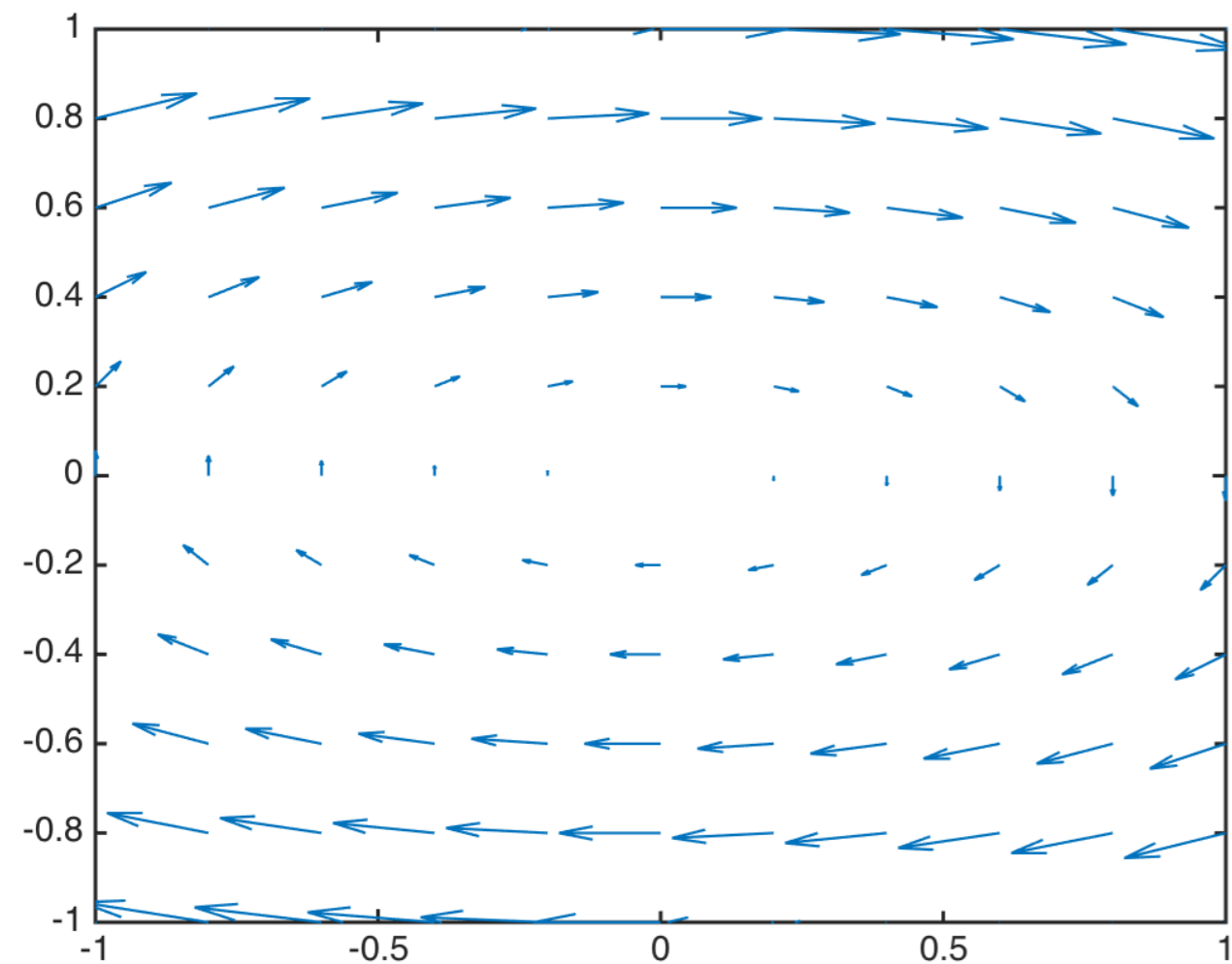
Saddle point

$d = \det$

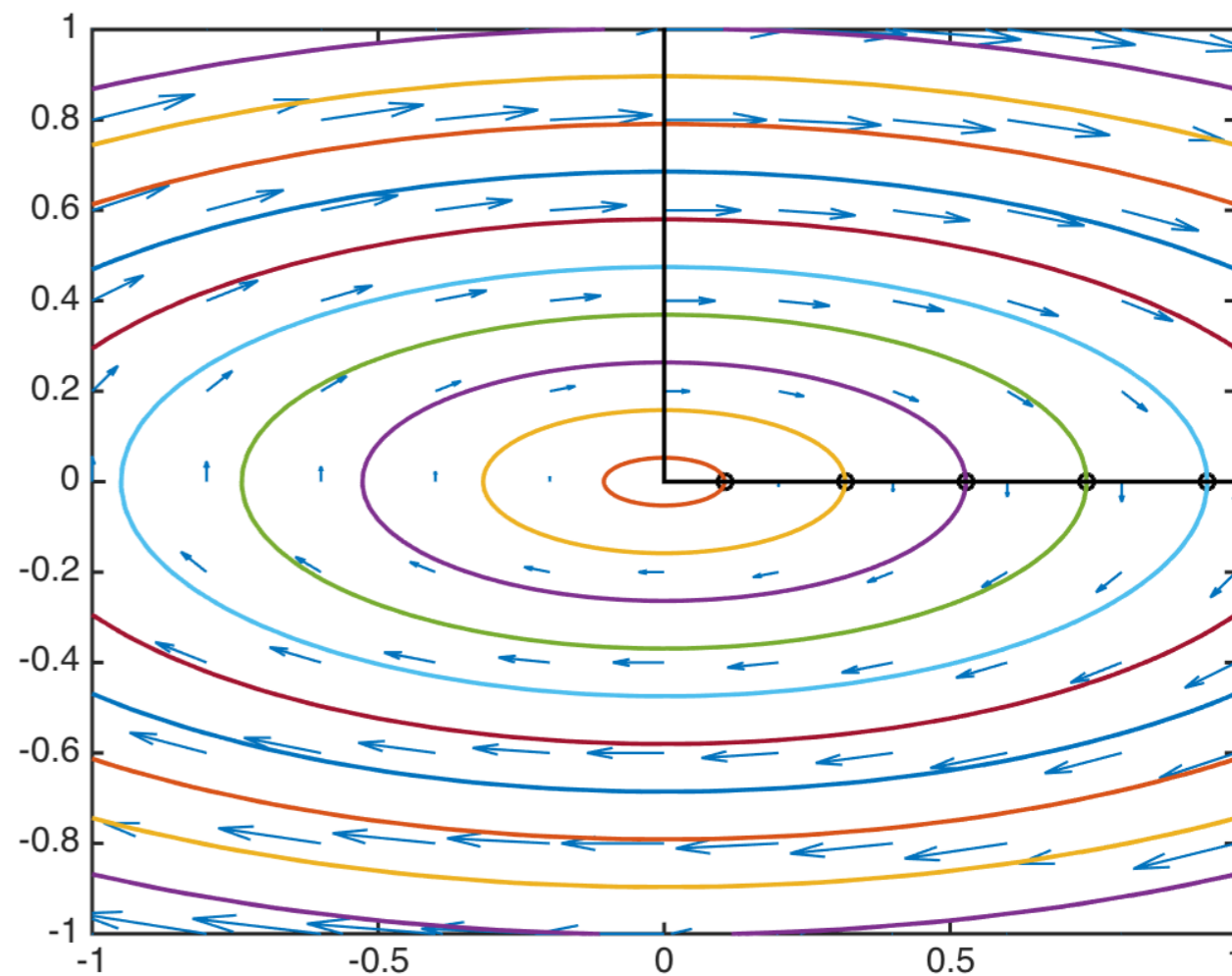


$$\lambda_{1,2} = \pm 2i$$

Vector Field



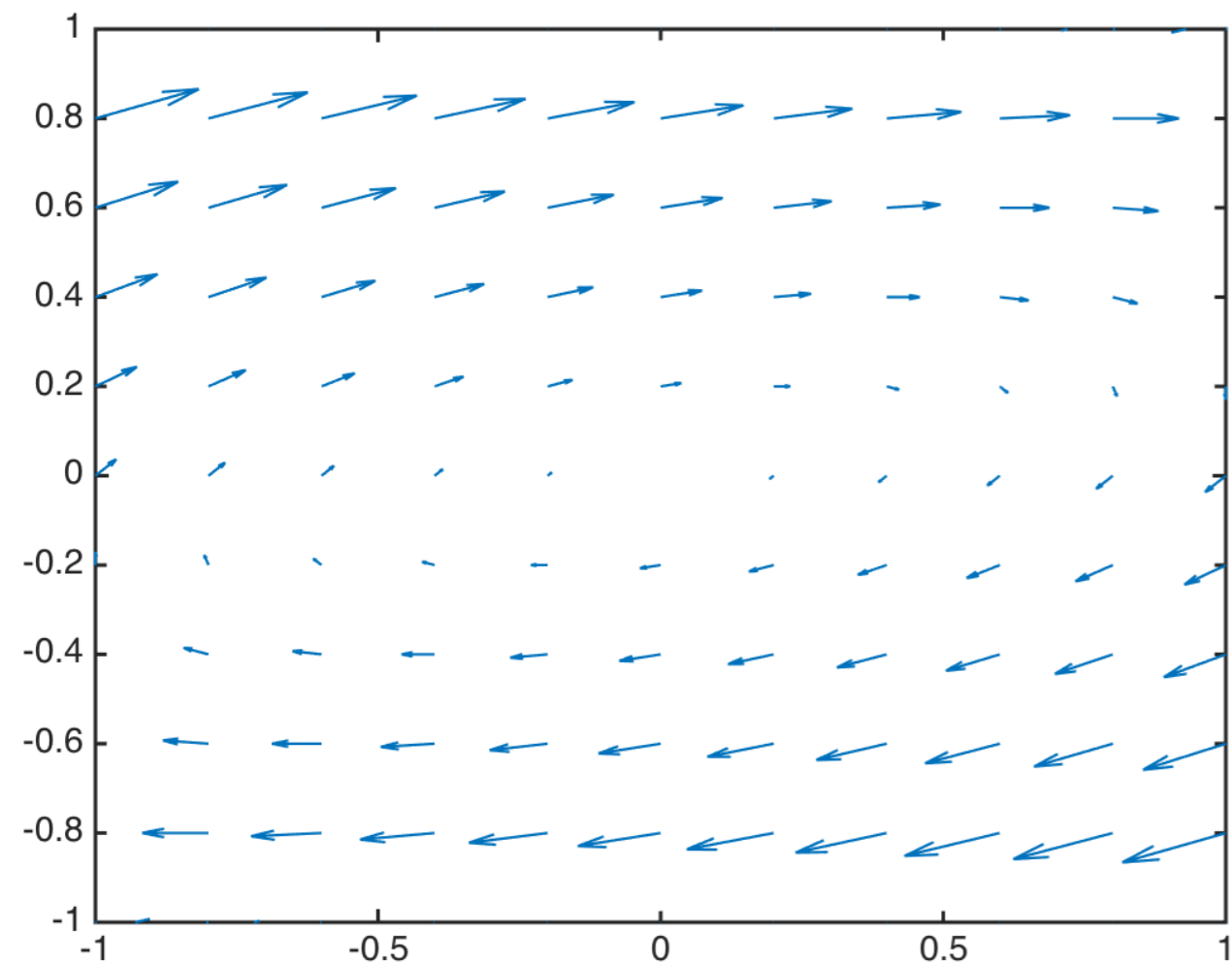
Phase portrait



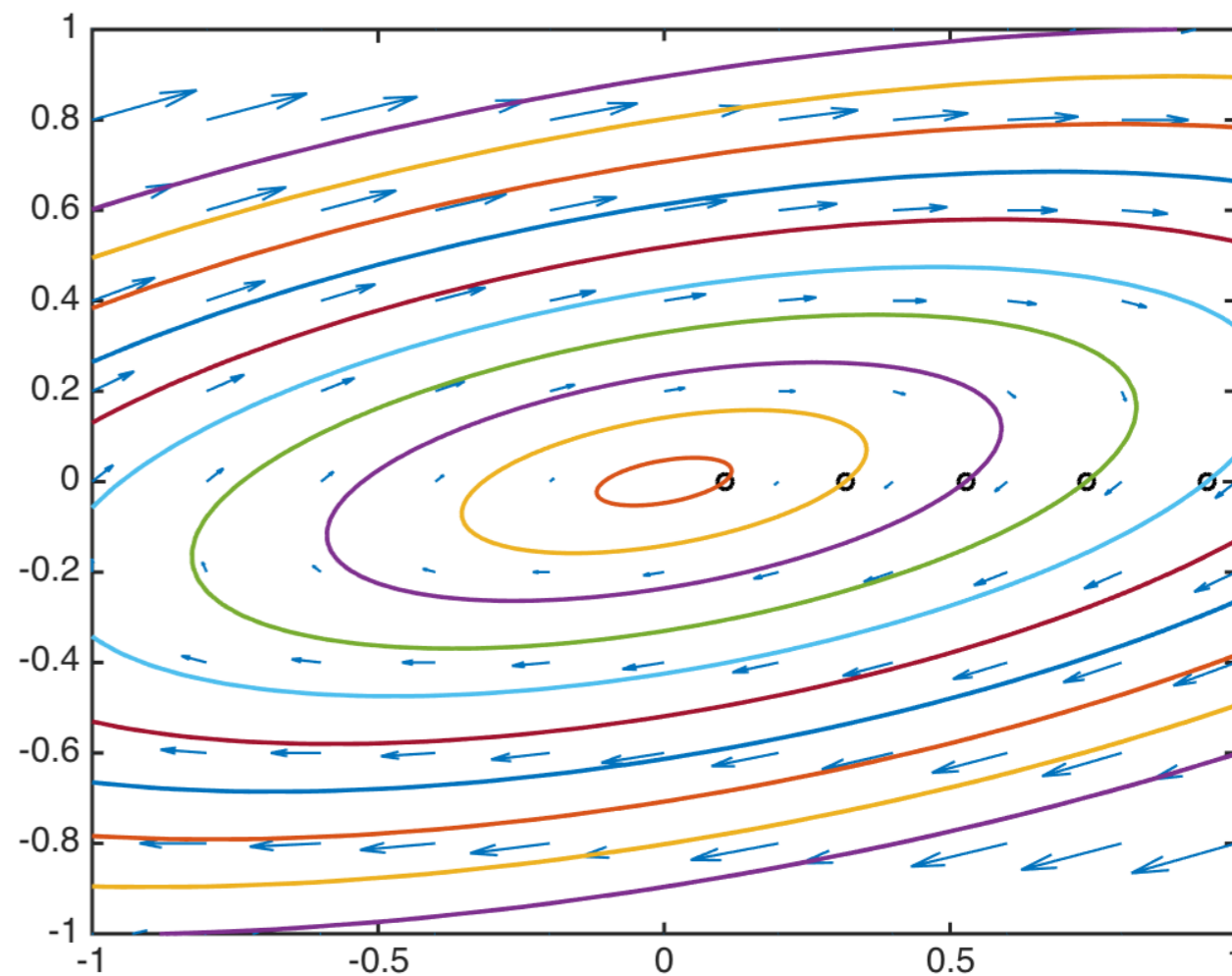
Center point

$$\lambda_{1,2} = \pm 2i$$

Vector Field

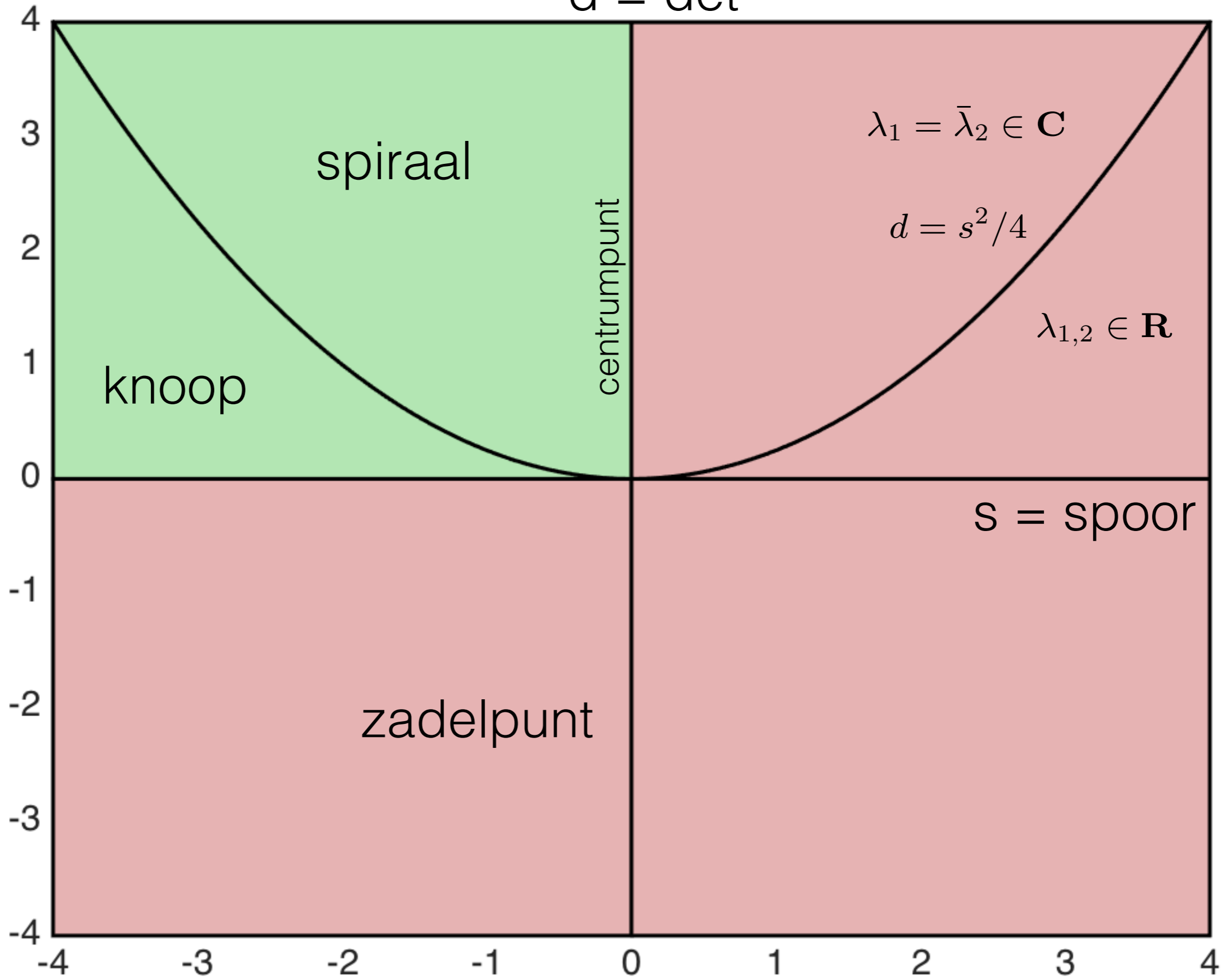


Phase portrait



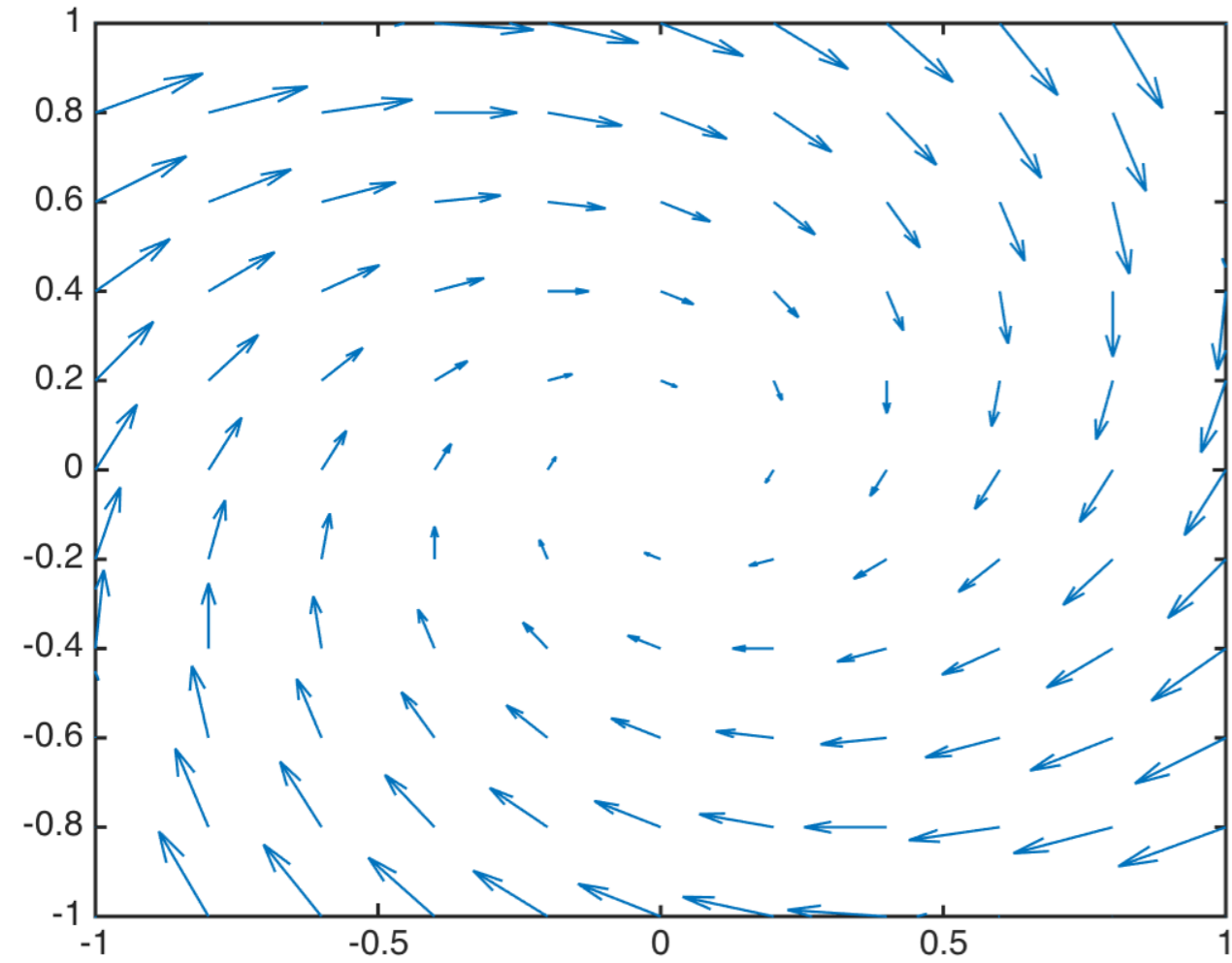
Center point

$d = \det$

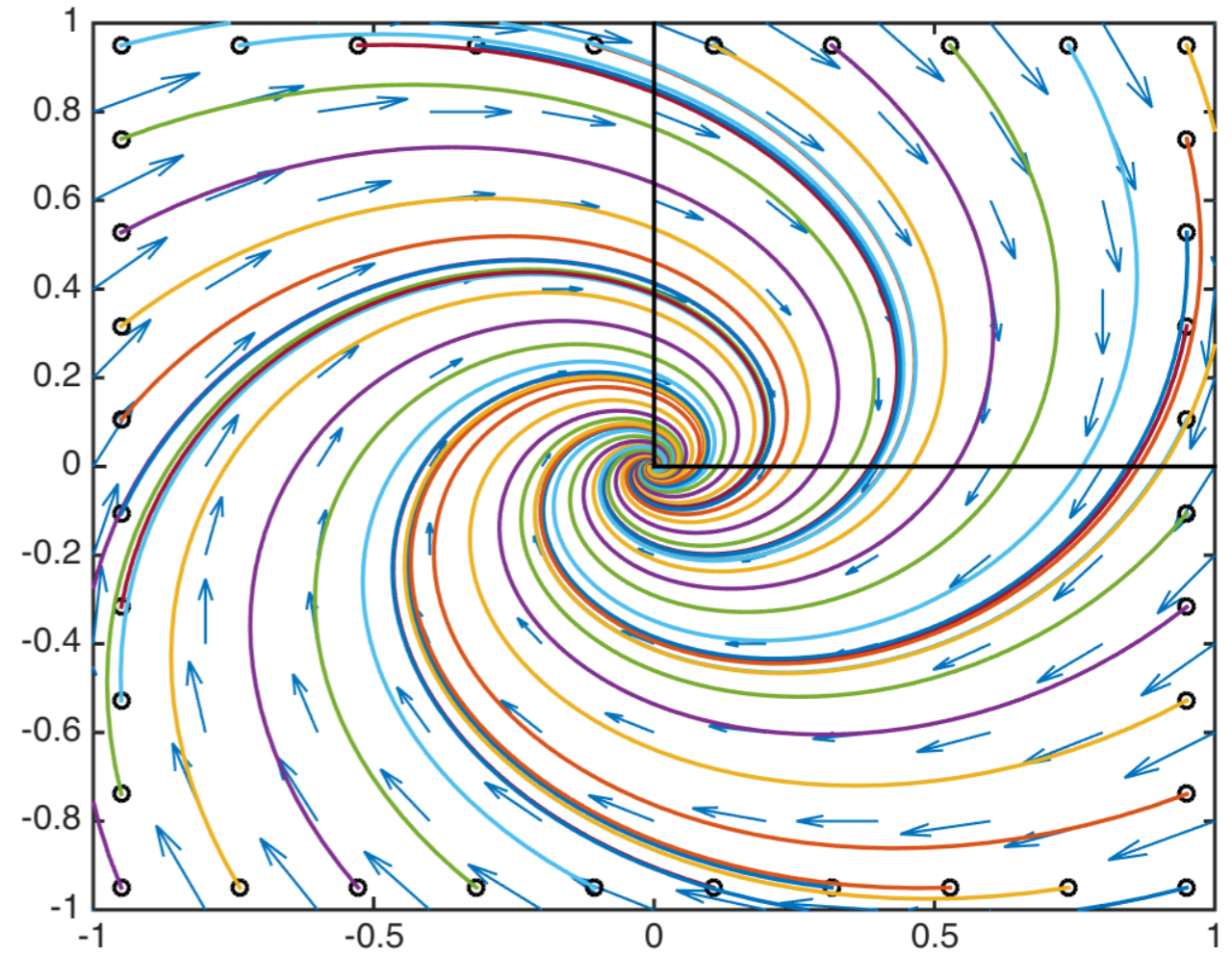


$$\lambda_{1,2} = -0.5 \pm 1.0 i$$

Vector Field



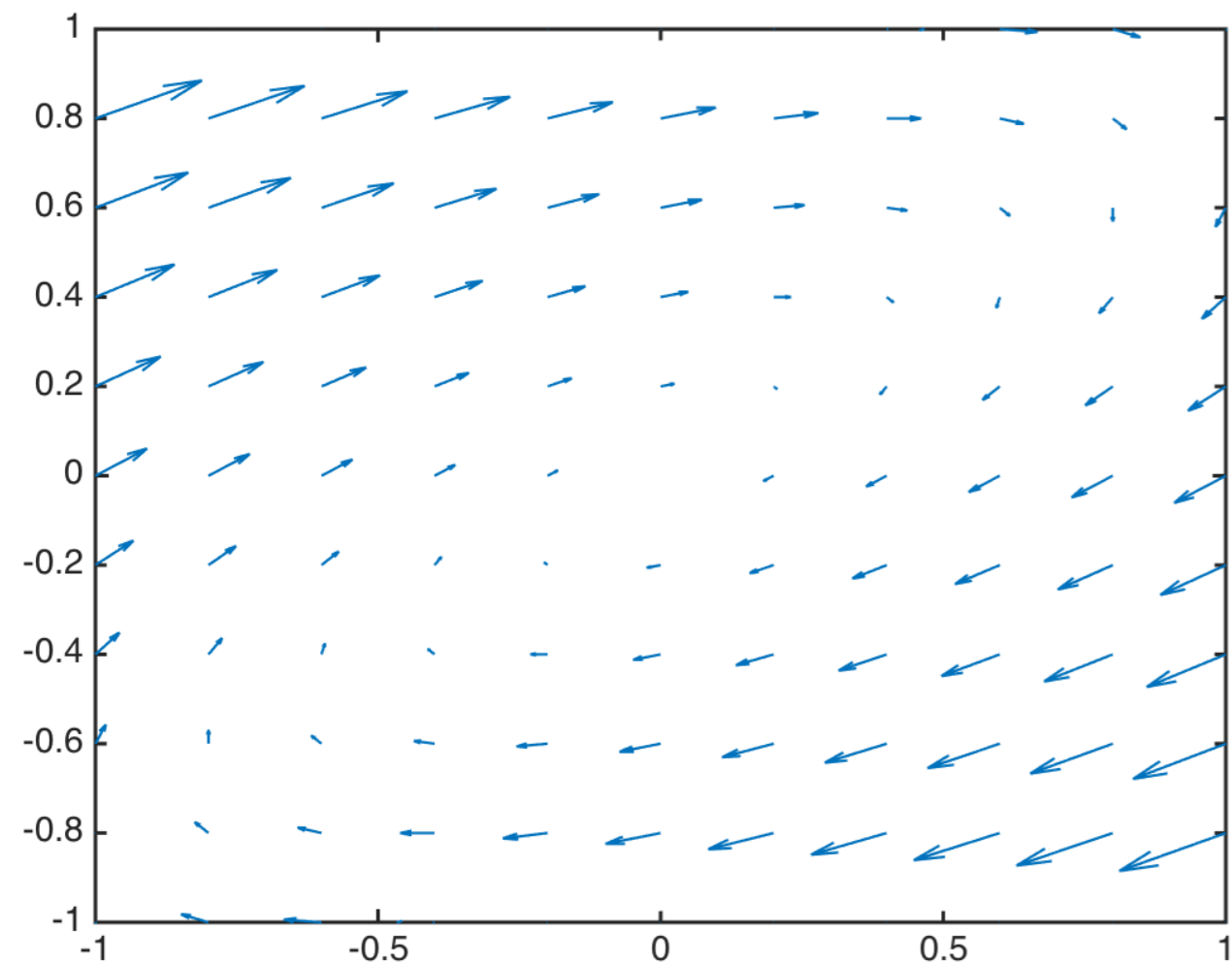
Phase portrait



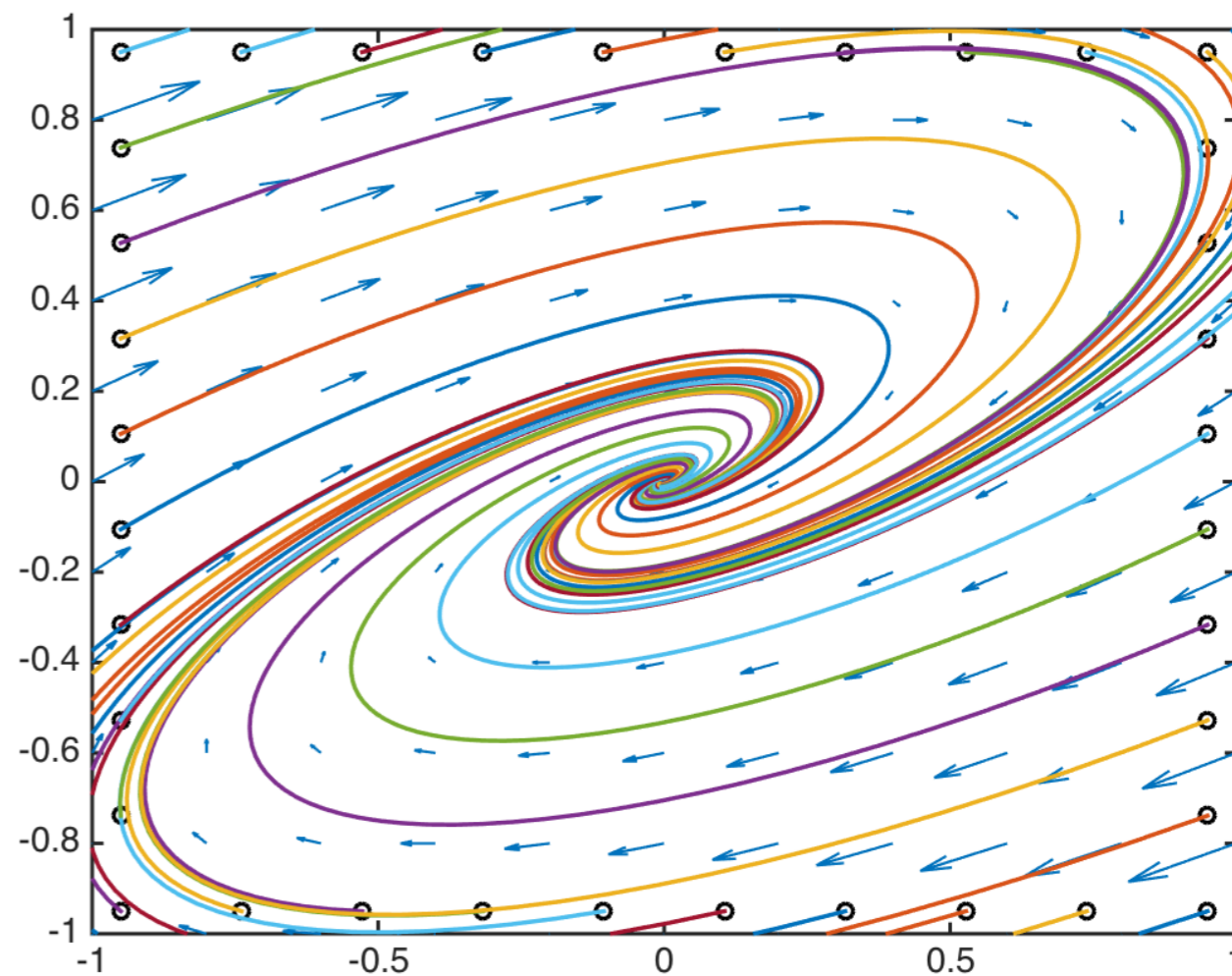
Stable spiral (focus)

$$\lambda_{1,2} = -0.5 \pm 1.0i$$

Vector Field

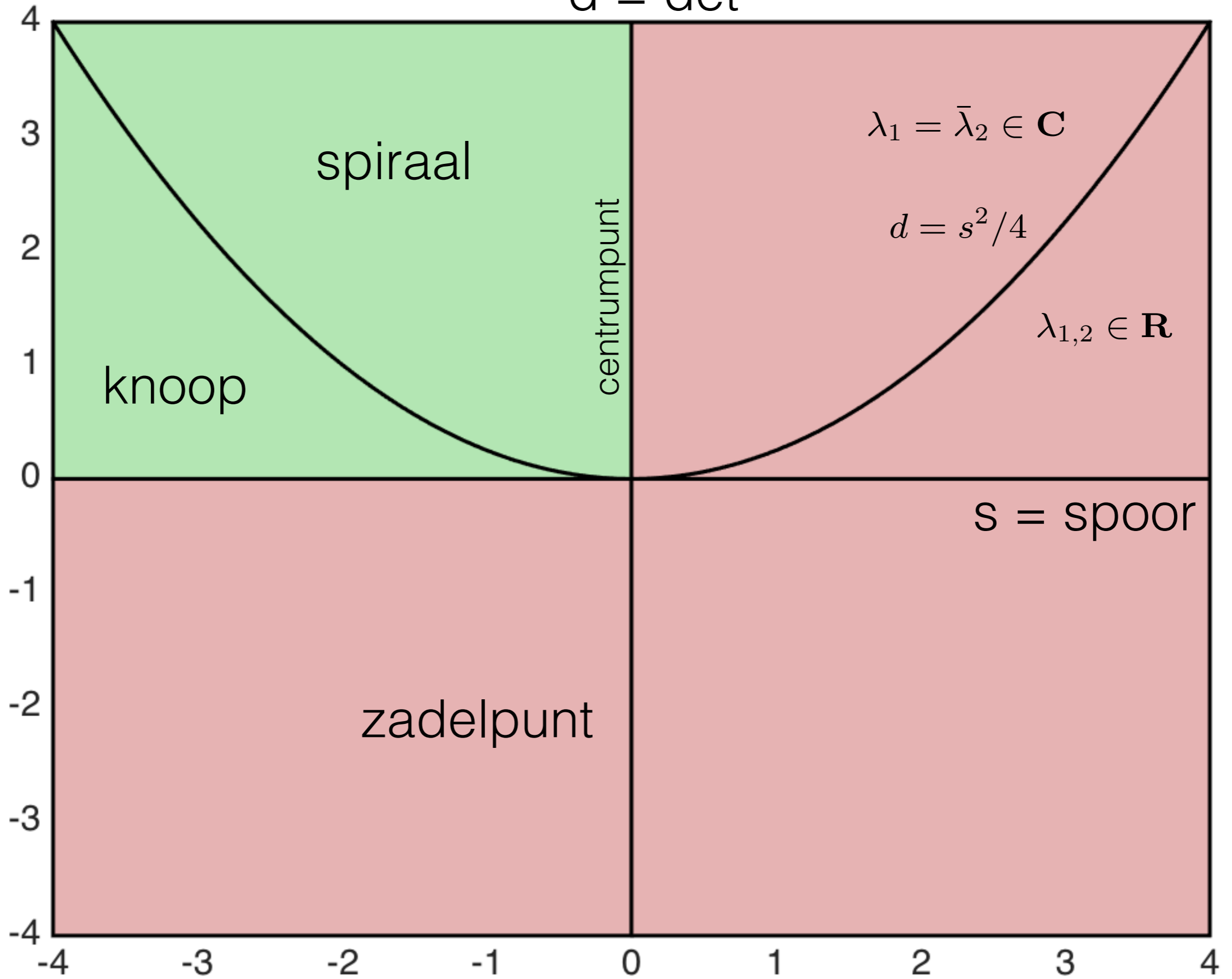


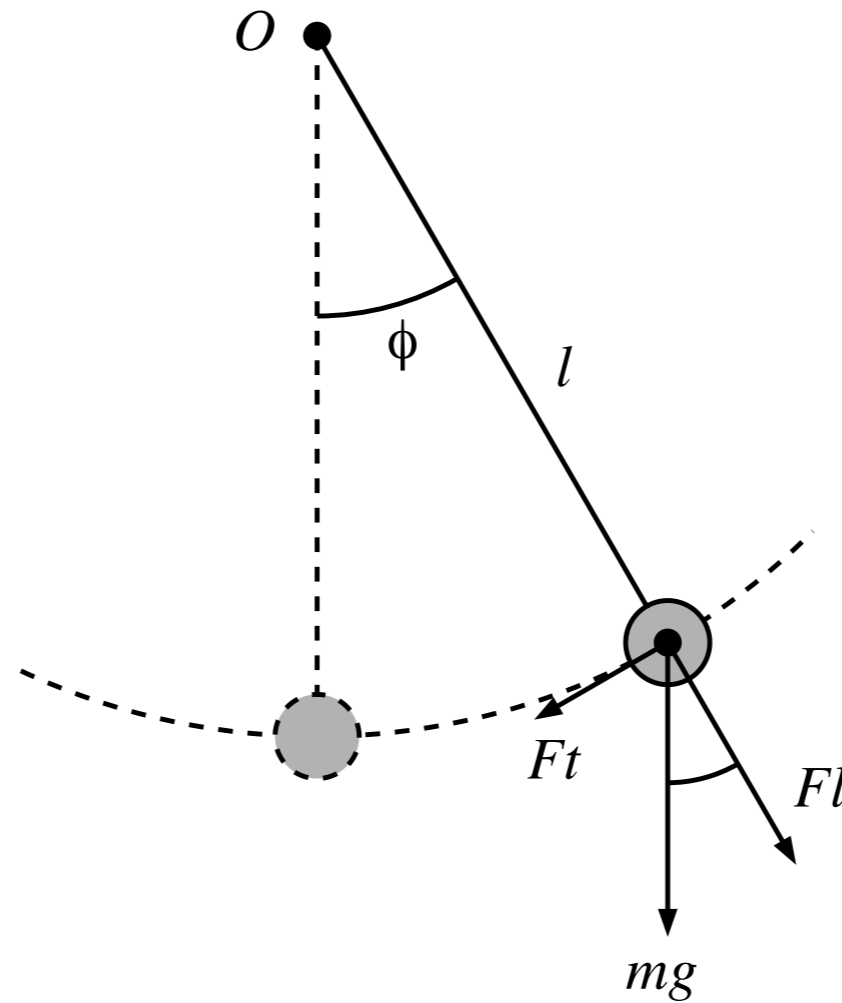
Phase portrait



Stable spiral (focus)

$d = \det$





Massa m slingert aan een gewichtsloos touw van lengte l .

Snelheid massa: $v = l \phi'$. Versnelling: $a = l \phi''$.

Effect zwaartekracht (in bewegingsrichting): $F_t = -m g \sin(\phi)$.

Wrijving: $F_w = -c l \phi'$. *Groote afremming evenredig snelheid.*

Newton. $F = ma \quad \Rightarrow \quad -m g \sin(\phi) - c l \phi' = m l \phi''$

Slinger

$$\phi'' = -\frac{c}{m} \phi' - \frac{g}{\ell} \phi$$

$$c = 0: \quad \lambda_1 = \overline{\lambda_2} = i\nu \quad \text{met} \quad \nu \equiv \sqrt{\frac{g}{\ell}}$$

$$\phi(t) = \operatorname{Re}(\gamma e^{i\nu t}) = |\gamma| \cos(\nu t + \delta)$$

voor zekere constante $\gamma \in \mathbb{C}$.

Omdat $\gamma \in \mathbb{C}$ is $\gamma = |\gamma| e^{i\delta}$ voor zekere $\delta \in [0, 2\pi)$.

Schrijf $C \equiv |\gamma|$. Dan $C \in \mathbb{R}$.

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voor zekere constanten $C \in [0, \infty)$ en $\delta \in [0, 2\pi)$.

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De oplossing ϕ is een **harmonische oscillatie**

met **frequentie** ν ,

fase δ en

amplitude C .

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voor zekere constanten $C \in [0, \infty)$ en $\delta \in [0, 2\pi)$.

C en δ hangen af van de **begin voorwaarden**.

Voorbeeld. Als $\phi(0) = \alpha_0$ en $\phi'(0) = 0$ dan

$$\phi(t) = \alpha_0 \cos(\nu t).$$

Slinger

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voor zekere constanten $C \in [0, \infty)$ en $\delta \in [0, 2\pi)$.

$$c > 0: \quad \lambda_{1,2} = -\frac{c}{2m} \pm \sqrt{\frac{c^2}{4m^2} - \nu^2} = -\rho \pm \sqrt{\rho^2 - \nu^2}$$

$$\text{met} \quad \rho \equiv \frac{c}{2m} = -\text{Re}(\lambda_1) \quad \text{en} \quad \nu \equiv \sqrt{\frac{g}{\ell}}.$$

λ_1 en λ_2 zijn de nulpunten van

$$p(\lambda) \equiv \lambda^2 + 2\rho\lambda + \nu^2.$$

Slinger

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$$\rho < \nu: \quad \sqrt{\rho^2 - \nu^2} = i\sqrt{\nu^2 - \rho^2} \quad \text{en}$$

$$\phi(t) = \text{Re} \left(\gamma \exp(-\rho t + i\sqrt{\nu^2 - \rho^2} t) \right)$$

$$\gamma = |\gamma| e^{i\delta} = C e^{i\delta}.$$

Slinger

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$$\rho < \nu: \quad \phi(t) = C \exp(-\rho t) \cos(\sqrt{\nu^2 - \rho^2} t + \delta)$$

Damping $\rho > 0$: **Oscillerende demping** als $\rho < \nu$

Amplitude verkleint in de tijd met de factor $e^{-\rho t}$

De **frequentie verschuift** van ν naar $\sqrt{\nu^2 - \rho^2}$.

Slinger

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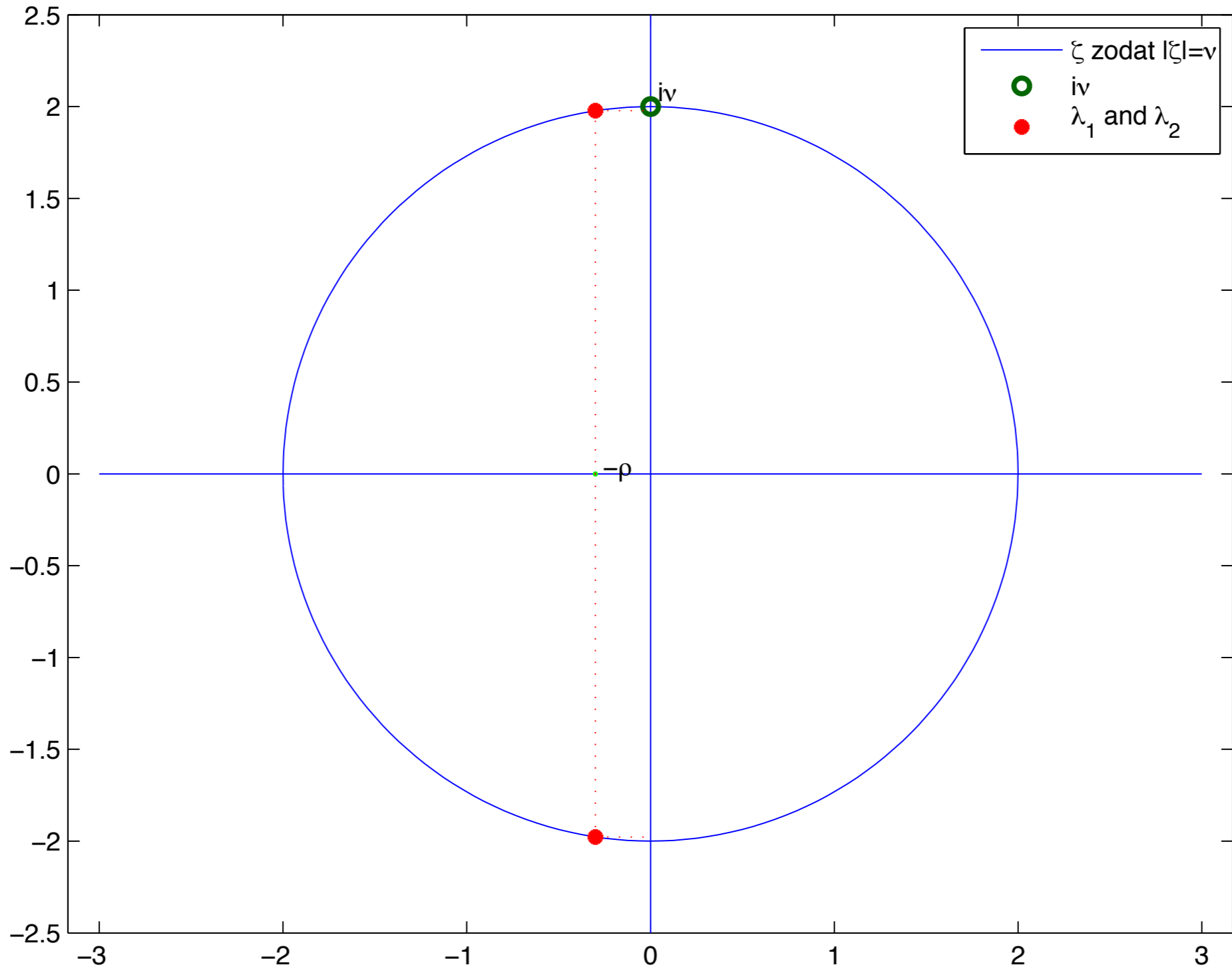
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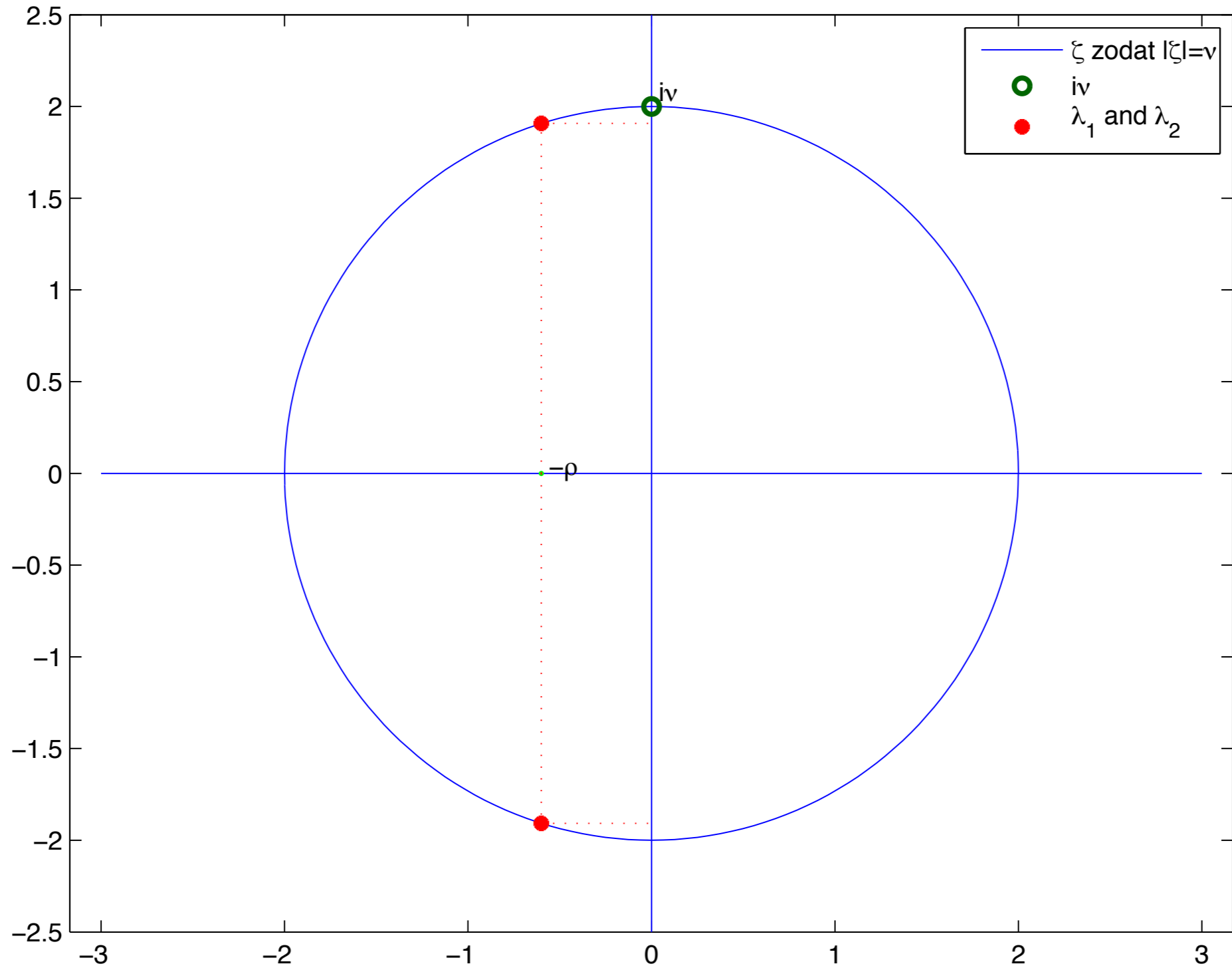
$$\rho < \nu: \quad \phi(t) = C \exp(-\rho t) \cos(\sqrt{\nu^2 - \rho^2} t + \delta)$$

Opmerking. $|\lambda_1| = |\lambda_2| = \nu$. Als $\rho \ll \nu$ dan $\sqrt{\nu^2 - \rho^2} \approx \nu$.

In het complexe vlak, λ_1 en λ_2 met $-\rho = \text{Re}(\lambda_1) < 0$ en $|\lambda_1| = v$



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$$\phi(t) = C \cos(\nu t + \delta)$$

voor zekere constanten $C \in [0, \infty)$ en $\delta \in [0, 2\pi)$.

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Opmerking. $|\lambda_1| = |\lambda_2| = \nu$. Als $\rho \ll \nu$ dan $\sqrt{\nu^2 - \rho^2} \approx \nu$.

Conclusie. *Damping is 'sterker' dan frequentieverschuiving.*

Vb. $\nu = 1$, $\frac{c}{m} = 0.1$. Dan $\sqrt{\rho^2 - \nu^2} = i\sqrt{0.9975} = i0.9987 \dots$

$$\phi(t) = C \exp(-0.05 t) \cos(0.9987 t + \delta).$$

Slinger

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voor zekere constanten $C \in [0, \infty)$ en $\delta \in [0, 2\pi)$.

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$$\rho < \nu: \quad \phi(t) = C \exp(-\rho t) \cos(\sqrt{\nu^2 - \rho^2} t + \delta)$$

$$\rho \geq \nu: \quad \sqrt{\rho^2 - \nu^2} \in \mathbb{R}$$

$$\phi(t) = C_1 \exp([- \rho + \sqrt{\rho^2 - \nu^2}]t) + C_2 \exp([- \rho - \sqrt{\rho^2 - \nu^2}]t)$$

$$\phi(t) \approx C_1 \exp([- \rho + \sqrt{\rho^2 - \nu^2}]t) \quad \text{voor } t \rightarrow \infty$$

Slinger

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voor zekere constanten $C \in [0, \infty)$ en $\delta \in [0, 2\pi)$.

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$$\rho < \nu: \quad \phi(t) = C \exp(-\rho t) \cos(\sqrt{\nu^2 - \rho^2} t + \delta)$$

$$\rho \geq \nu: \quad \phi(t) \approx C_1 \exp\left([- \rho + \sqrt{\rho^2 - \nu^2}]t\right) \quad \text{voor } t \rightarrow \infty$$

Damping $\rho > 0$: **Kritische demping** als $\rho = \nu$.

Overdamping als $\rho > \nu$

Slinger

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voor zekere constanten $C \in [0, \infty)$ en $\delta \in [0, 2\pi)$.

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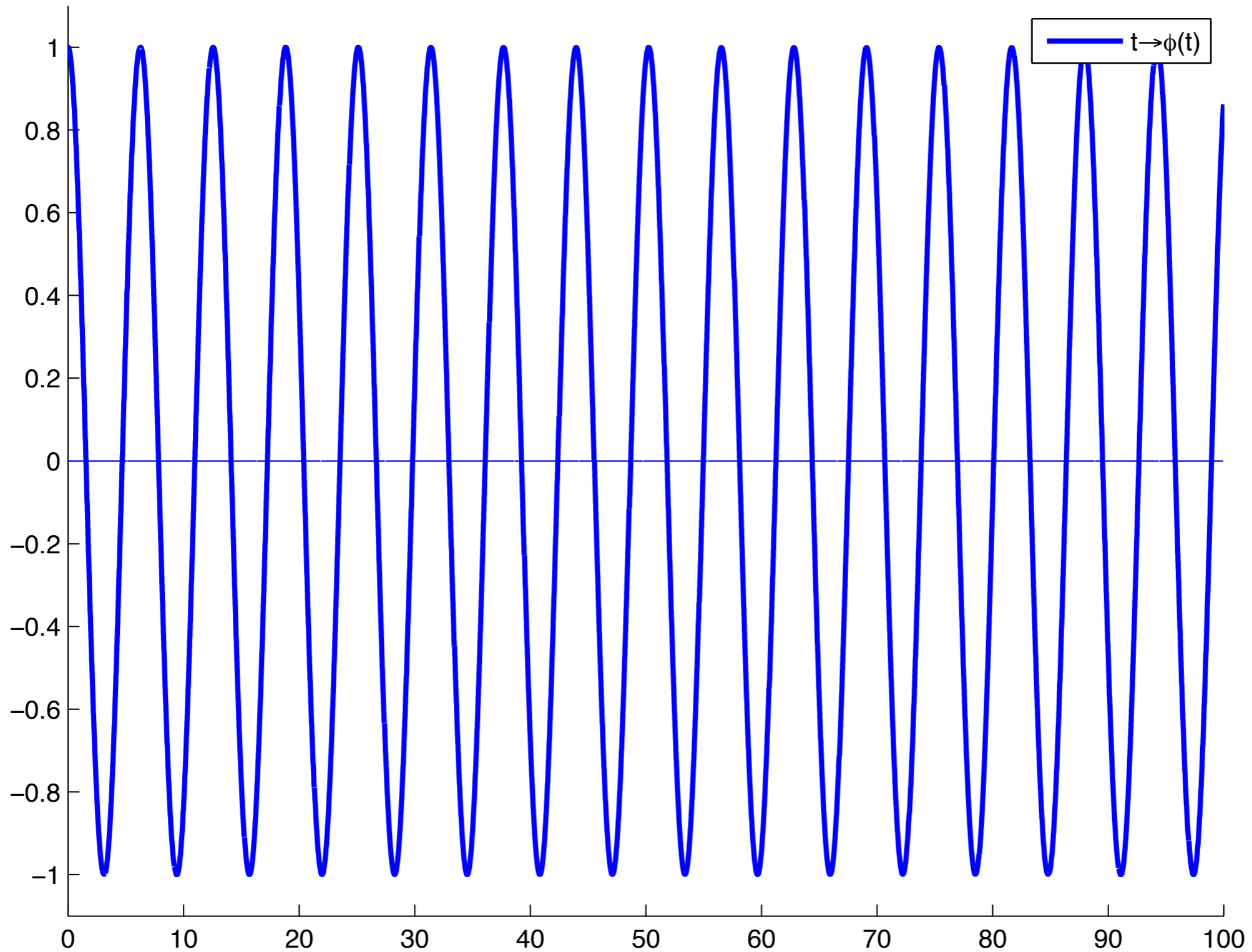
$$\rho < \nu: \quad \phi(t) = C \exp(-\rho t) \cos(\sqrt{\nu^2 - \rho^2} t + \delta)$$

Damping $\rho > 0$: **oscillerende demping** als $\rho < \nu$
 kritische demping als $\rho = \nu$
 overdemping als $\rho > \nu$

Kritische demping: “snelst” terug naar de rust situatie

Geen demping, harmonische oscillatie ($0 = \rho$)

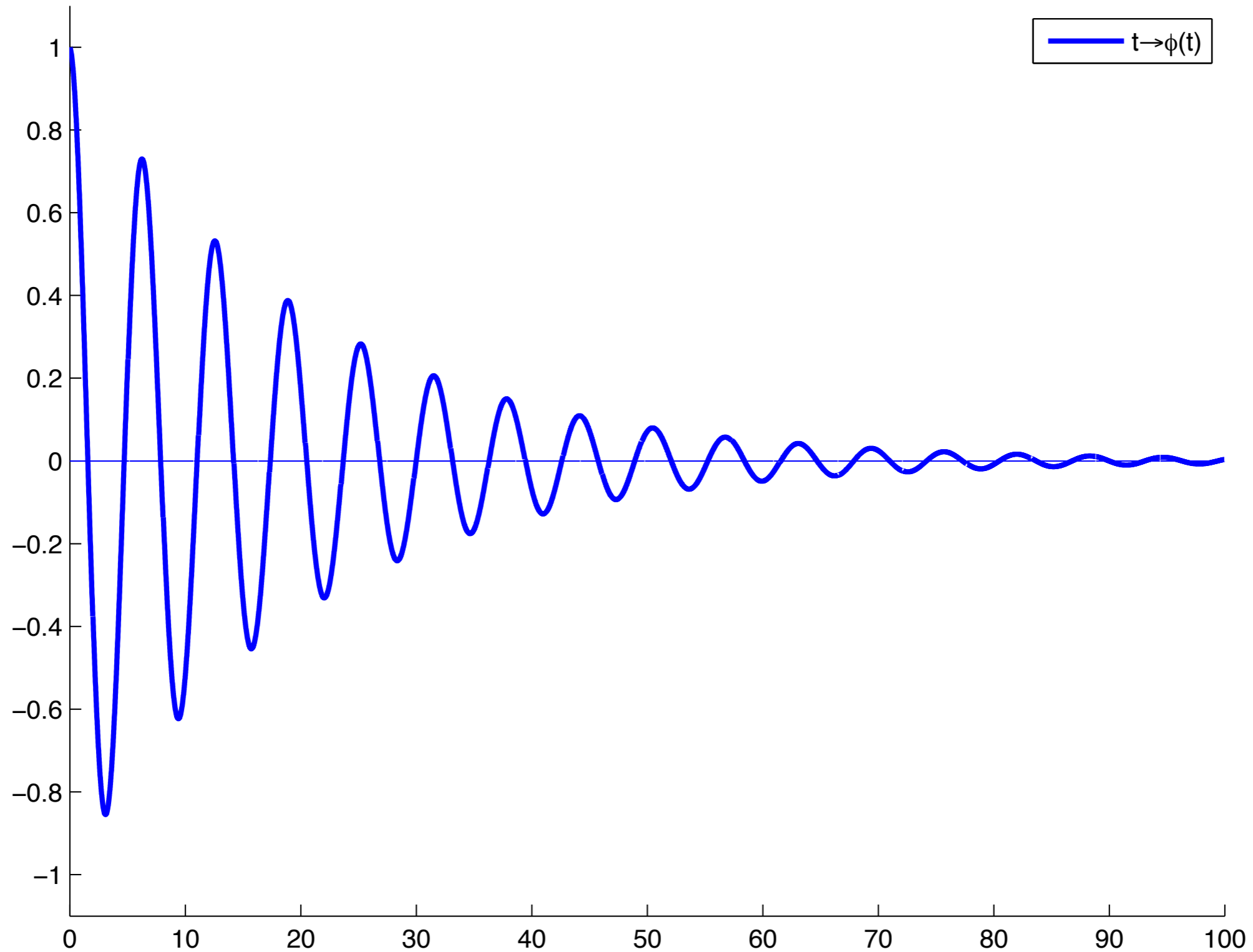
Gedempte slinger, $\nu=1$, $c/l=0$



Oscillerende demping

$$(0 < \rho < \nu)$$

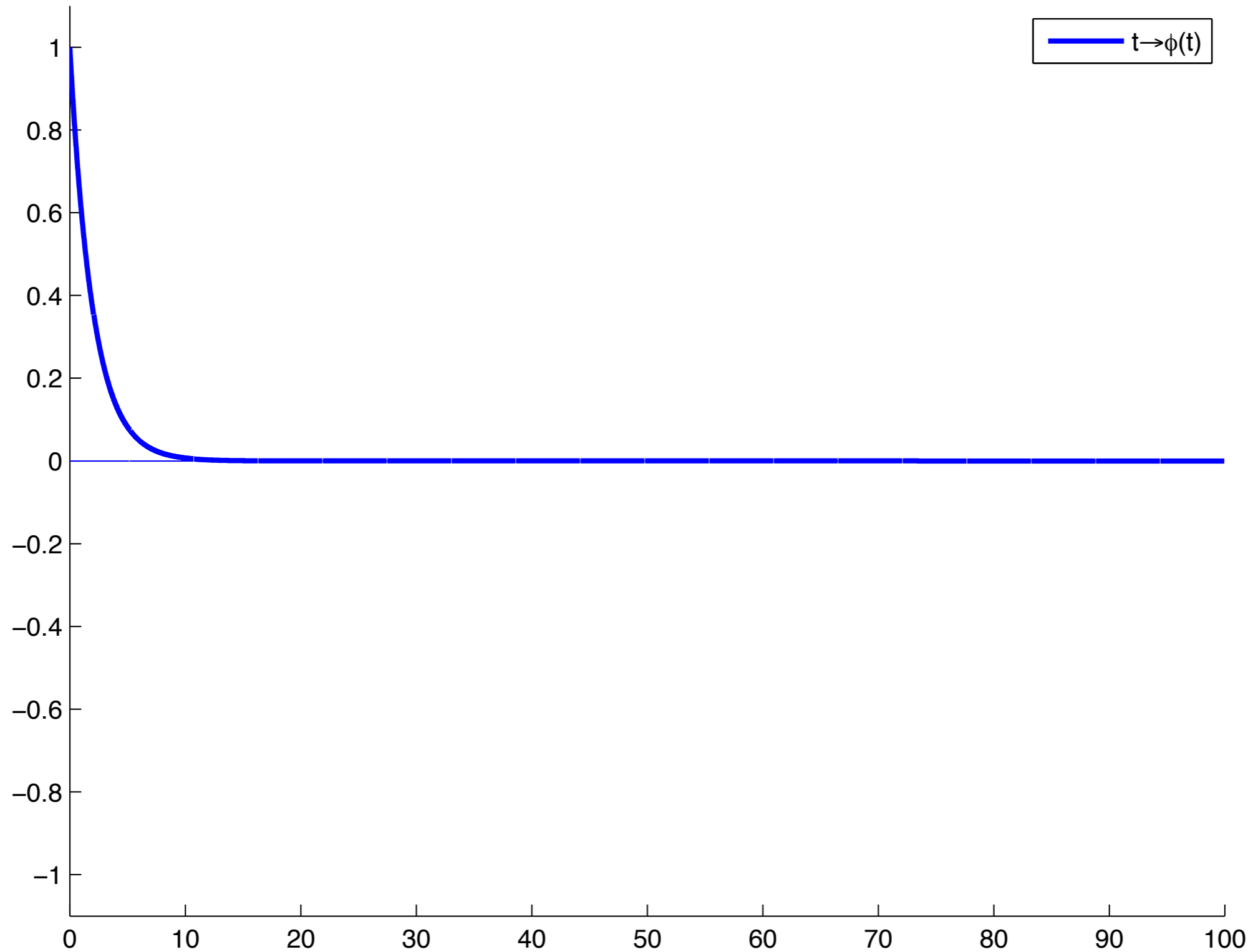
Gedempte slinger, $\nu=1$, $c/l=0.1$



Kritische demping

$$(\rho = \nu)$$

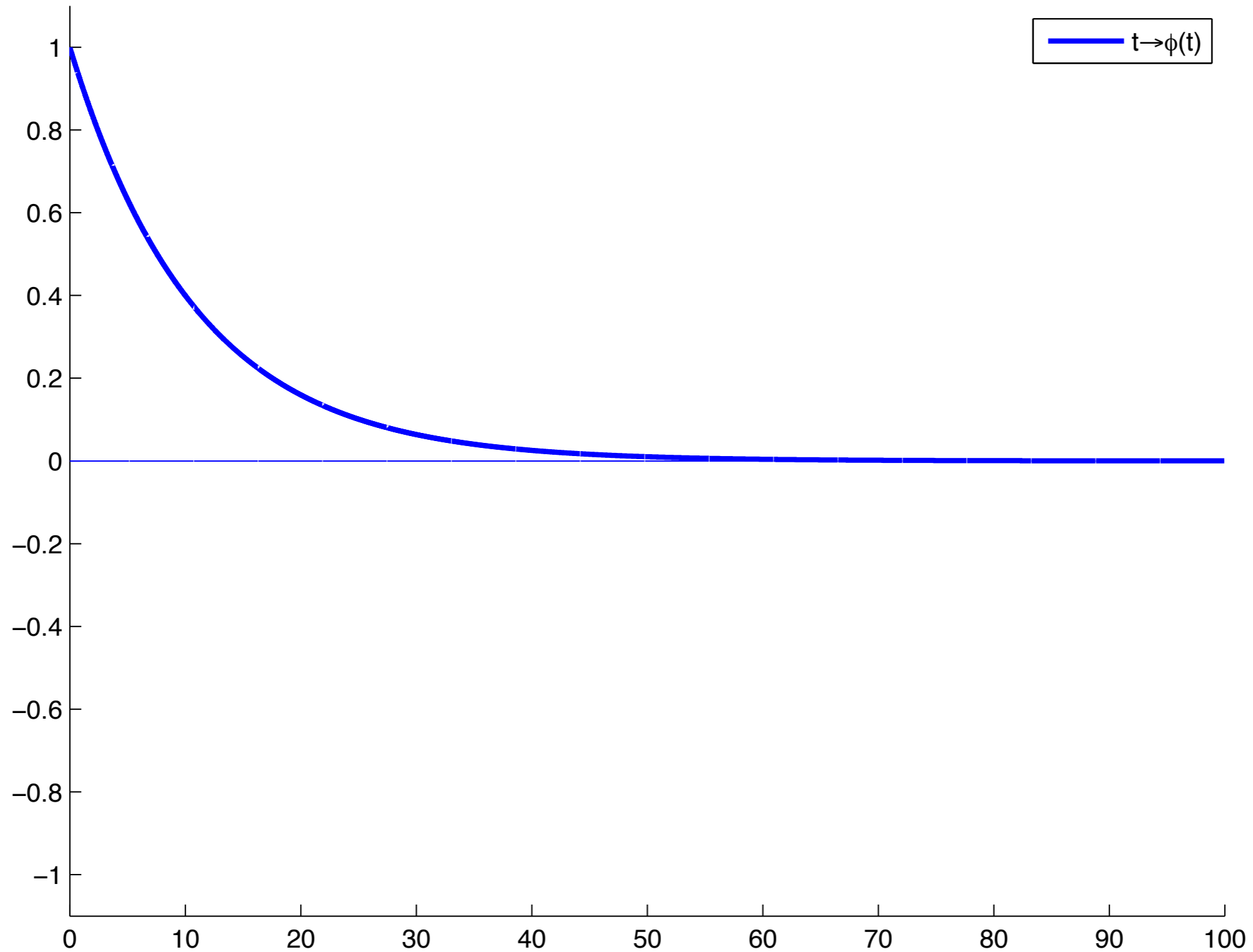
Gedempte slinger, $\nu=1$, $c/l=1$



Overdamping

$$(\rho > \nu)$$

Gedempte slinger, $\nu=1$, $c/l=1.1$



Slinger

$$\phi'' = -\frac{c}{m}\phi' - \frac{g}{\ell}\phi$$

Wiskunde: λ_i **eigenwaarden** (van $\mathbf{A} = \begin{bmatrix} -\frac{c}{m} & -\frac{g}{\ell} \\ 1 & 0 \end{bmatrix}$).

Dagelijks taalgebruik:

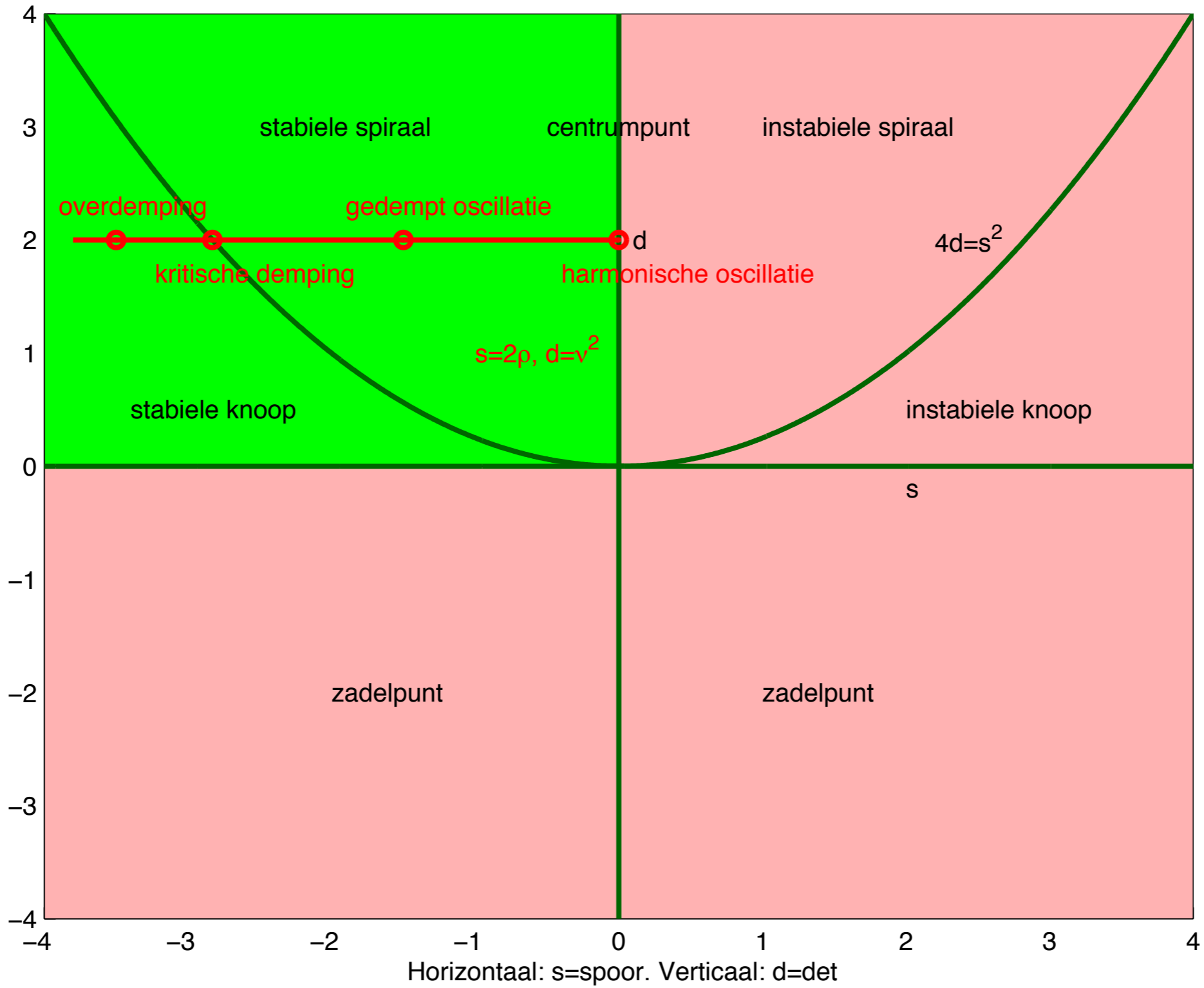
$\text{Im}(\lambda_i)$ **eigenfrequentie** van het systeem.

$$p(\lambda) \equiv \lambda^2 + \frac{c}{m}\lambda + \frac{g}{\ell} = 0$$

$$s = \text{spoor} = -\frac{c}{m} = -2\rho \quad \text{en} \quad d = \det = \frac{g}{\ell} = \nu^2.$$

c verhogen (= wrijving verhogen = demping verhogen)
betekent spoor verlagen.

De Slinger: s en d bij oplopende wrijving (demping)



Slinger

$$\phi'' = -\frac{c}{m} \phi' - \frac{g}{\ell} \phi$$

Opmerkingen.

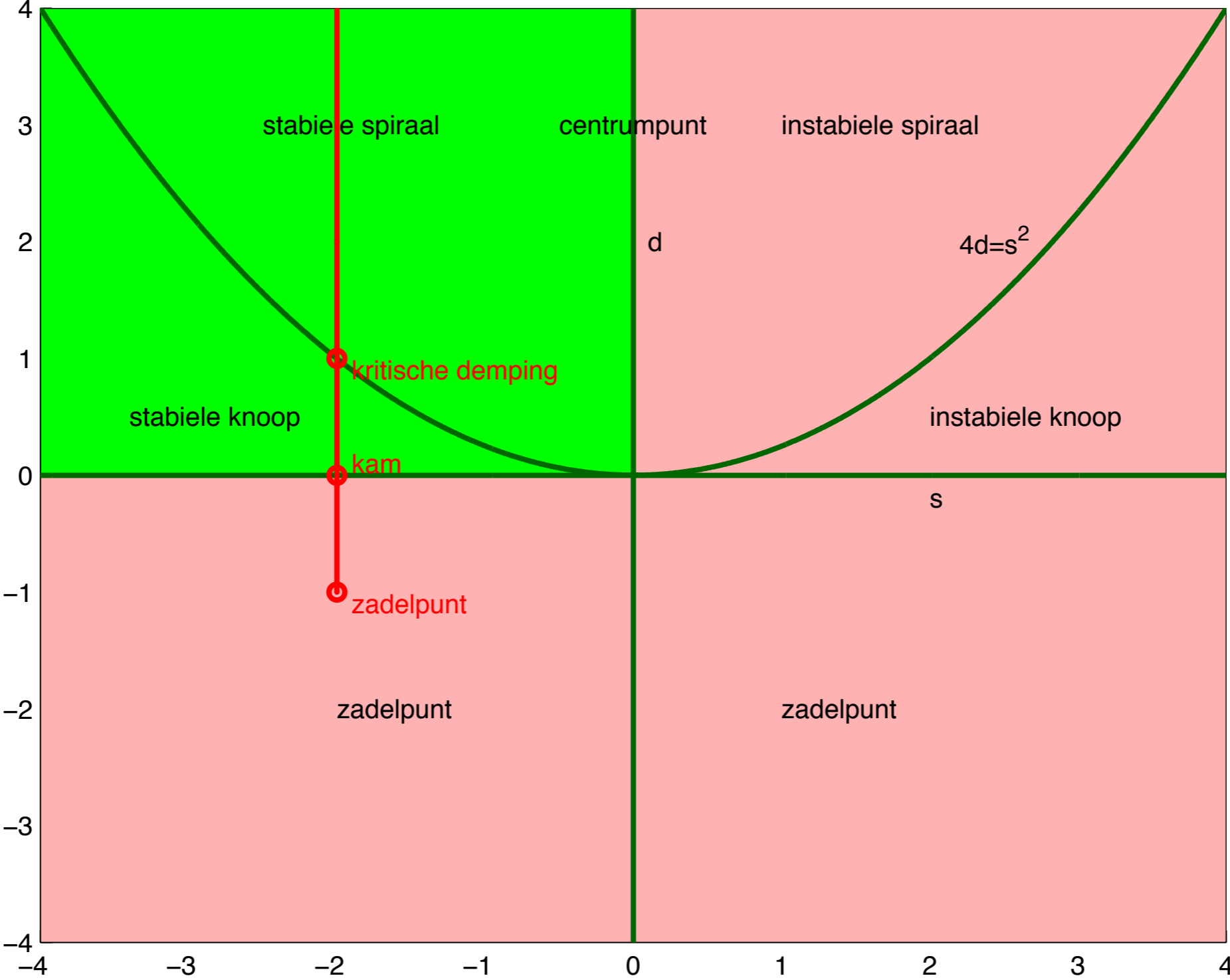
- In de bovenstaande analyse hebben we aangenomen dat $\phi(t)$ voor iedere t zo klein is dat $\sin(\phi(t)) \approx \phi(t)$.
(In een volgende les laten we deze aanname vallen.)
- Verder hebben we aangenomen dat de grootte van de wrijving evenredig is met de grootte van de snelheid. In geval van luchtweerstand is dit maar alleen correct bij lage snelheden (de wet van Stokes). Bij grotere snelheden is de wrijving door luchtweerstand evenredig met de snelheid in het kwadraat (wet van Rayleigh).

Vast spoor, oplopende determinant

$$\mathbf{A} = \begin{bmatrix} -1 & 1 \\ \varepsilon & -1 \end{bmatrix}$$

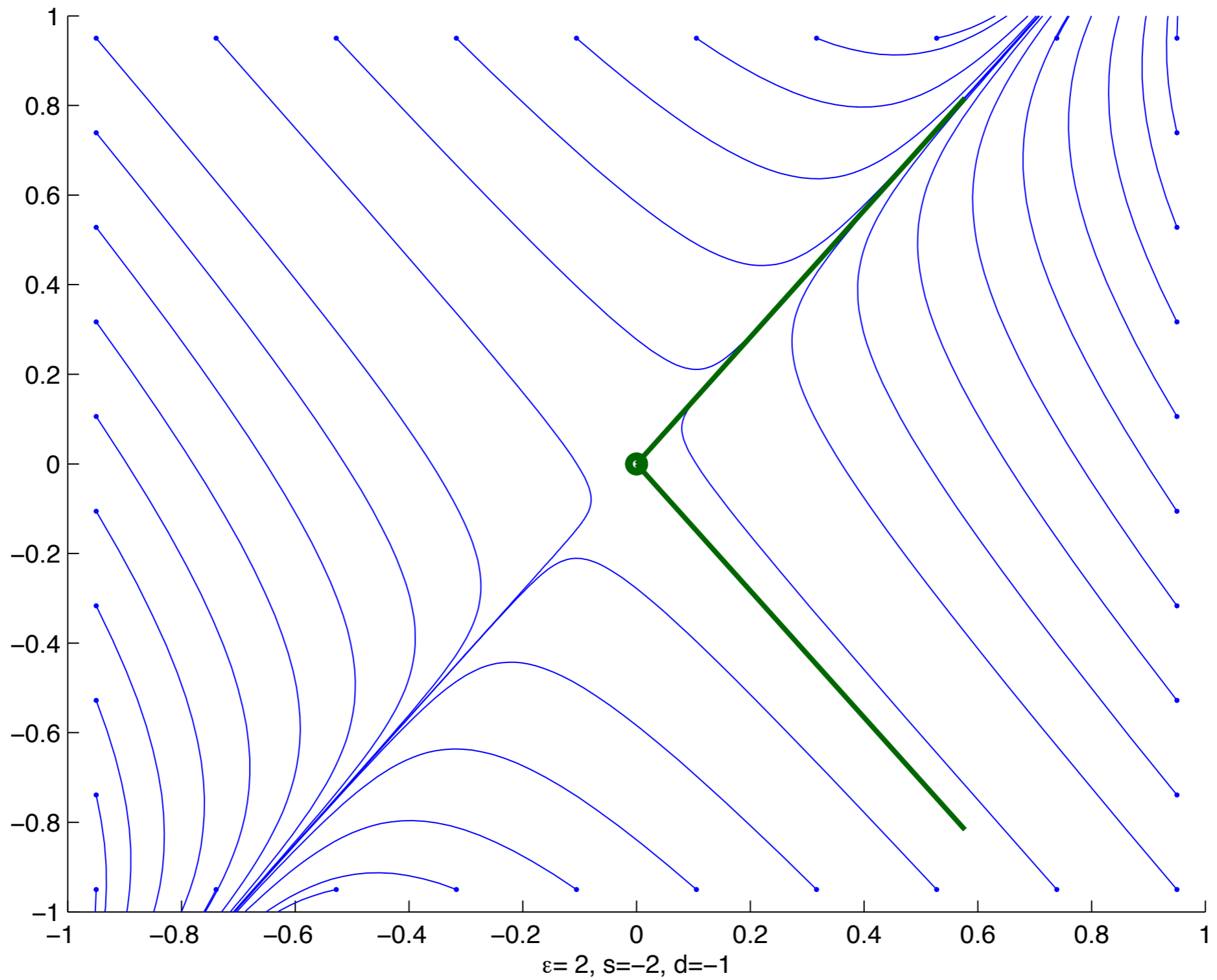
$$\Rightarrow \quad s = \text{spoor} = -2, \quad d = \text{det} = 1 - \varepsilon$$

s en d bij oplopende d

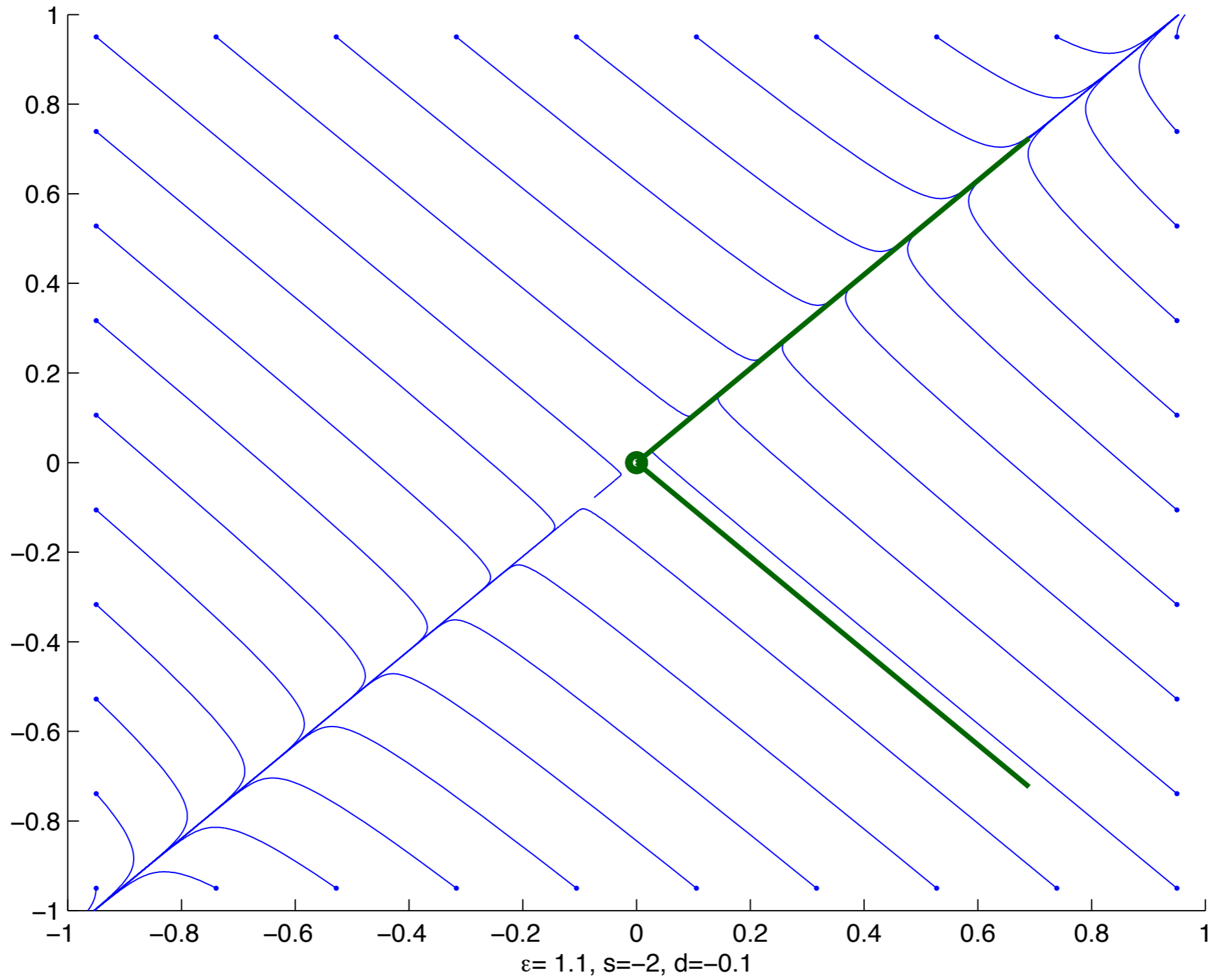


Horizontaal: s=spoor. Verticaal: d=det

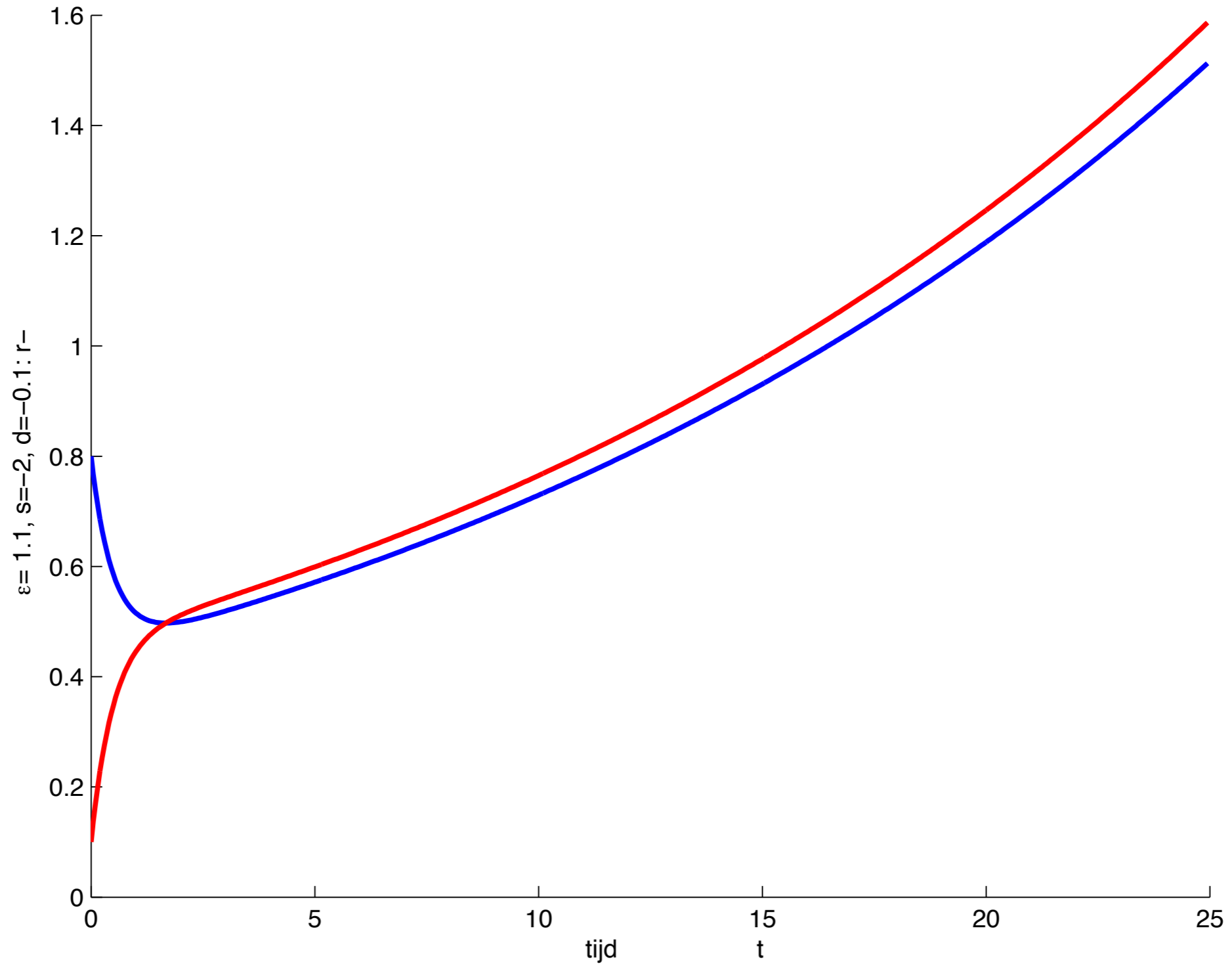
2-d Lineaire differentiaal vergelijking met $\lambda_1=0.414214$, $\lambda_2=-2.41421$



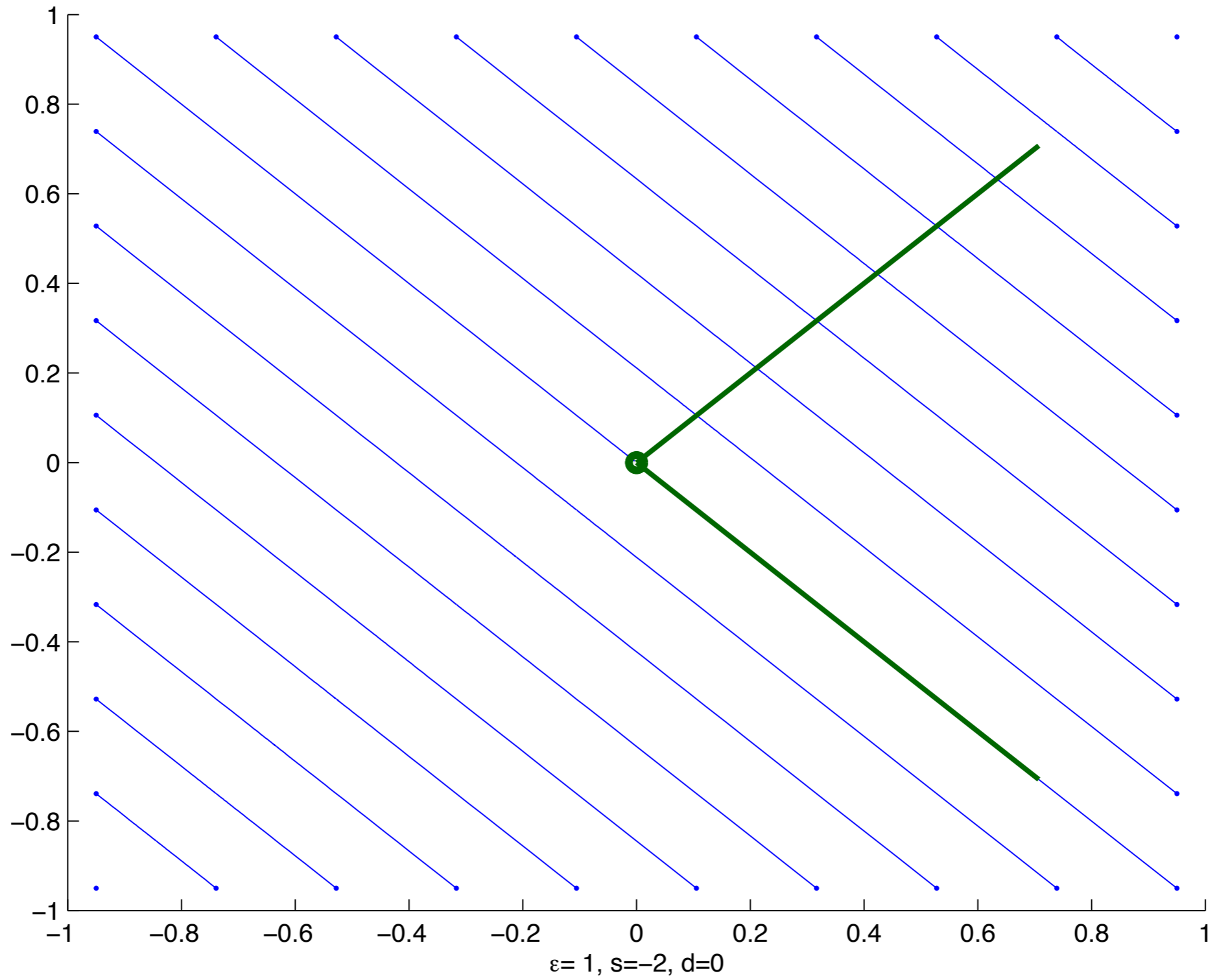
2-d Lineaire differentiaal vergelijking met $\lambda_1=0.0488088$, $\lambda_2=-2.04881$



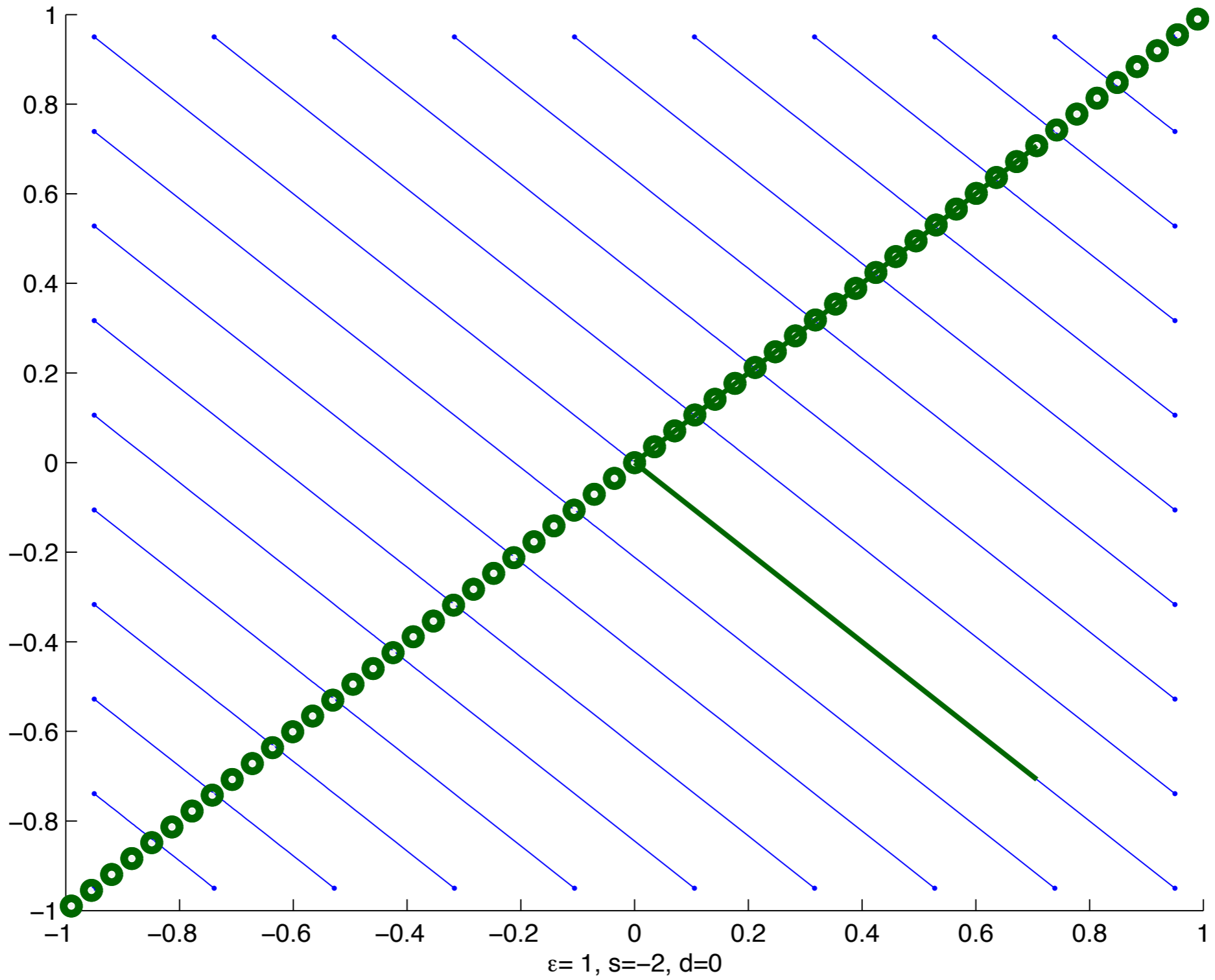
2-d Lineaire differentiaal vergelijking met $\lambda_1 = -2.04881$, $\lambda_2 = 0.0488088$



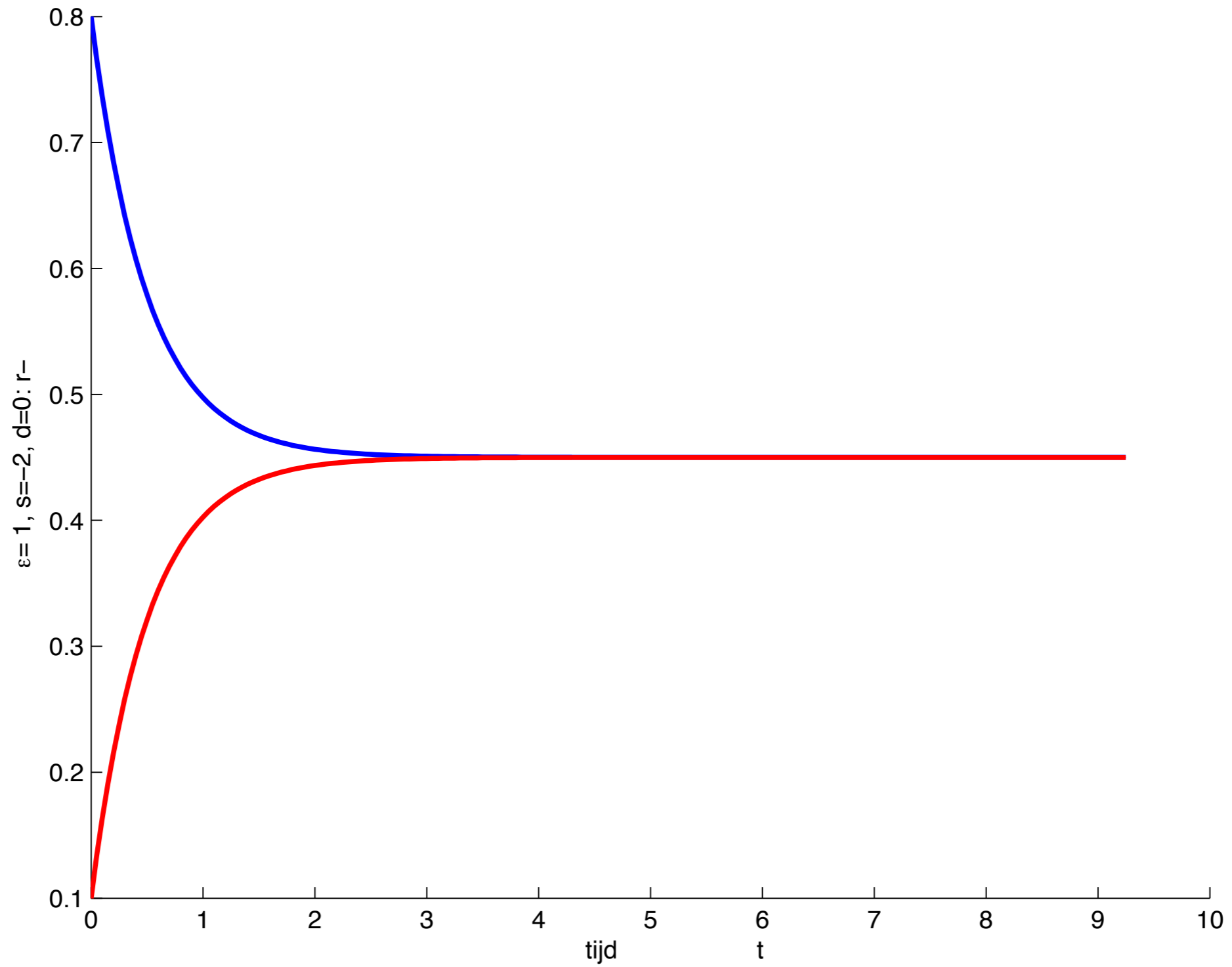
2-d Lineaire differentiaal vergelijking met $\lambda_1=-2$, $\lambda_2=0$



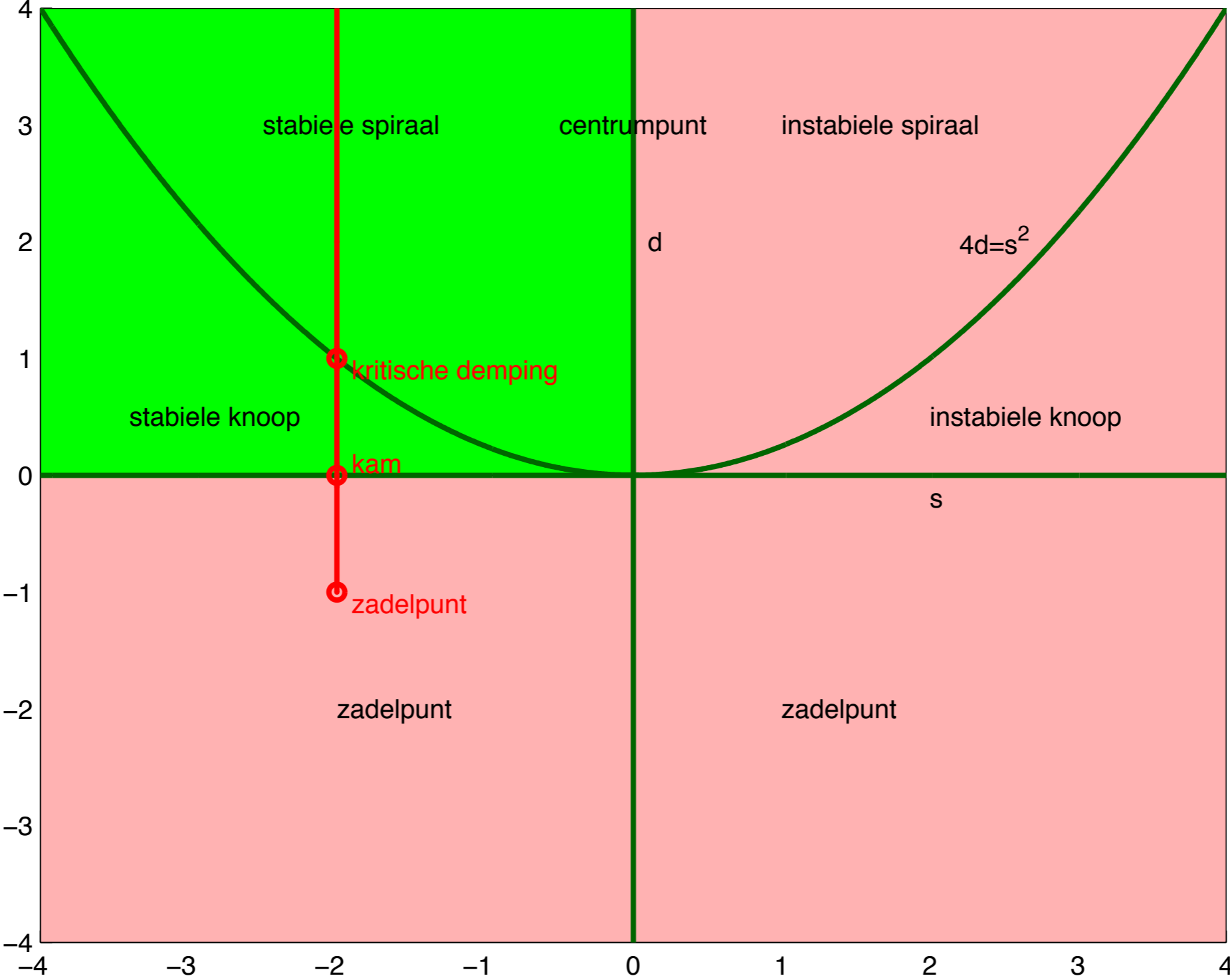
2-d Lineaire differentiaal vergelijking met $\lambda_1=-2, \lambda_2=0$



2-d Lineaire differentiaal vergelijking met $\lambda_1=-2$, $\lambda_2=0$

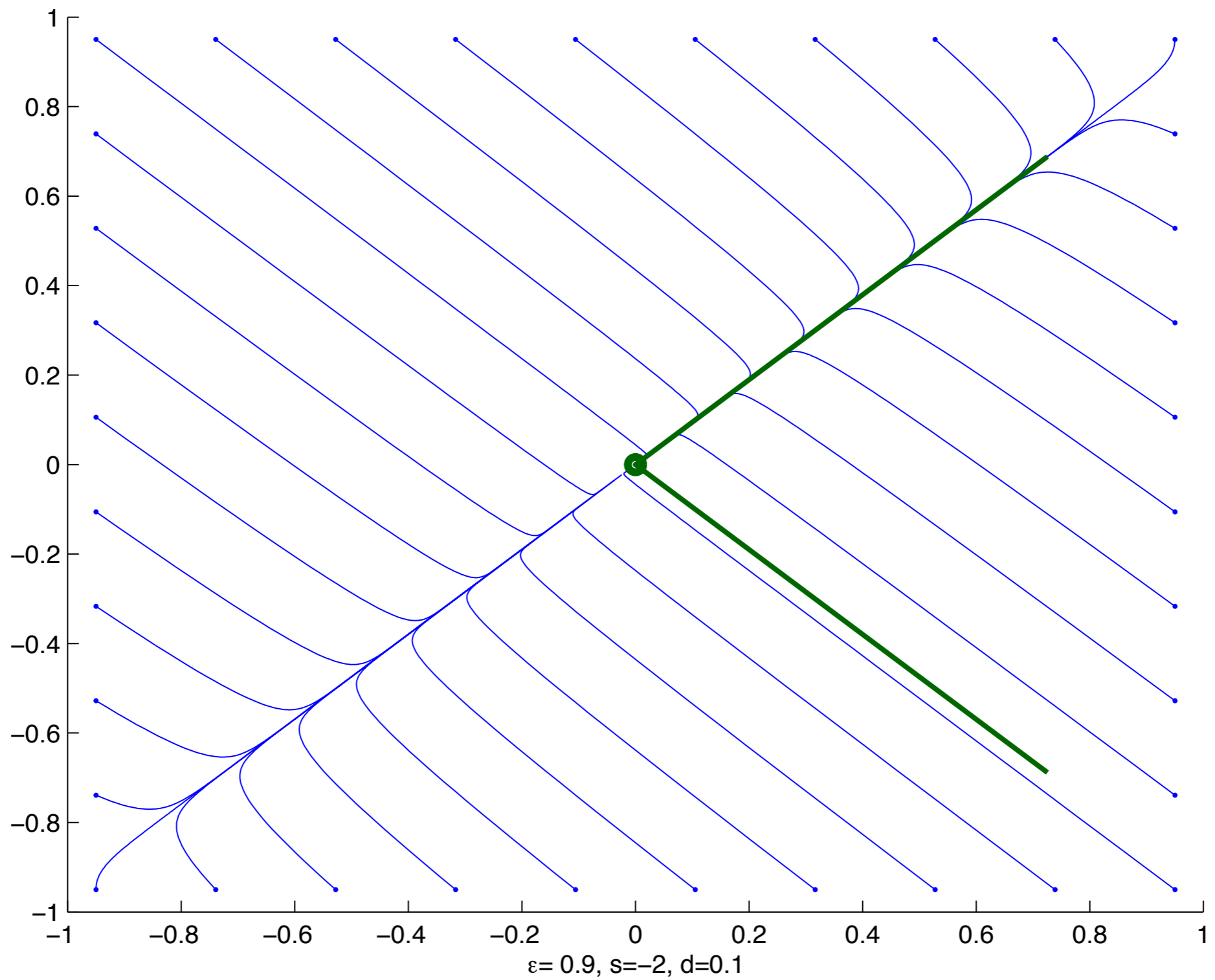


s en d bij oplopende d

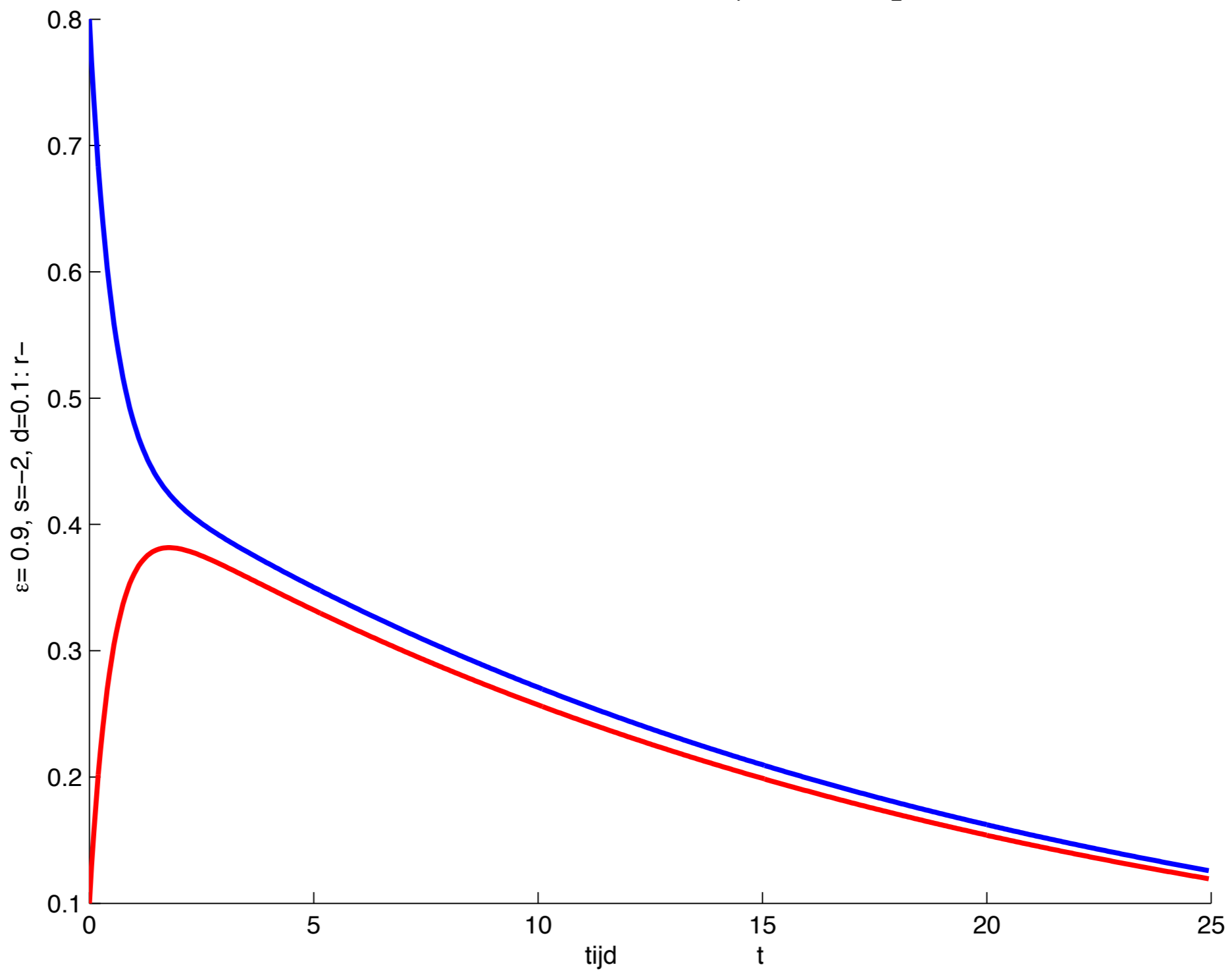


Horizontaal: s =spoor. Verticaal: d =det

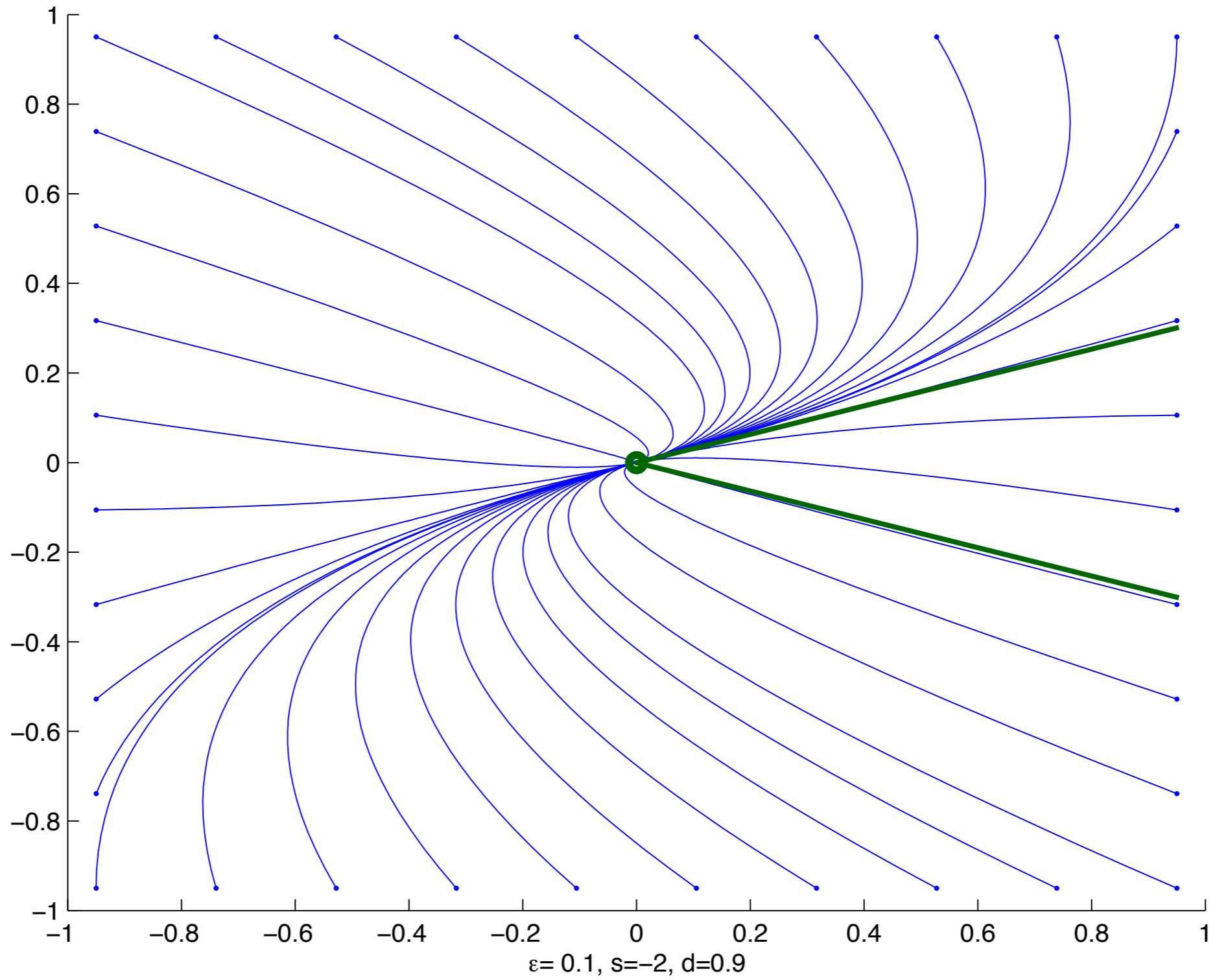
2-d Lineaire differentiaal vergelijking met $\lambda_1=-0.0513167$, $\lambda_2=-1.94868$



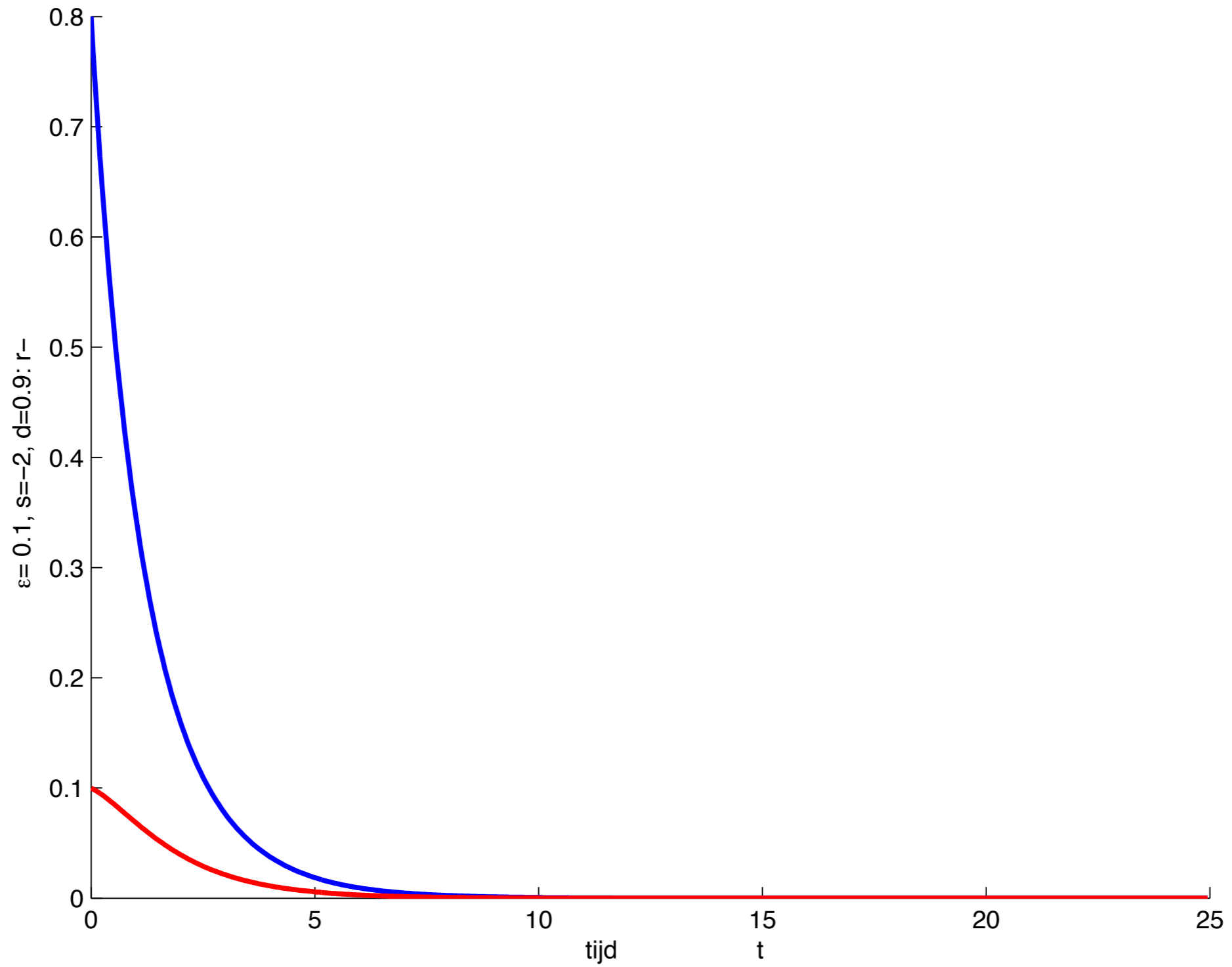
2-d Lineaire differentiaal vergelijking met $\lambda_1=-1.94868$, $\lambda_2=-0.0513167$



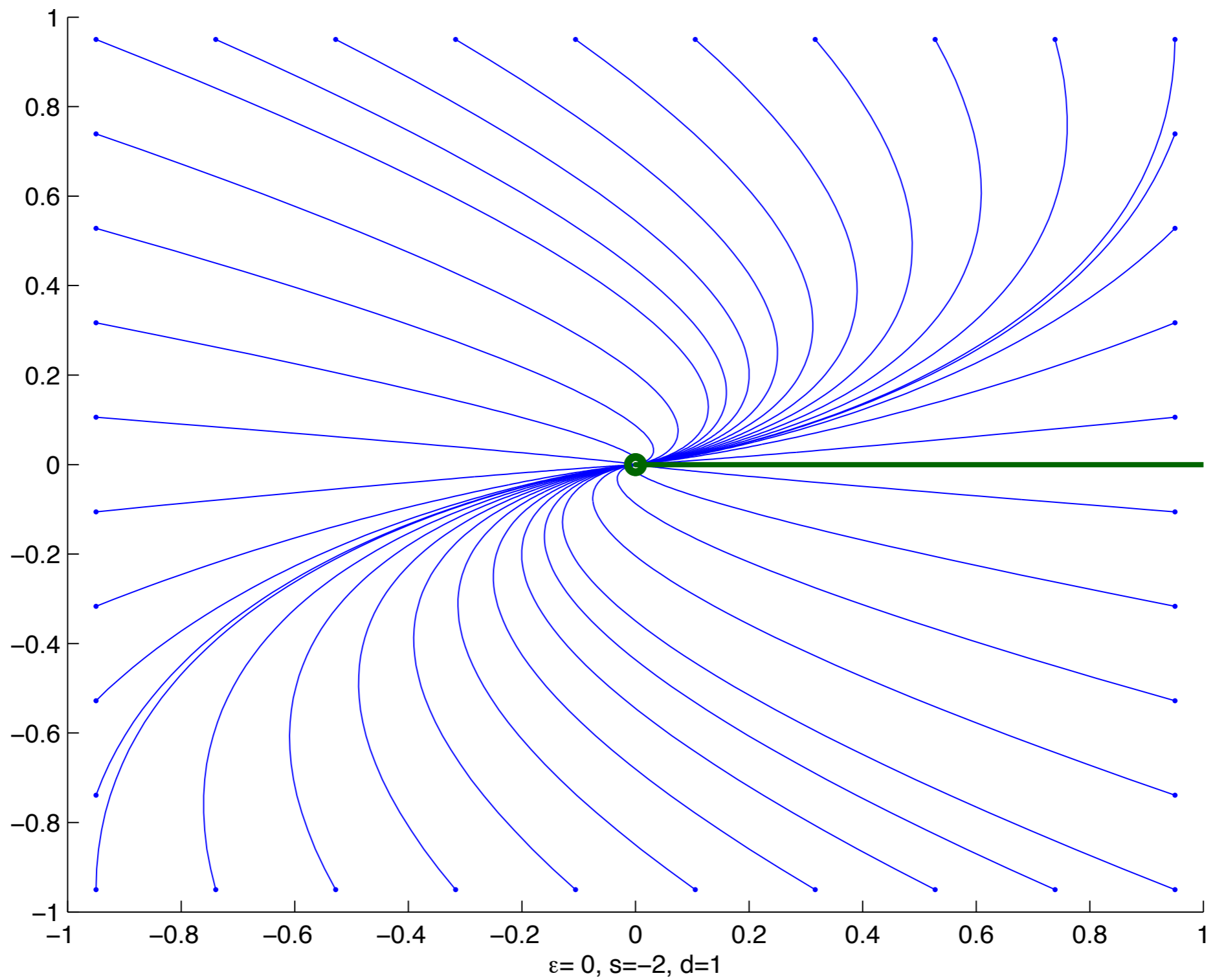
2-d Lineaire differentiaal vergelijking met $\lambda_1=-0.683772$, $\lambda_2=-1.31623$



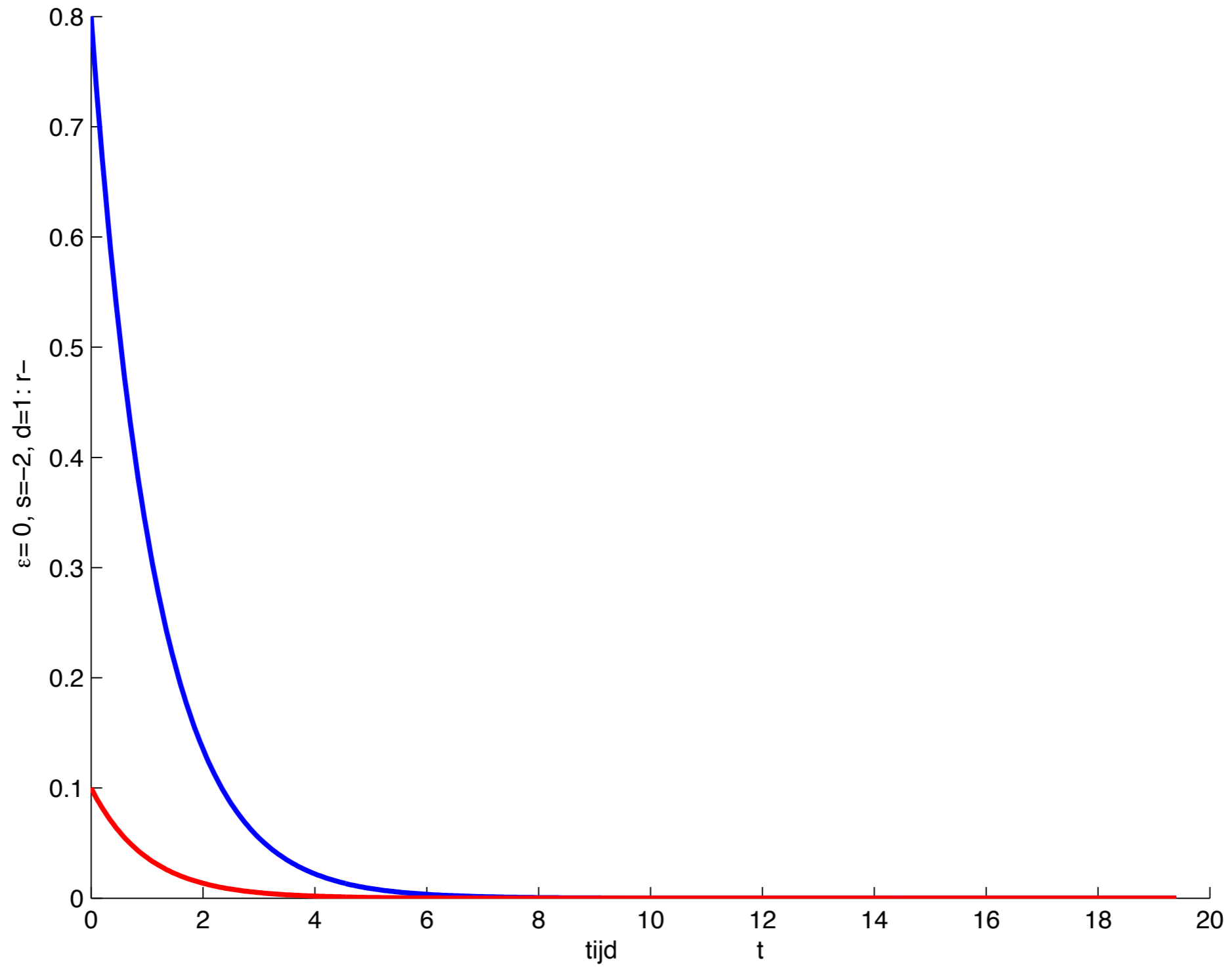
2-d Lineaire differentiaal vergelijking met $\lambda_1 = -1.31623$, $\lambda_2 = -0.683772$



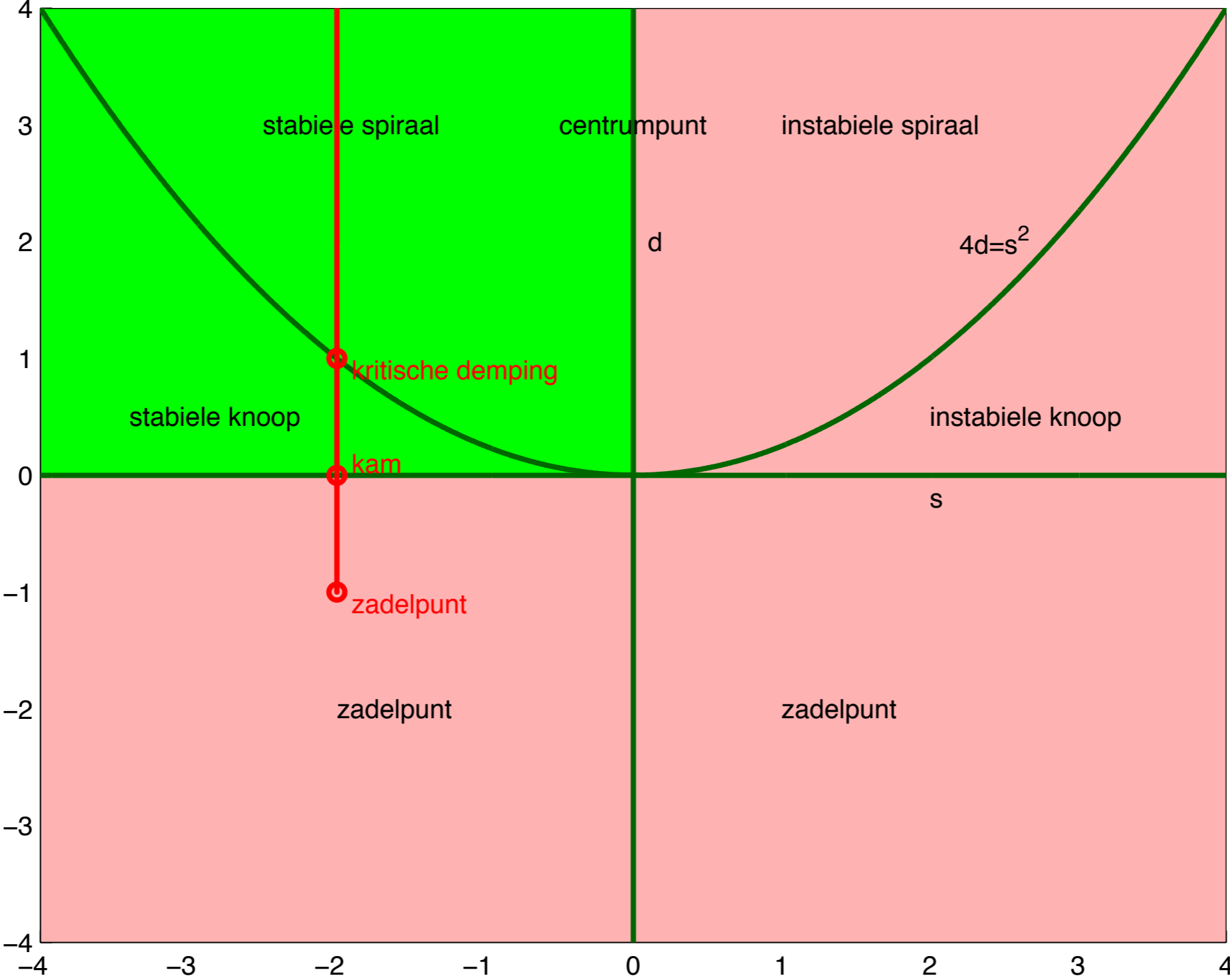
2-d Lineaire differentiaal vergelijking met $\lambda_1=-1, \lambda_2=-1$



2-d Lineaire differentiaal vergelijking met $\lambda_1=-1, \lambda_2=-1$

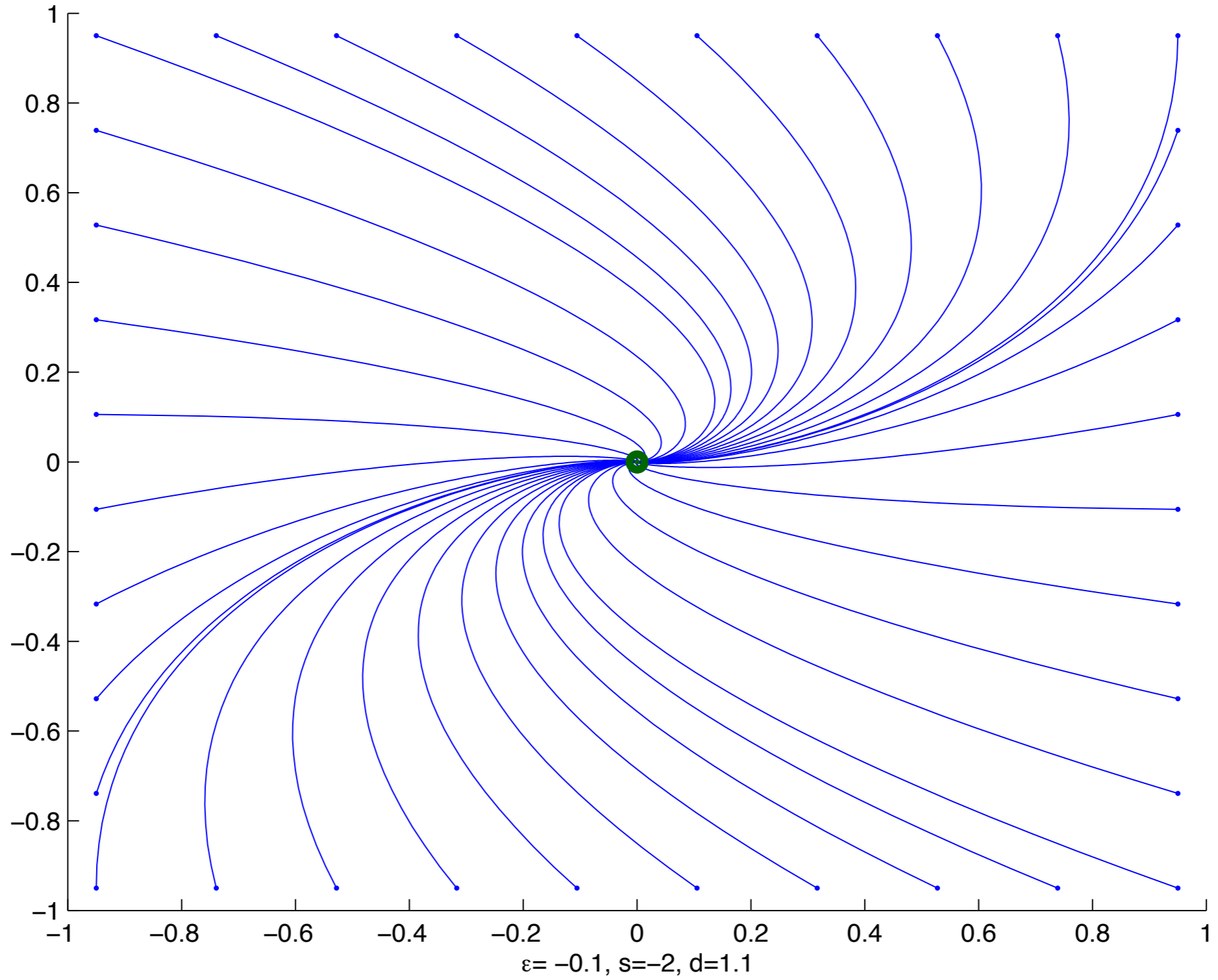


s en d bij oplopende d

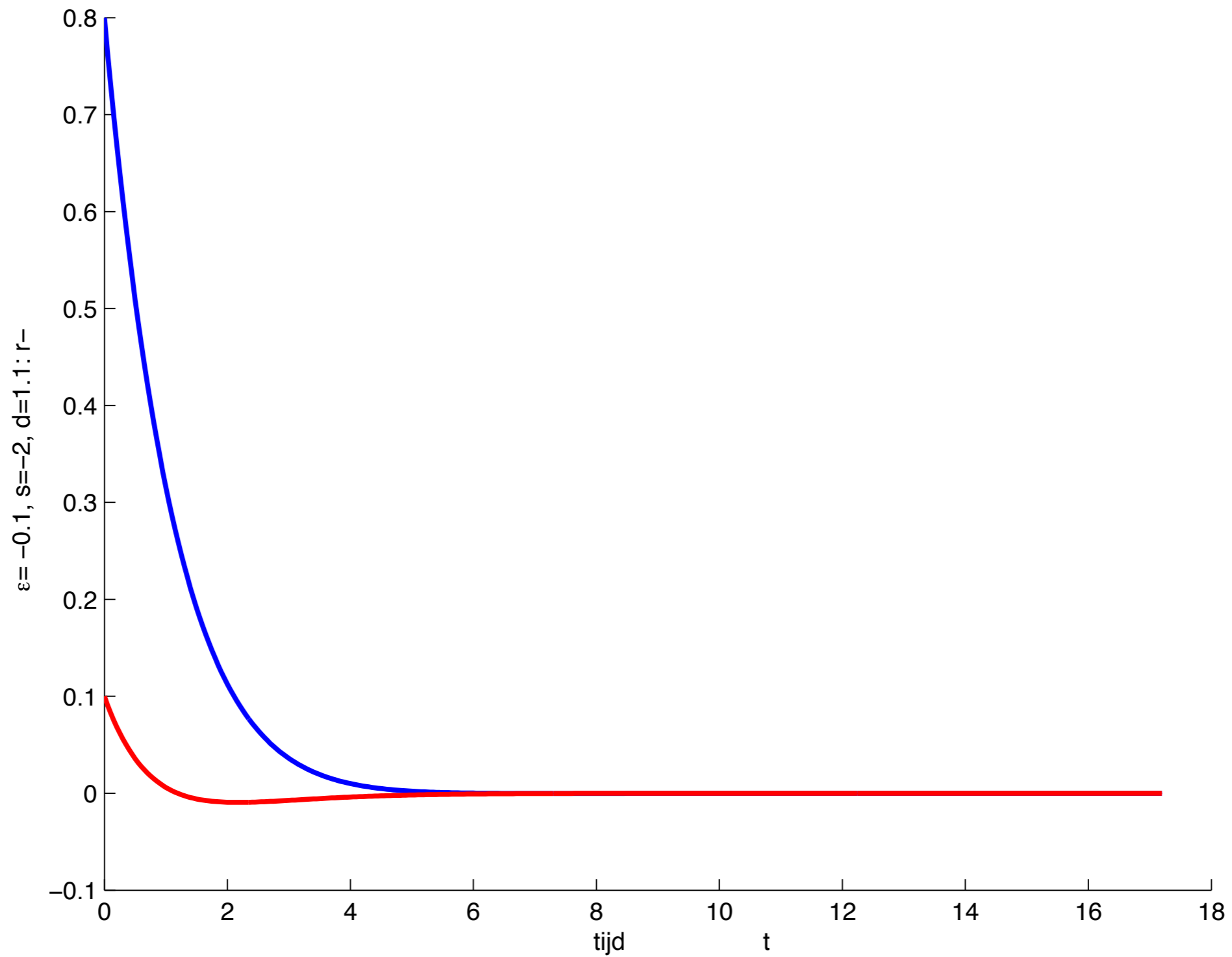


Horizontaal: s =spoor. Verticaal: d =det

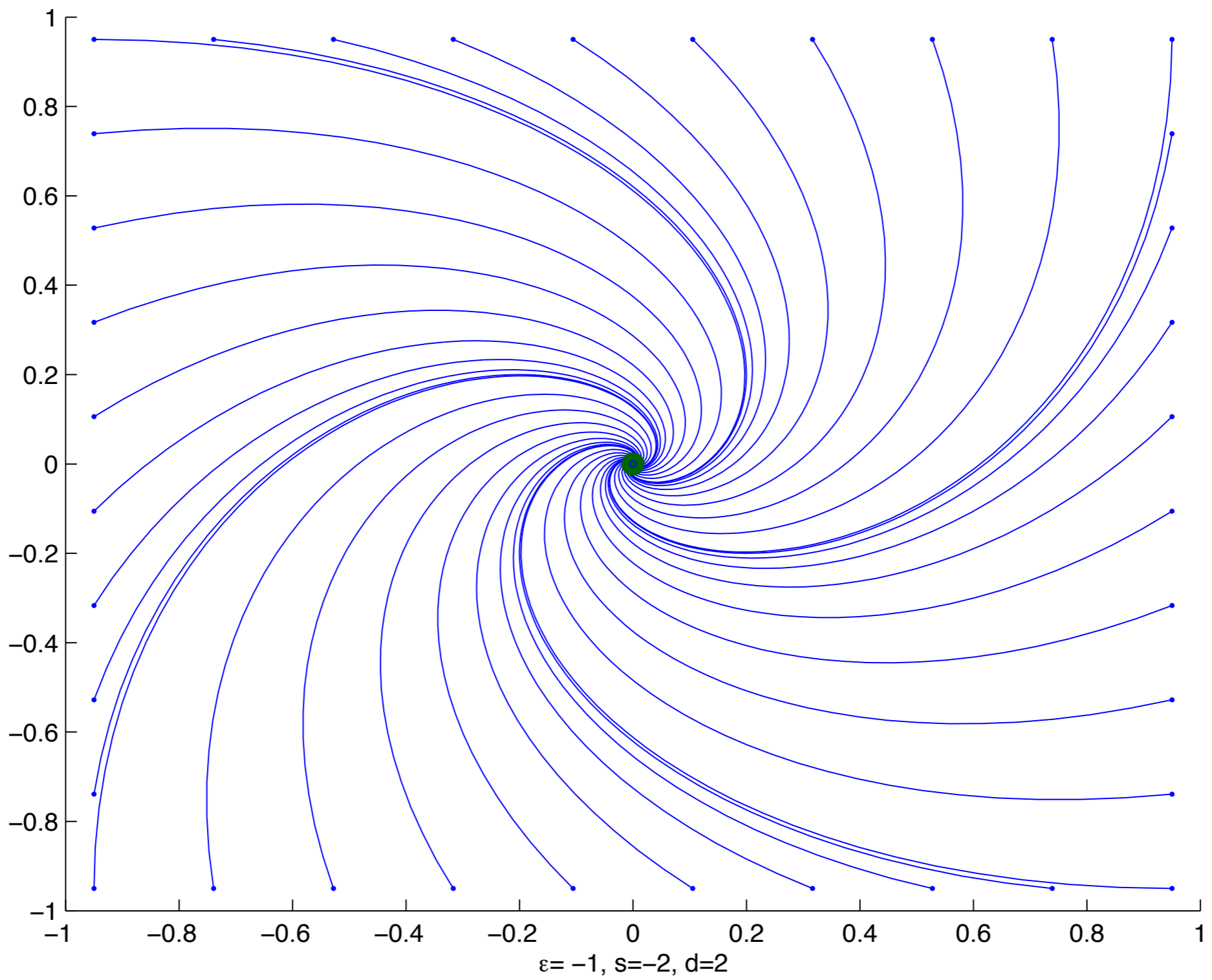
2-d Lineaire differentiaal vergelijking met $\lambda_1 = -1 + 0.316228 i$, $\lambda_2 = -1 - 0.316228 i$



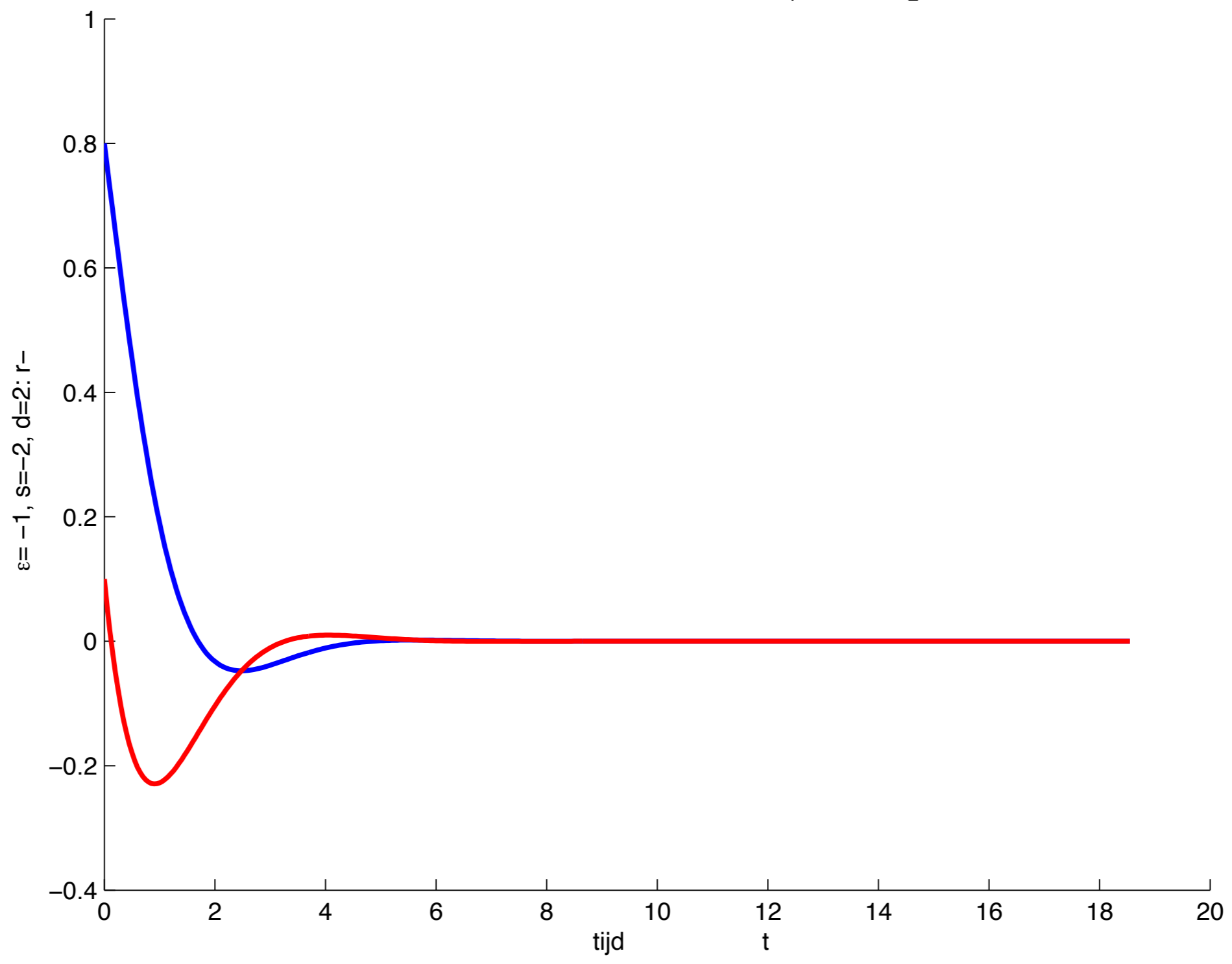
2-d Lineaire differentiaal vergelijking met $\lambda_1 = -1 - 0.316228 i$, $\lambda_2 = -1 + 0.316228 i$



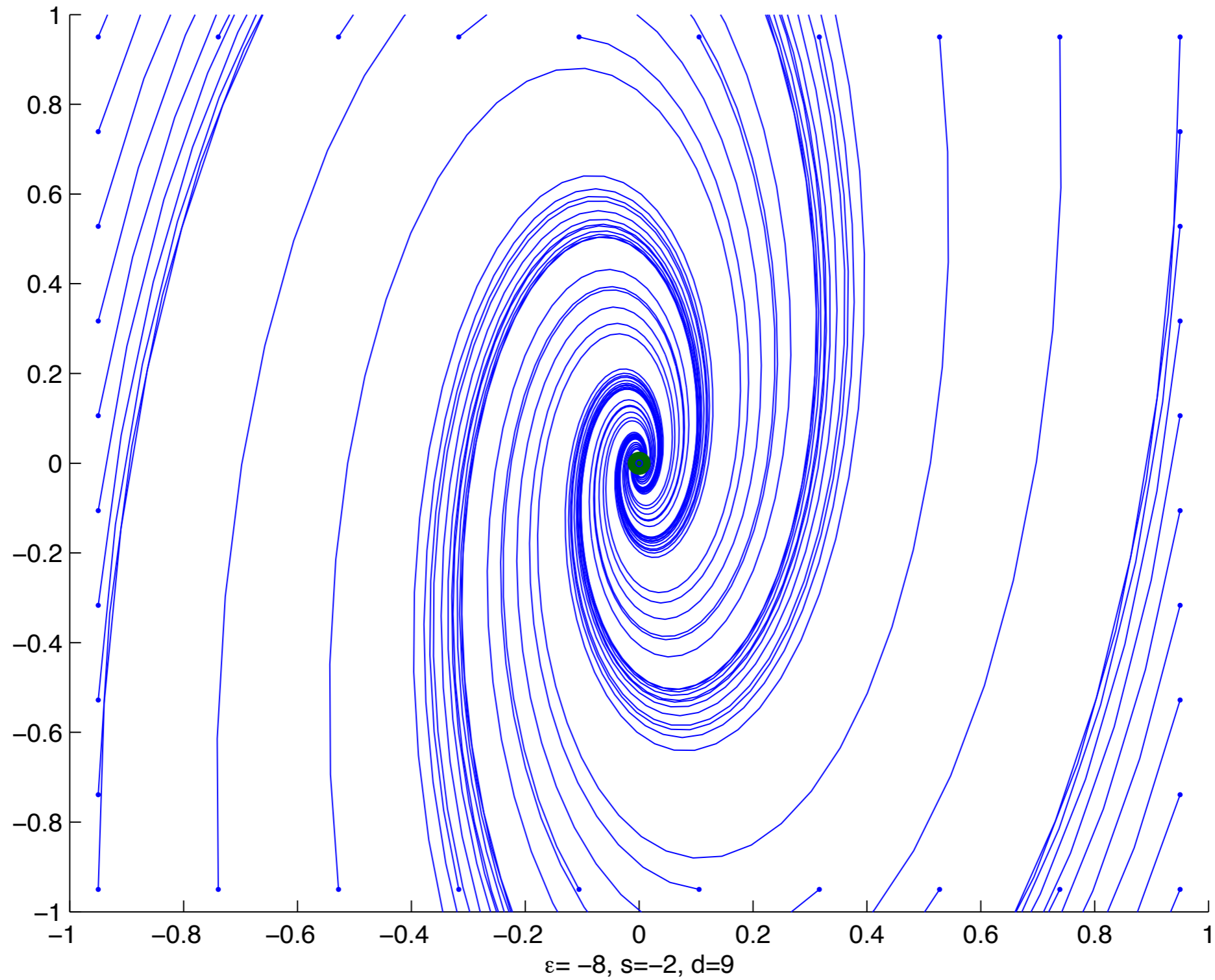
2-d Lineaire differentiaal vergelijking met $\lambda_1 = -1 + 1 i$, $\lambda_2 = -1 - 1 i$



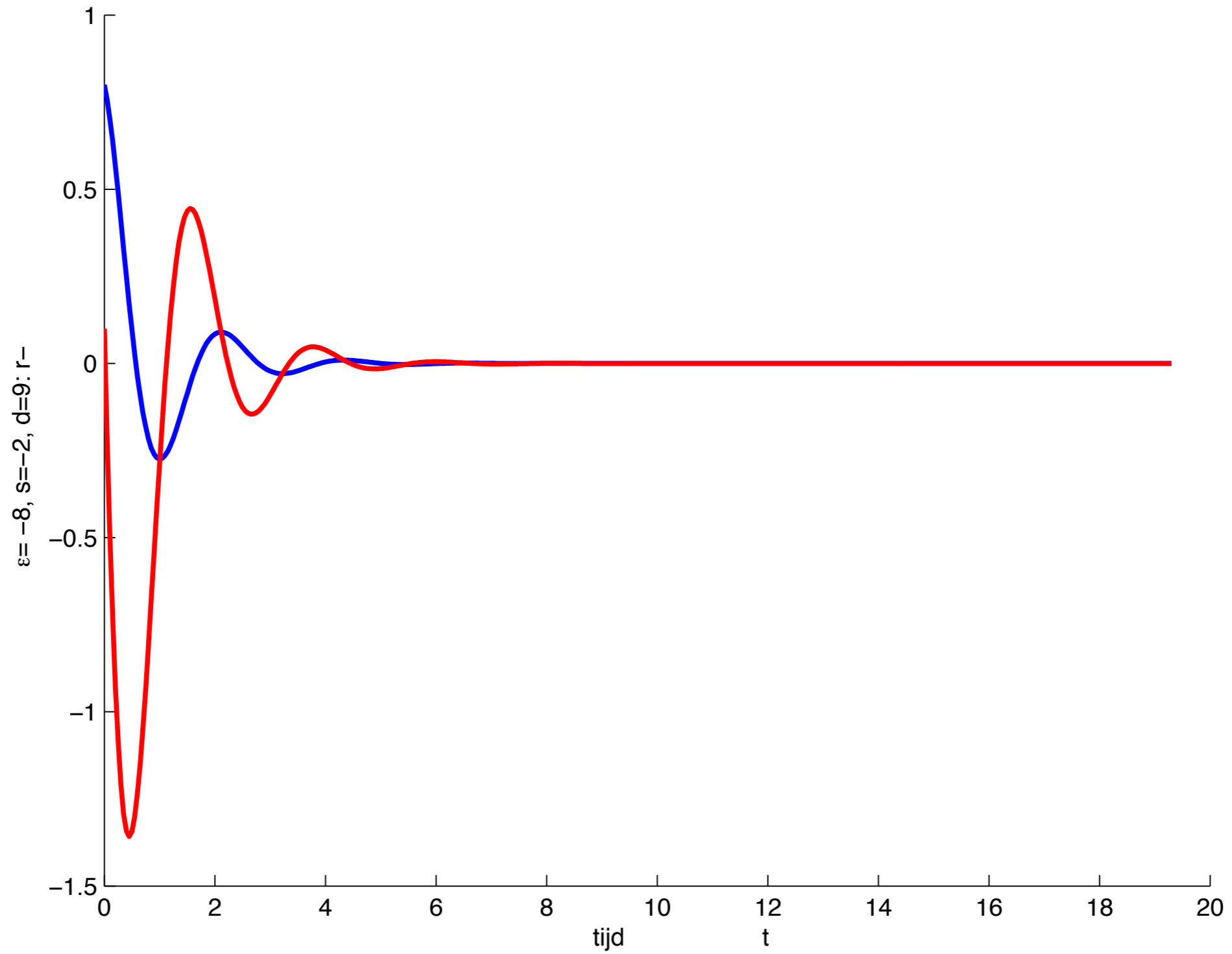
2-d Lineaire differentiaal vergelijking met $\lambda_1 = -1 - 1i$, $\lambda_2 = -1 + 1i$



2-d Lineaire differentiaal vergelijking met $\lambda_1 = -1 + 2.82843 i$, $\lambda_2 = -1 - 2.82843 i$



2-d Lineaire differentiaal vergelijking met $\lambda_1 = -1 - 2.82843 i$, $\lambda_2 = -1 + 2.82843 i$



Vast determinant, aflopende spoor

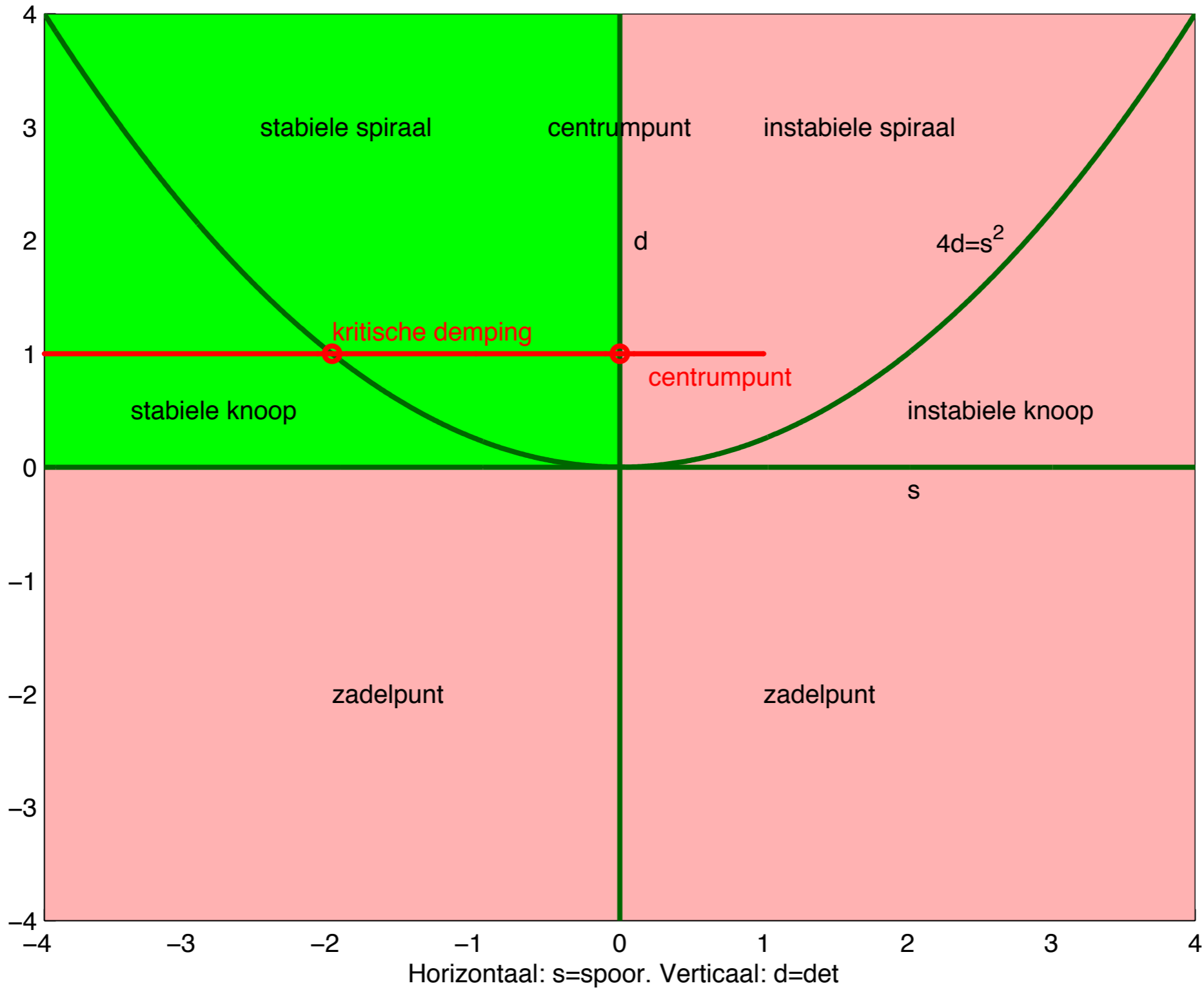
$$\mathbf{A} = \begin{bmatrix} \varepsilon & -1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow \quad s = \text{spoor} = \varepsilon, \quad d = \det = 1$$

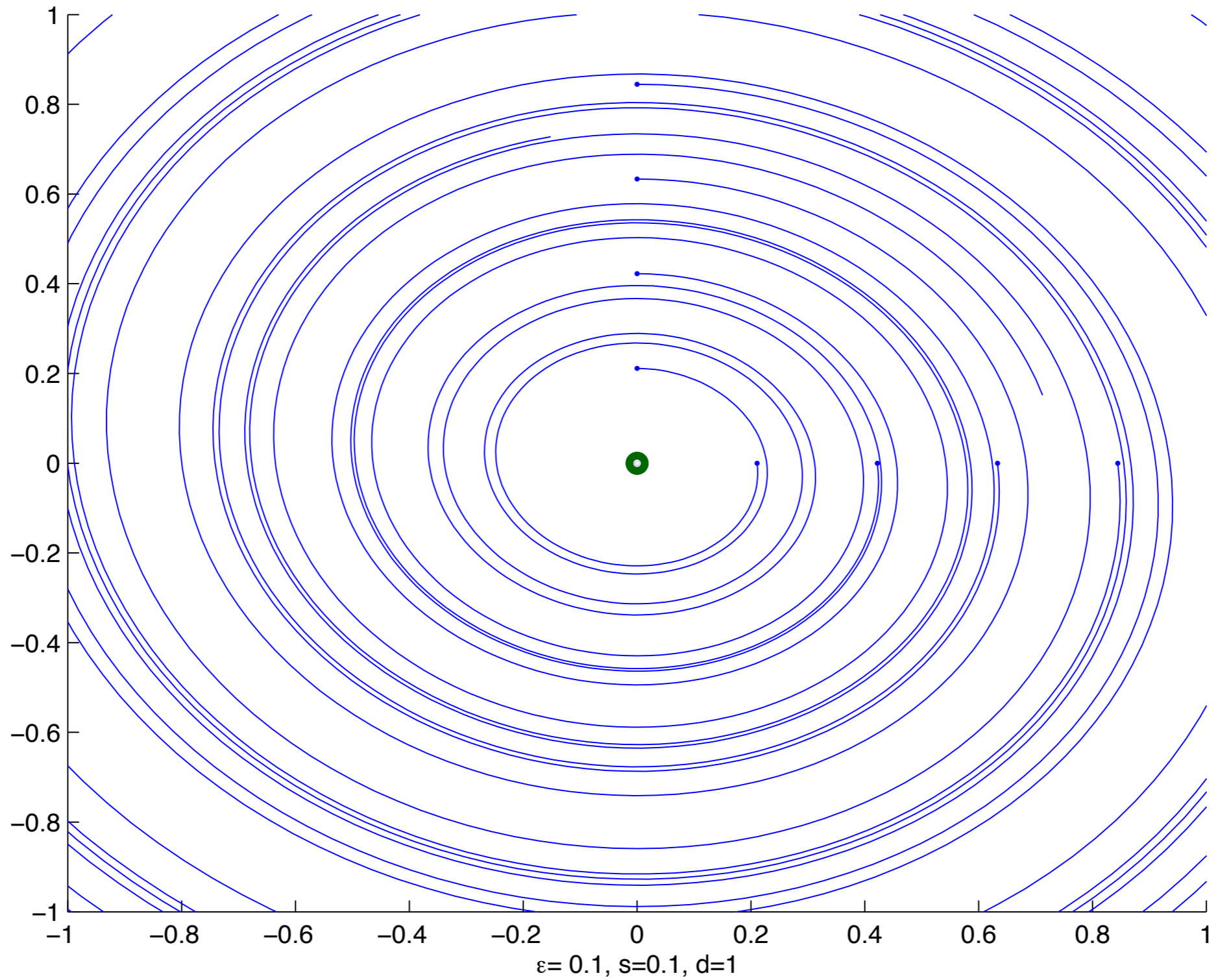
Voorbeeld. Slinger. Met $y = \phi, \quad x' = \phi'$

$$\phi'' = -\frac{c}{m}\phi' - \frac{g}{\ell}\phi \quad \Leftrightarrow \quad \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} -\frac{c}{m} & -\frac{g}{\ell} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

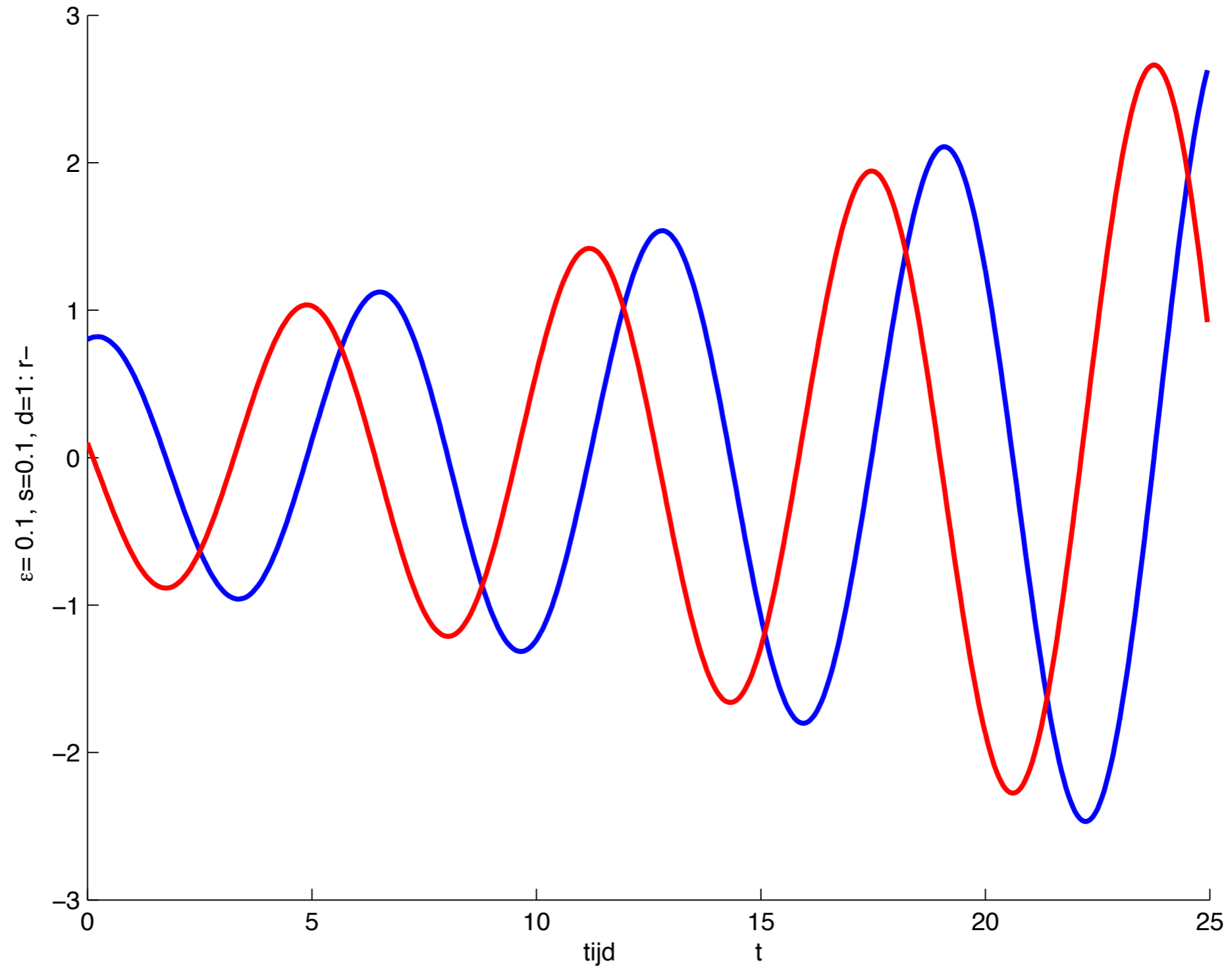
s en d bij aflopende s



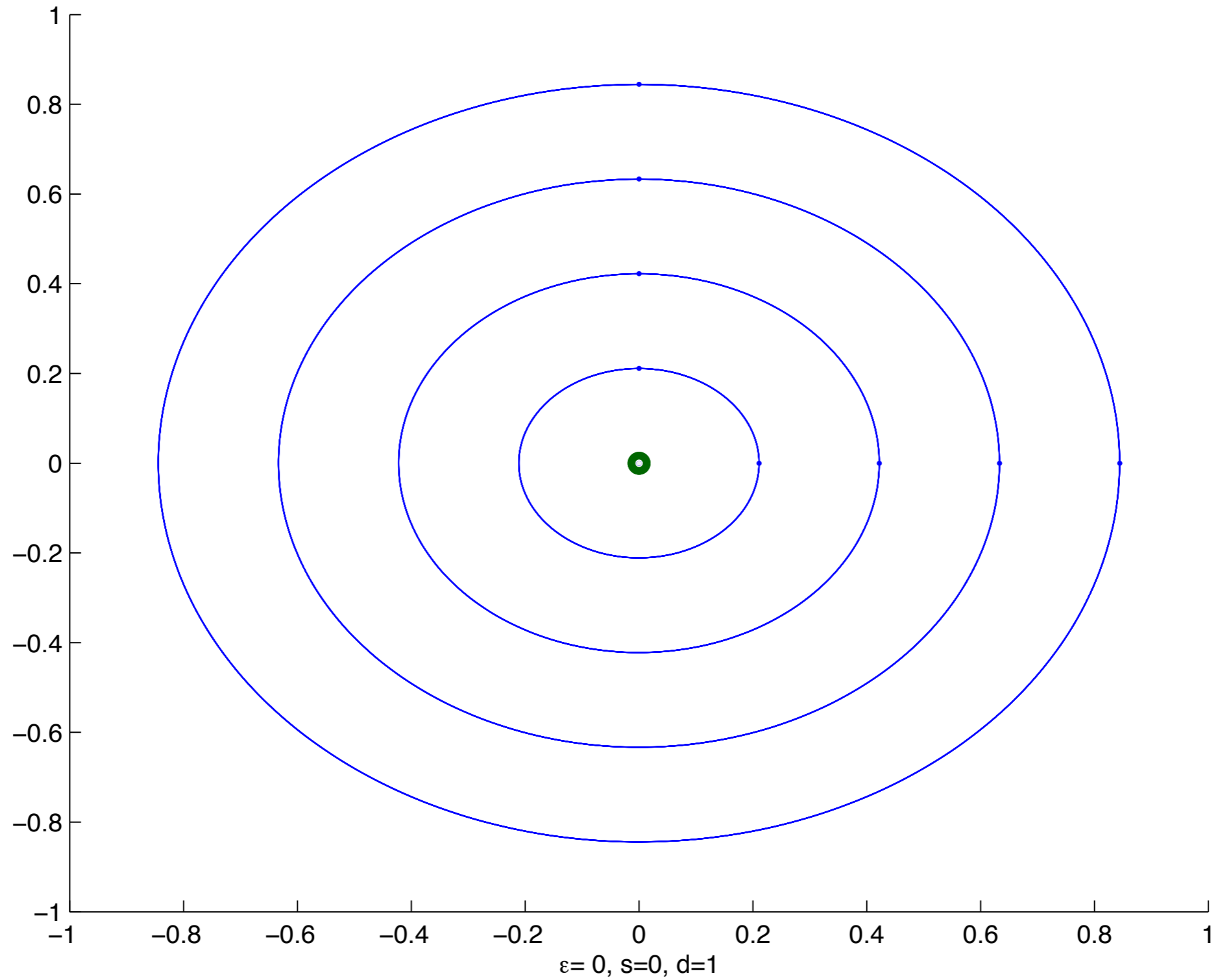
2-d Lineaire differentiaal vergelijking met $\lambda_1=0.05+0.998749 i$, $\lambda_2=0.05-0.998749 i$



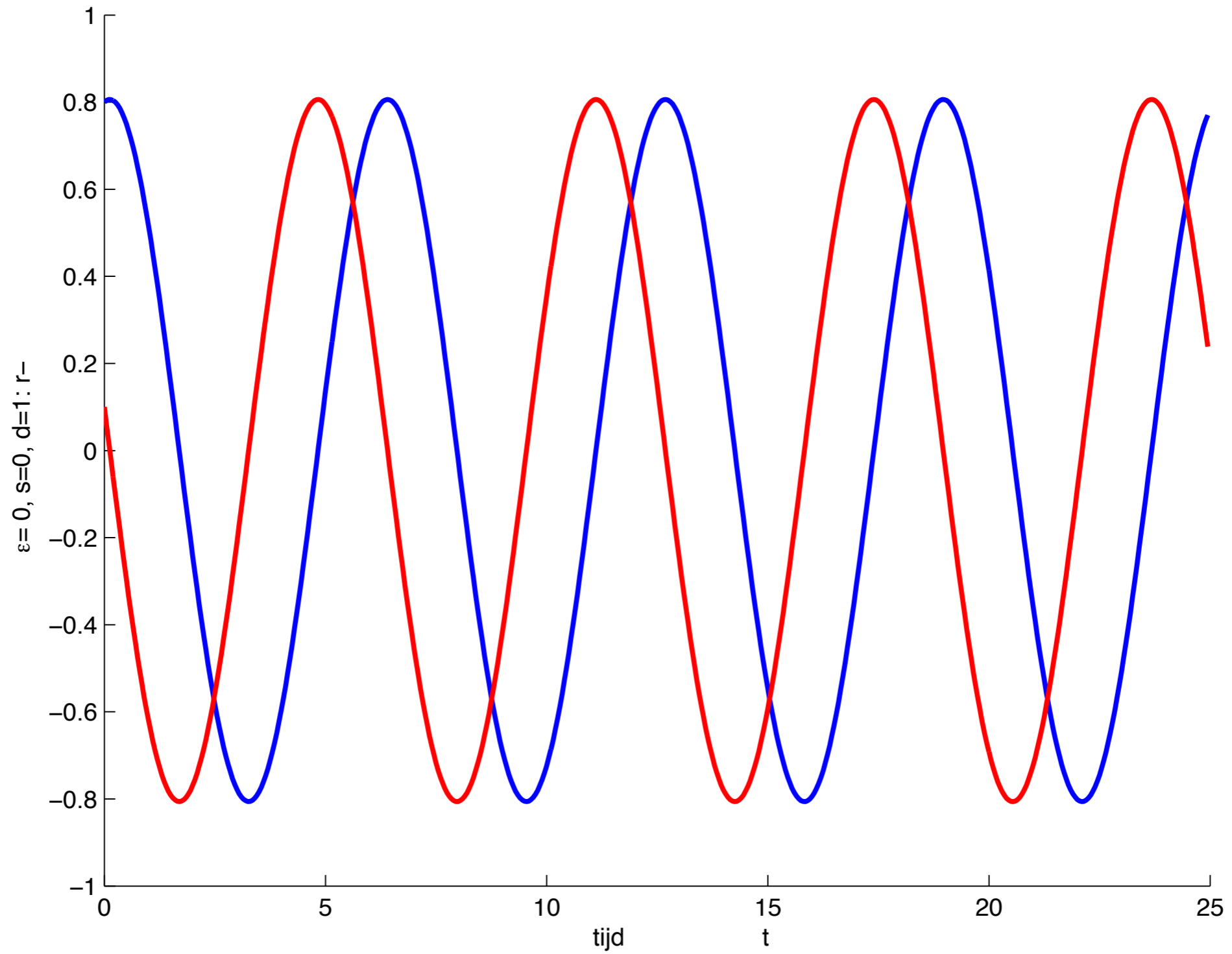
2-d Lineaire differentiaal vergelijking met $\lambda_1=0.05-0.998749 i$, $\lambda_2=0.05+0.998749 i$



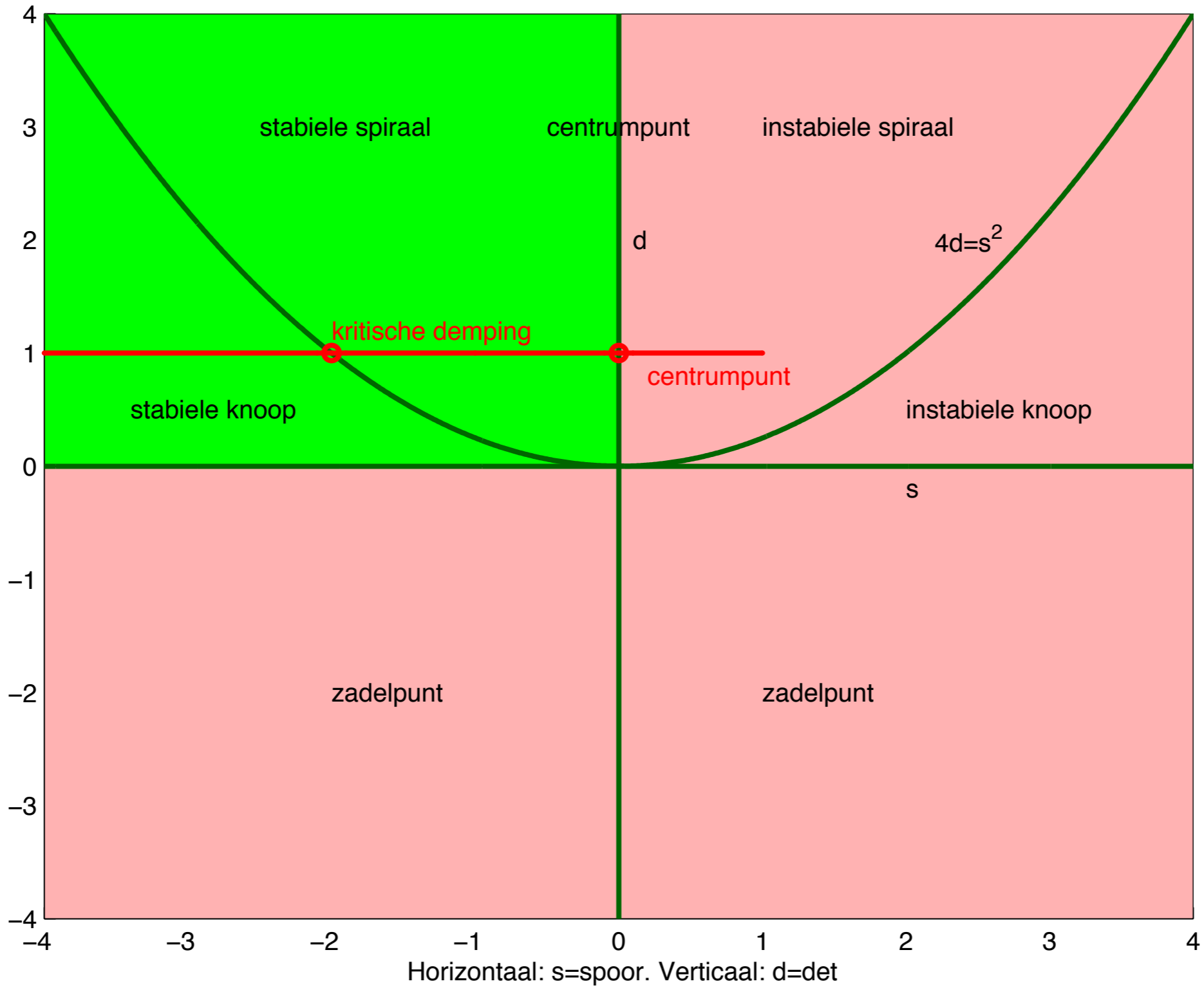
2-d Lineaire differentiaal vergelijking met $\lambda_1=1 i$, $\lambda_2=-1 i$



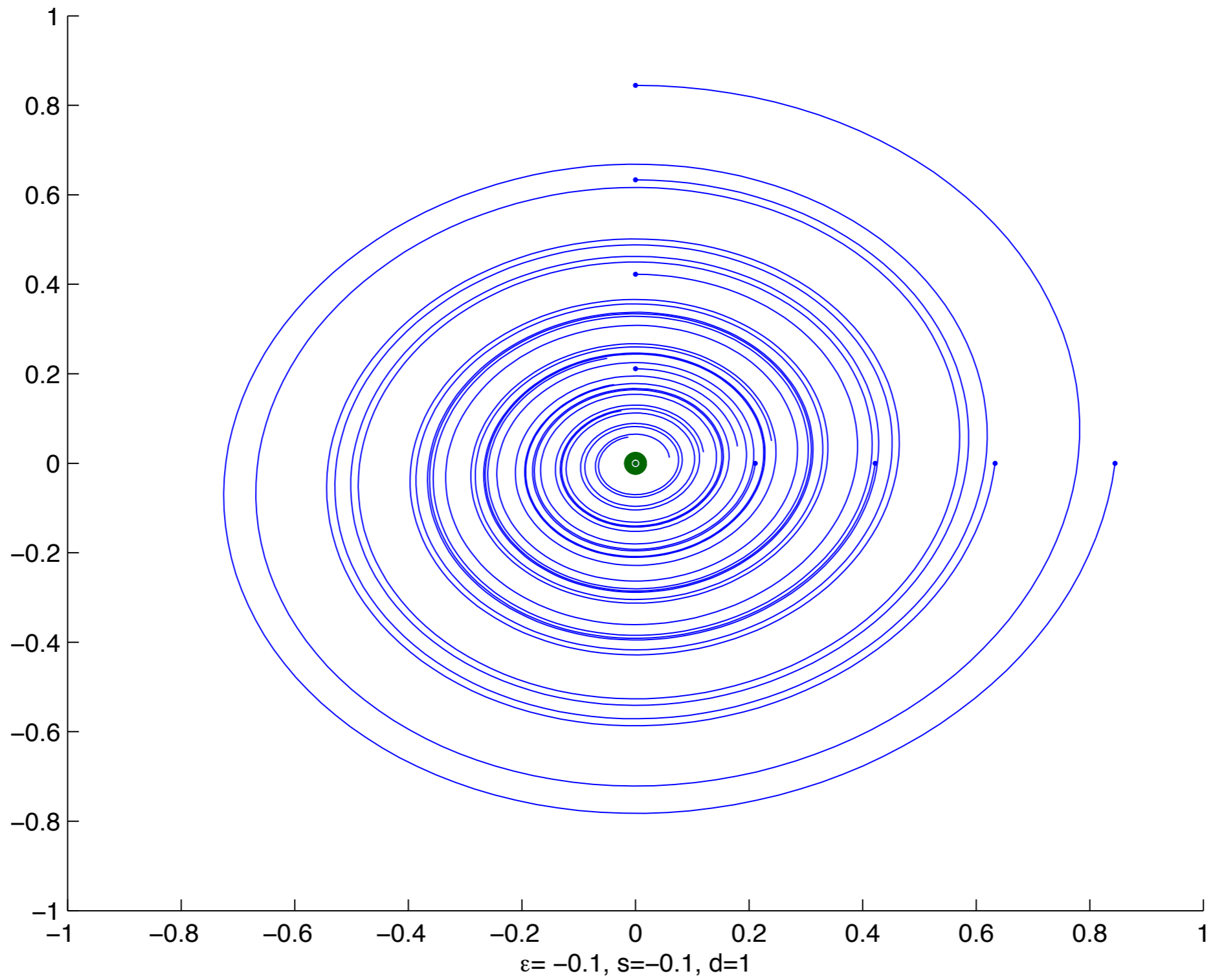
2-d Lineaire differentiaal vergelijking met $\lambda_1 = -1 i$, $\lambda_2 = 1 i$



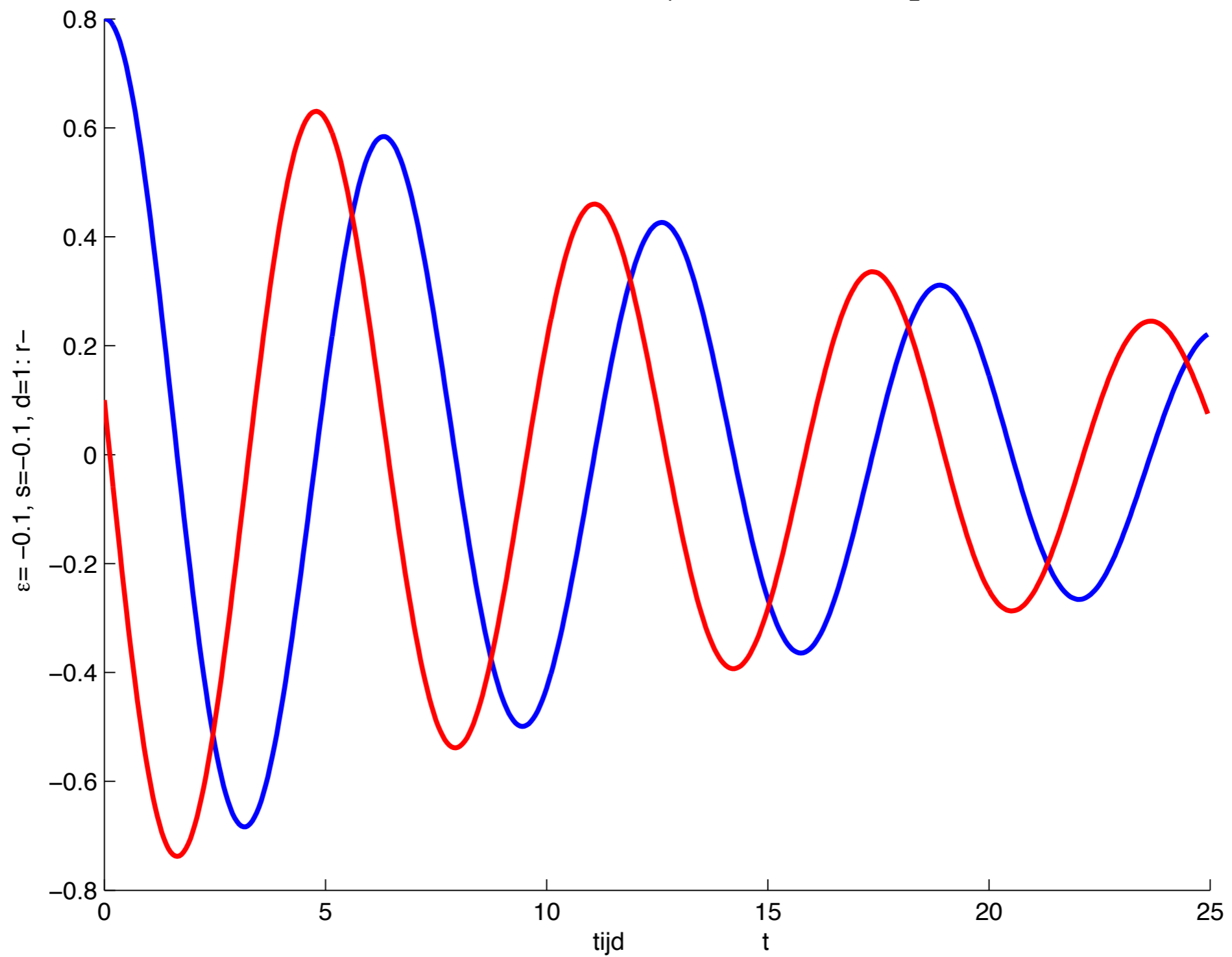
s en d bij aflopende s



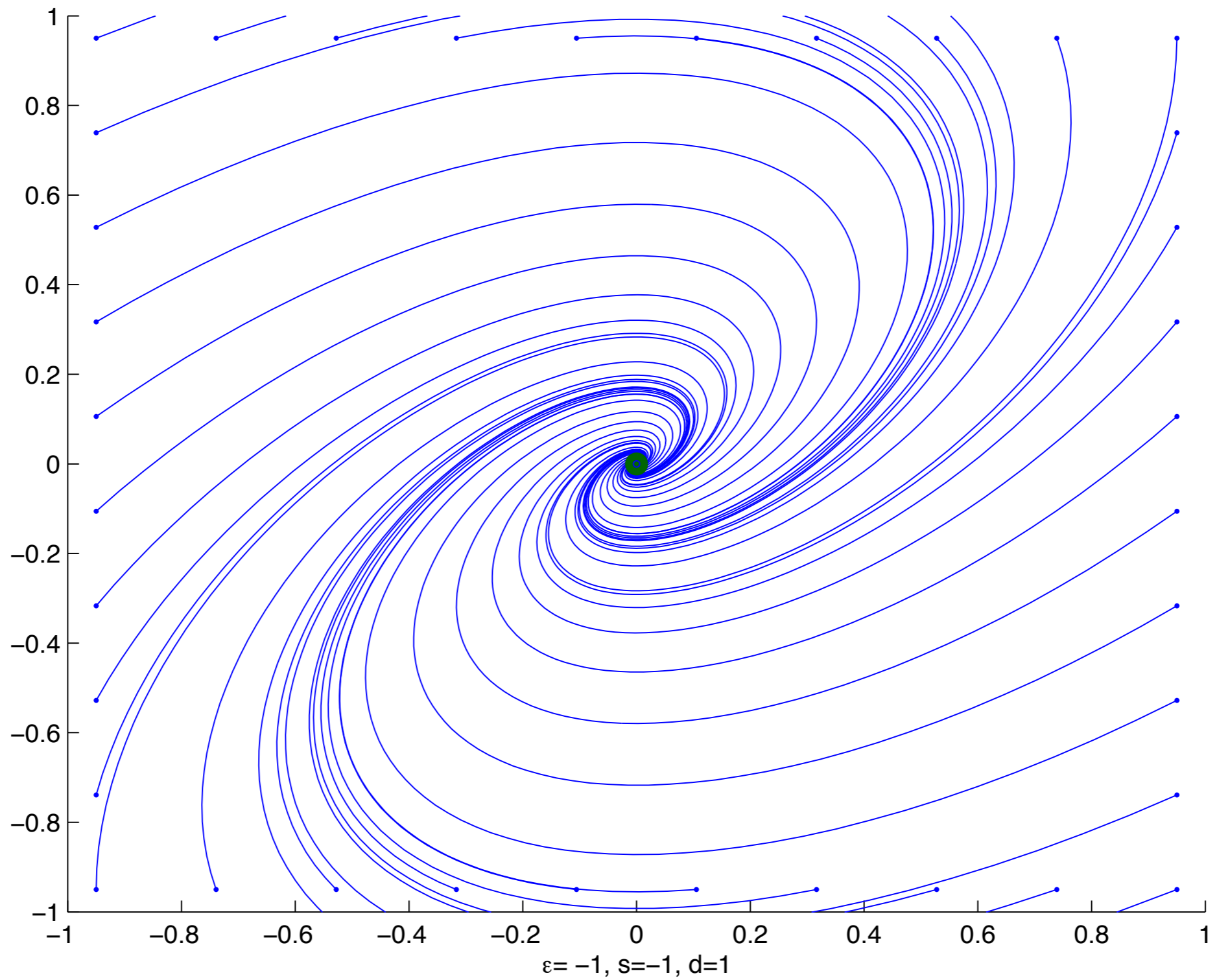
2-d Lineaire differentiaal vergelijking met $\lambda_1 = -0.05 + 0.998749 i$, $\lambda_2 = -0.05 - 0.998749 i$



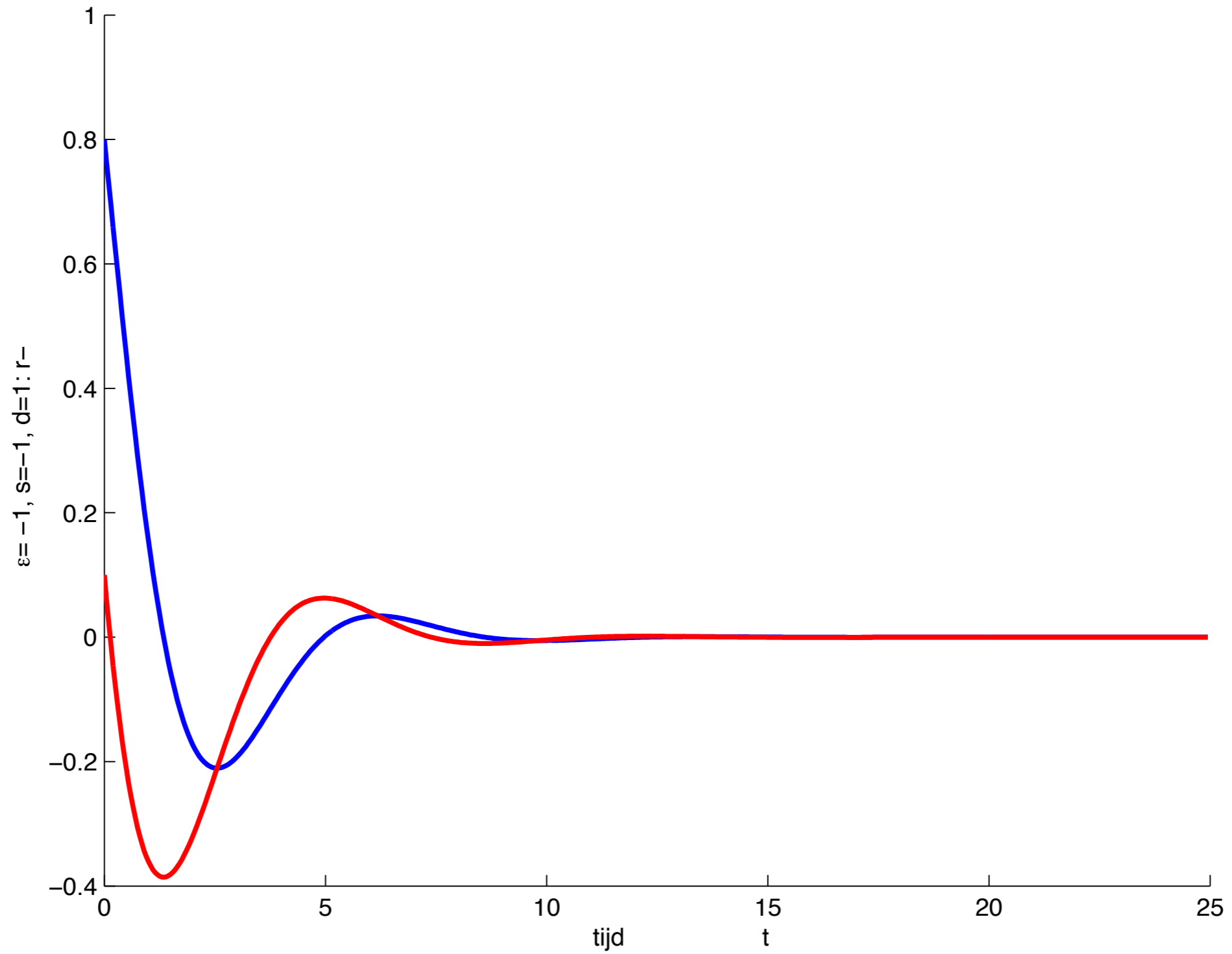
2-d Lineaire differentiaal vergelijking met $\lambda_1 = -0.05 - 0.998749 i$, $\lambda_2 = -0.05 + 0.998749 i$



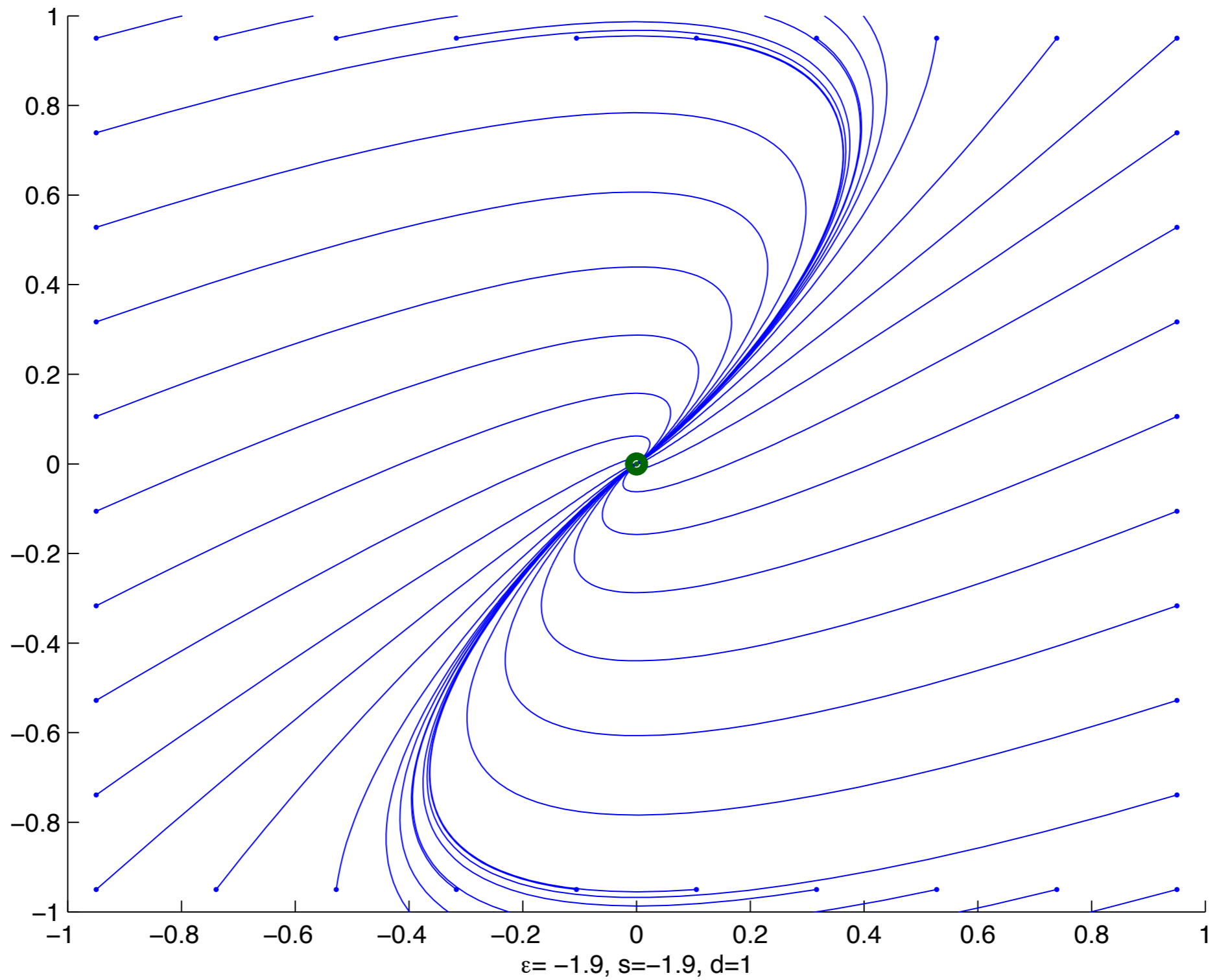
2-d Lineaire differentiaal vergelijking met $\lambda_1 = -0.5 + 0.866025 i$, $\lambda_2 = -0.5 - 0.866025 i$



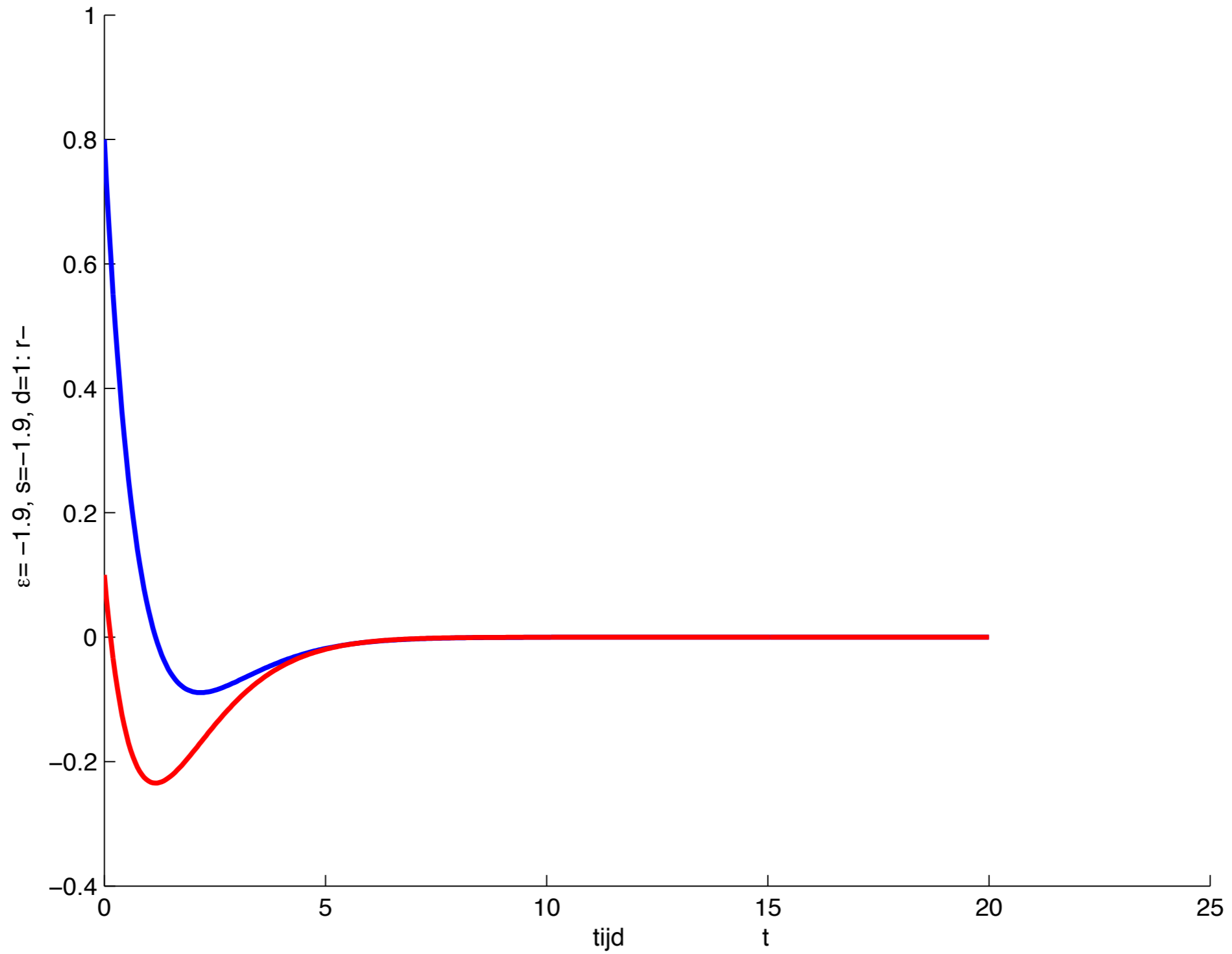
2-d Lineaire differentiaal vergelijking met $\lambda_1 = -0.5 - 0.866025 i$, $\lambda_2 = -0.5 + 0.866025 i$



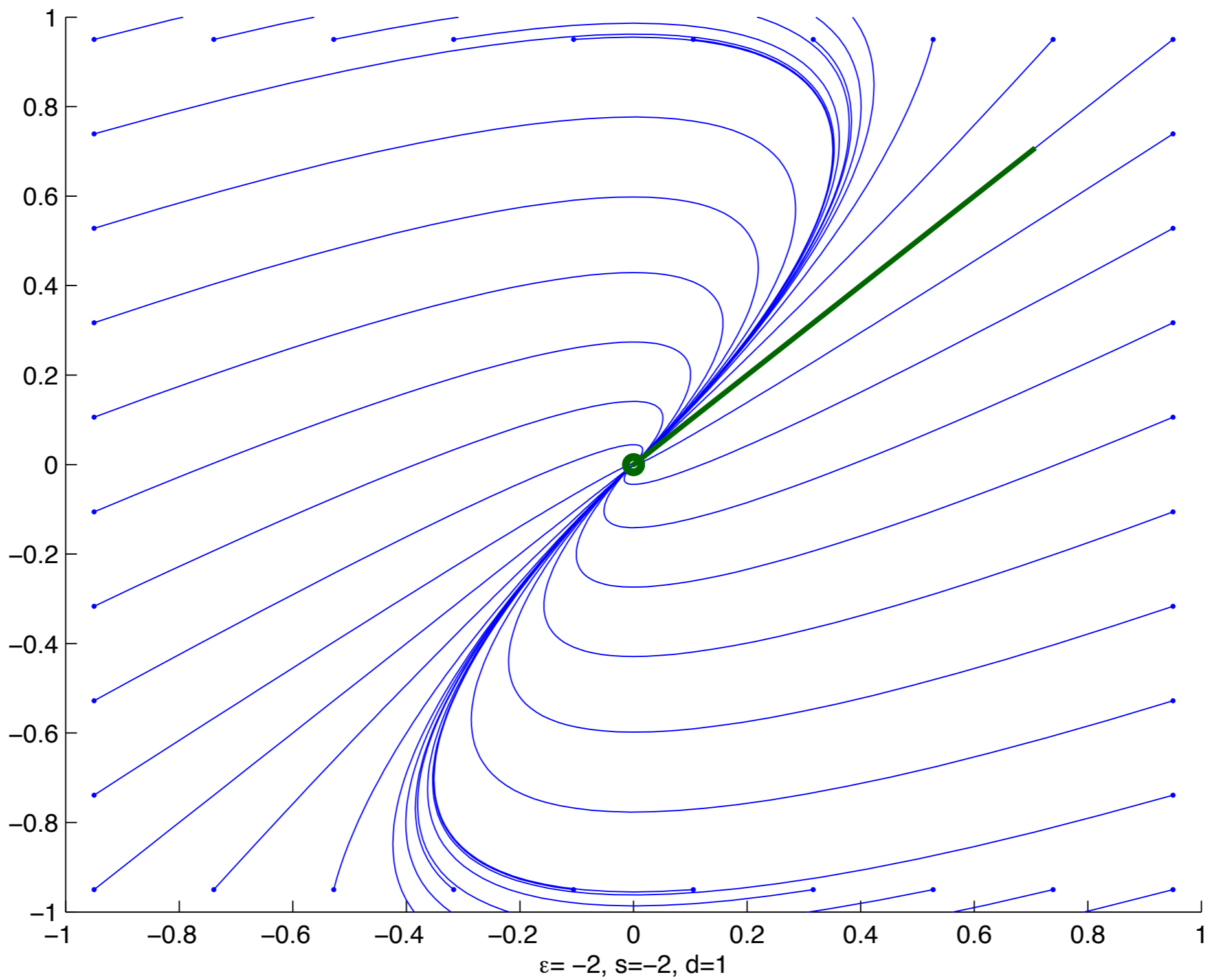
2-d Lineaire differentiaal vergelijking met $\lambda_1 = -0.95 + 0.31225 i$, $\lambda_2 = -0.95 - 0.31225 i$



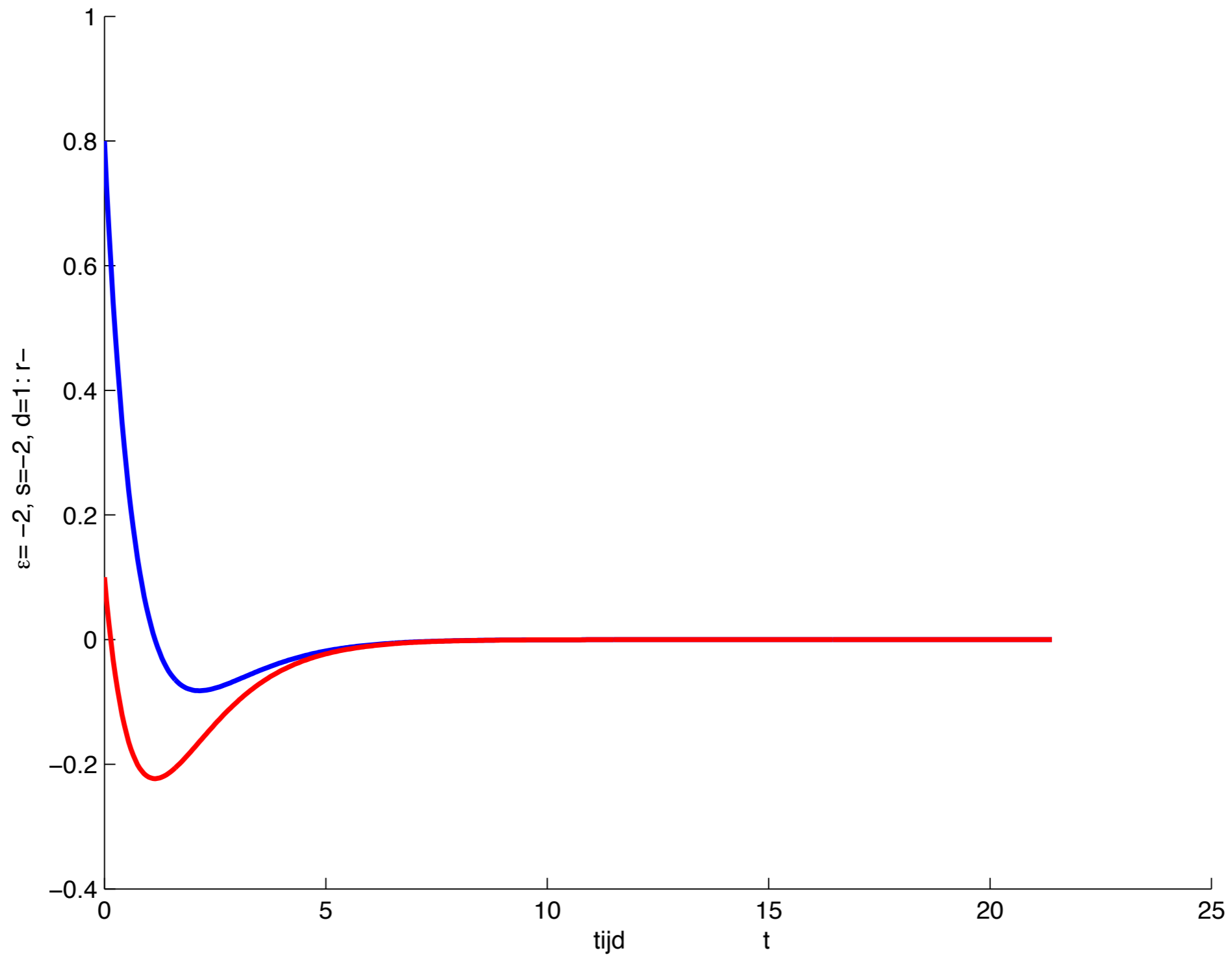
2-d Lineaire differentiaal vergelijking met $\lambda_1 = -0.95 - 0.31225 i$, $\lambda_2 = -0.95 + 0.31225 i$



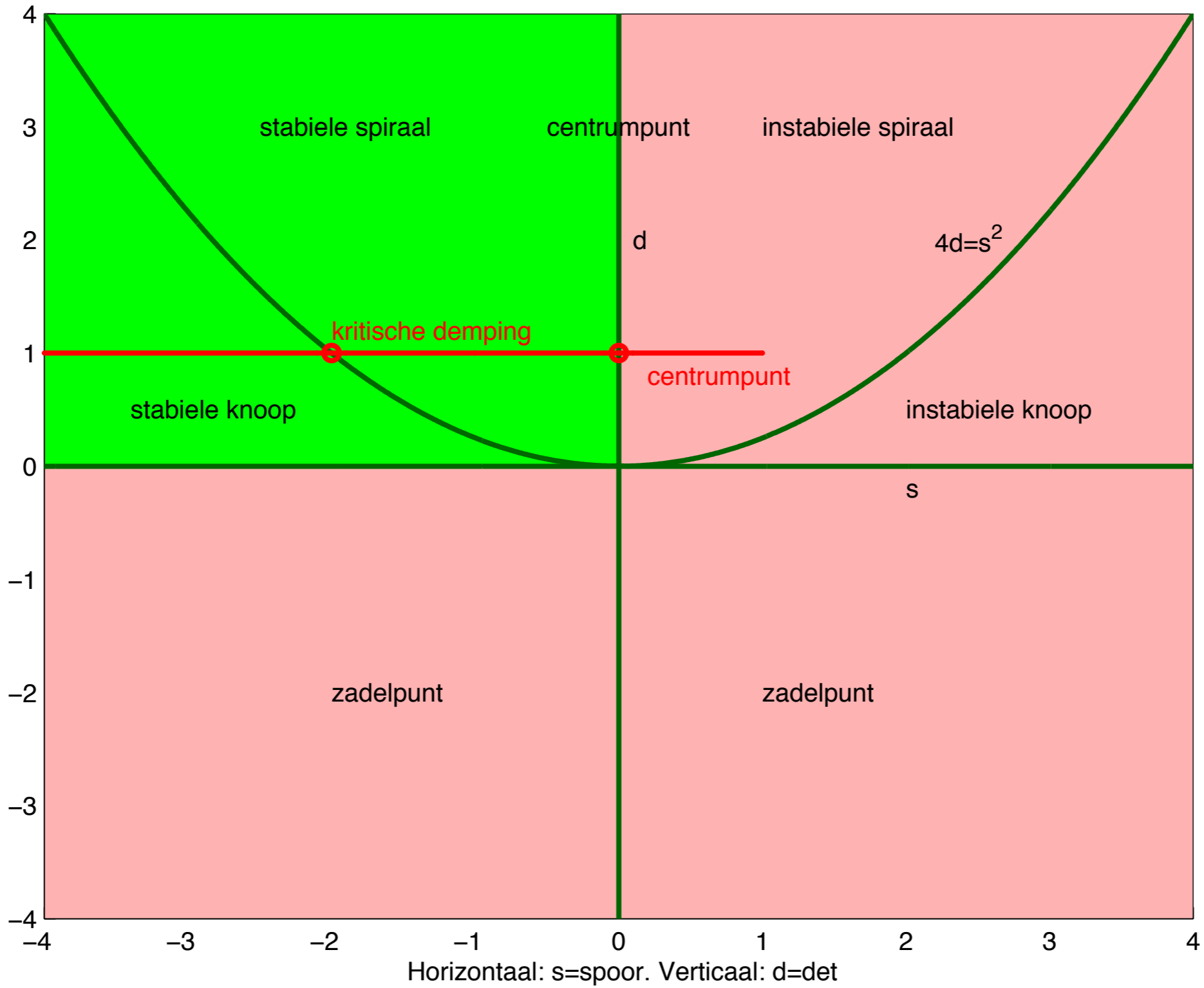
2-d Lineaire differentiaal vergelijking met $\lambda_1=-1, \lambda_2=-1$



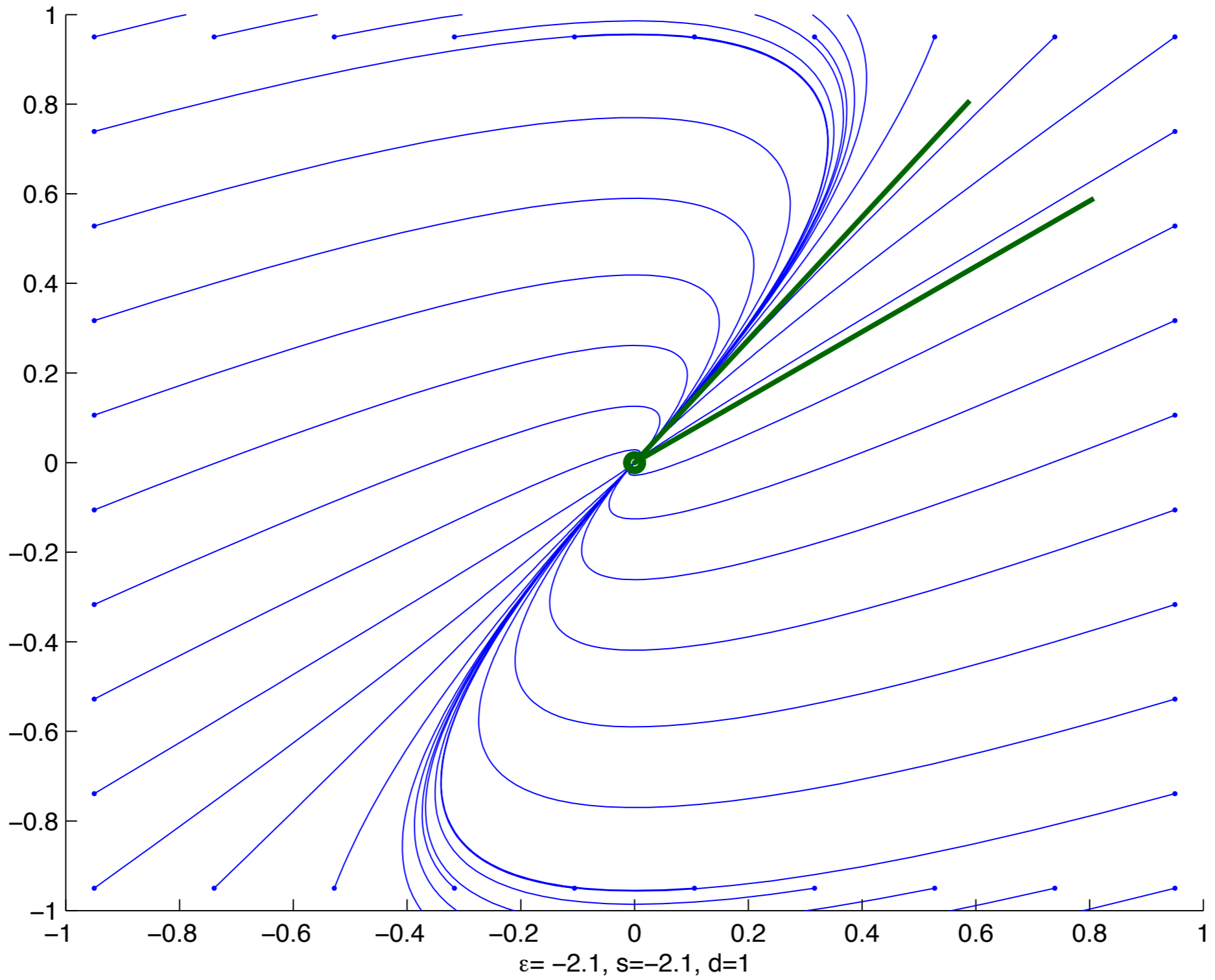
2-d Lineaire differentiaal vergelijking met $\lambda_1=-1, \lambda_2=-1$



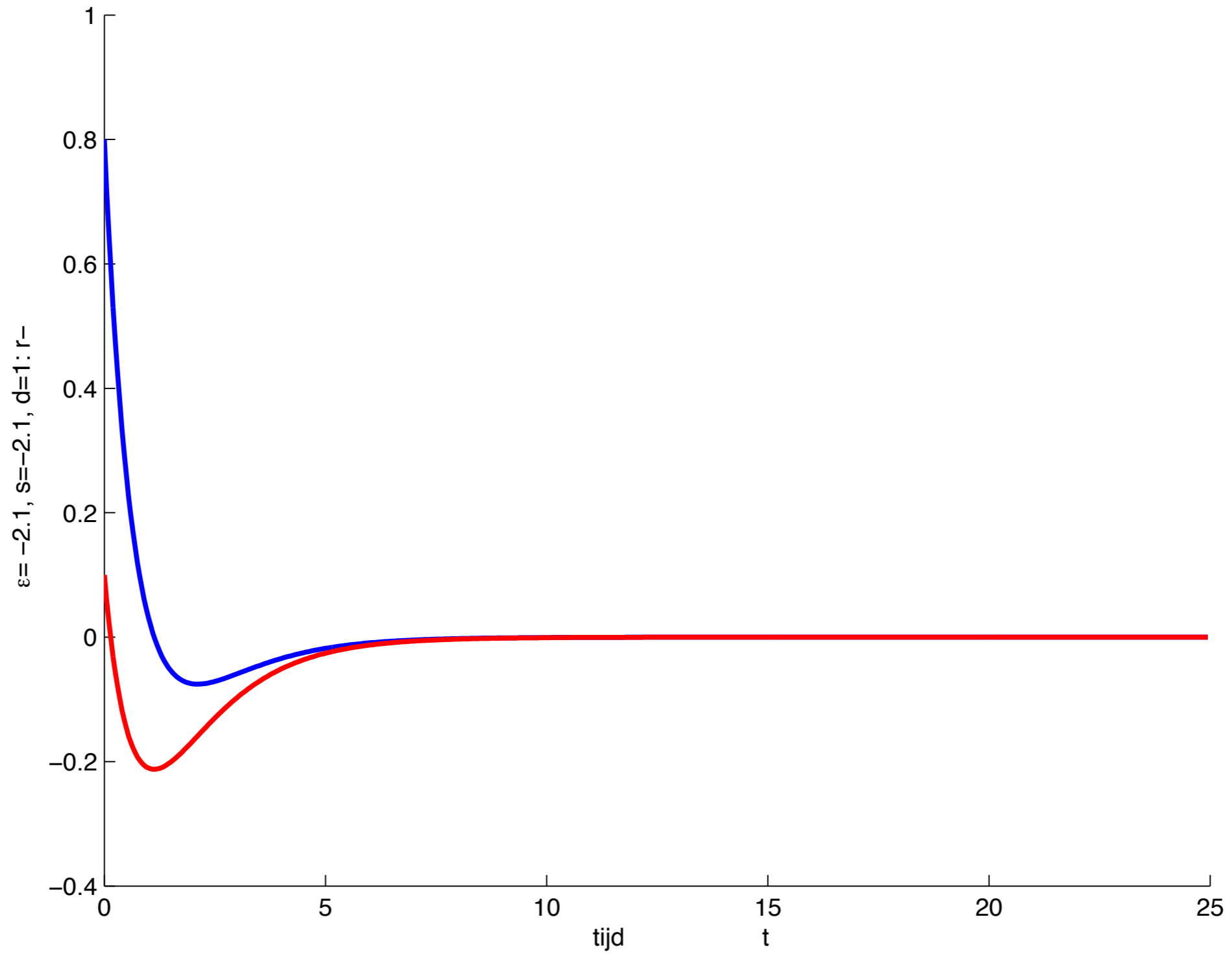
s en d bij aflopende s



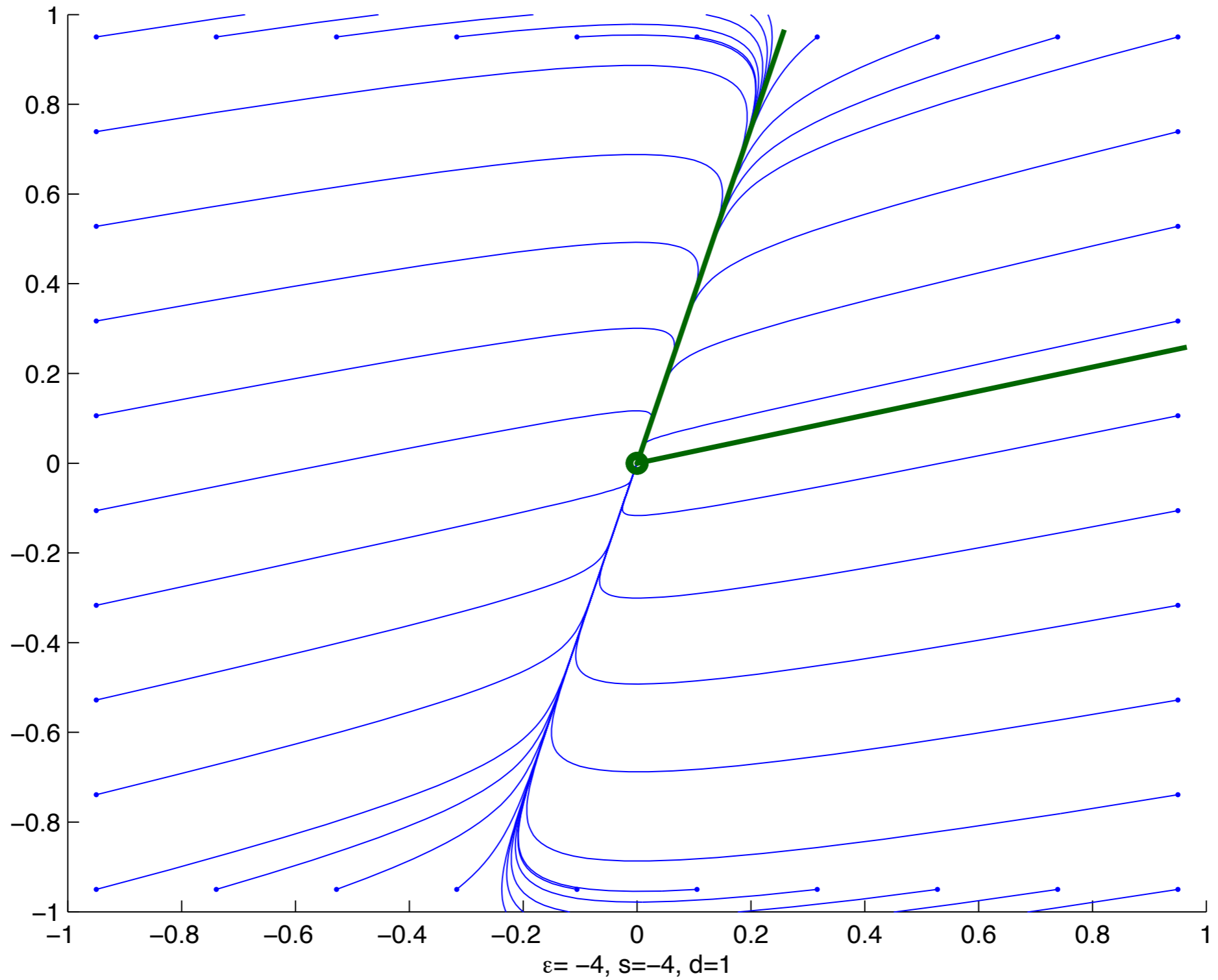
2-d Lineaire differentiaal vergelijking met $\lambda_1=-1.37016$, $\lambda_2=-0.729844$



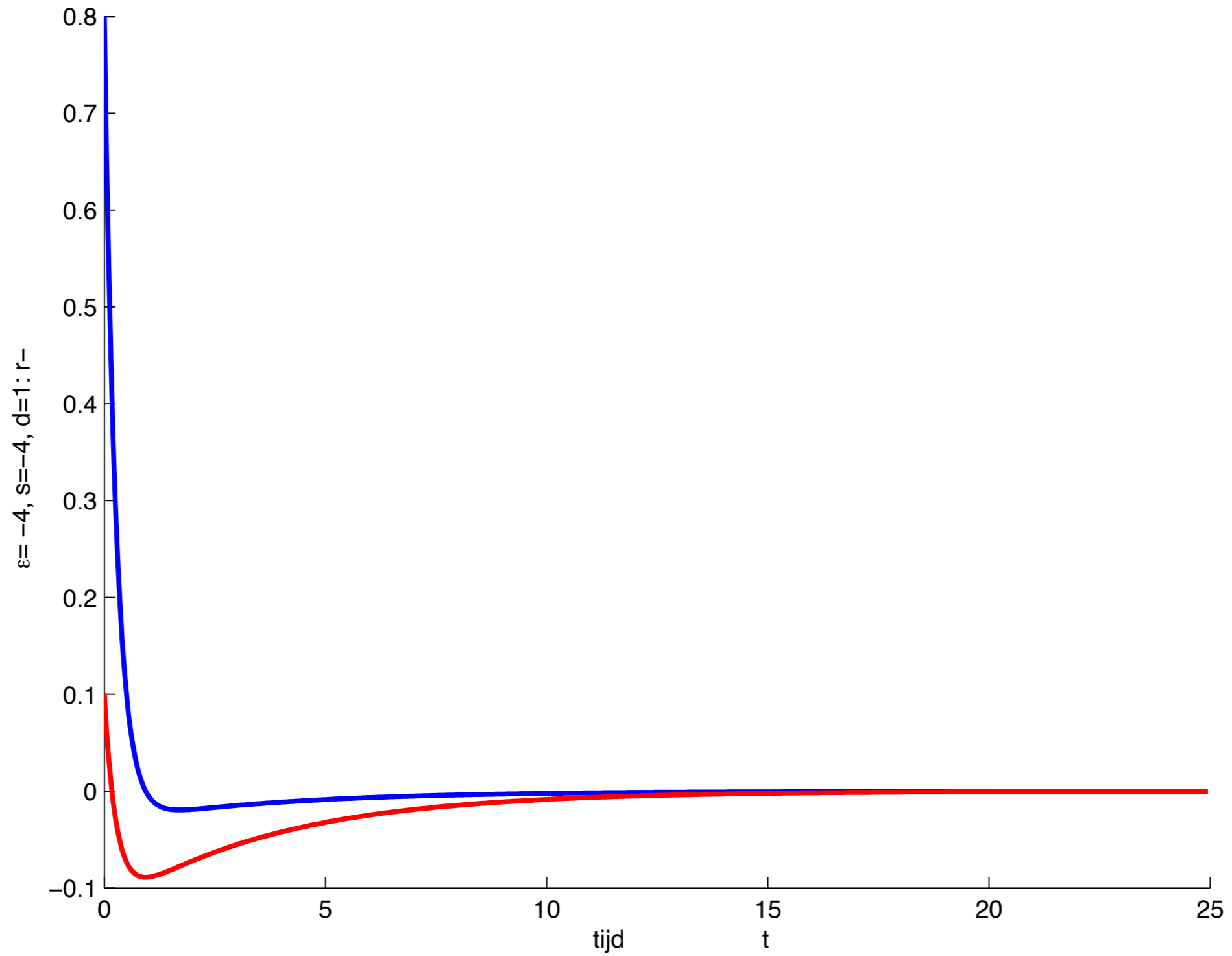
2-d Lineaire differentiaal vergelijking met $\lambda_1 = -1.37016$, $\lambda_2 = -0.729844$



2-d Lineaire differentiaal vergelijking met $\lambda_1=-3.73205$, $\lambda_2=-0.267949$



2-d Lineaire differentiaal vergelijking met $\lambda_1 = -3.73205$, $\lambda_2 = -0.267949$



Werkcollege voor vandaag

- **Geen opdrachten** - je mag te tijd gebruiken om aan het tweede verslag te werken, en eventuele vragen aan de werkcollege begeleiding te stellen