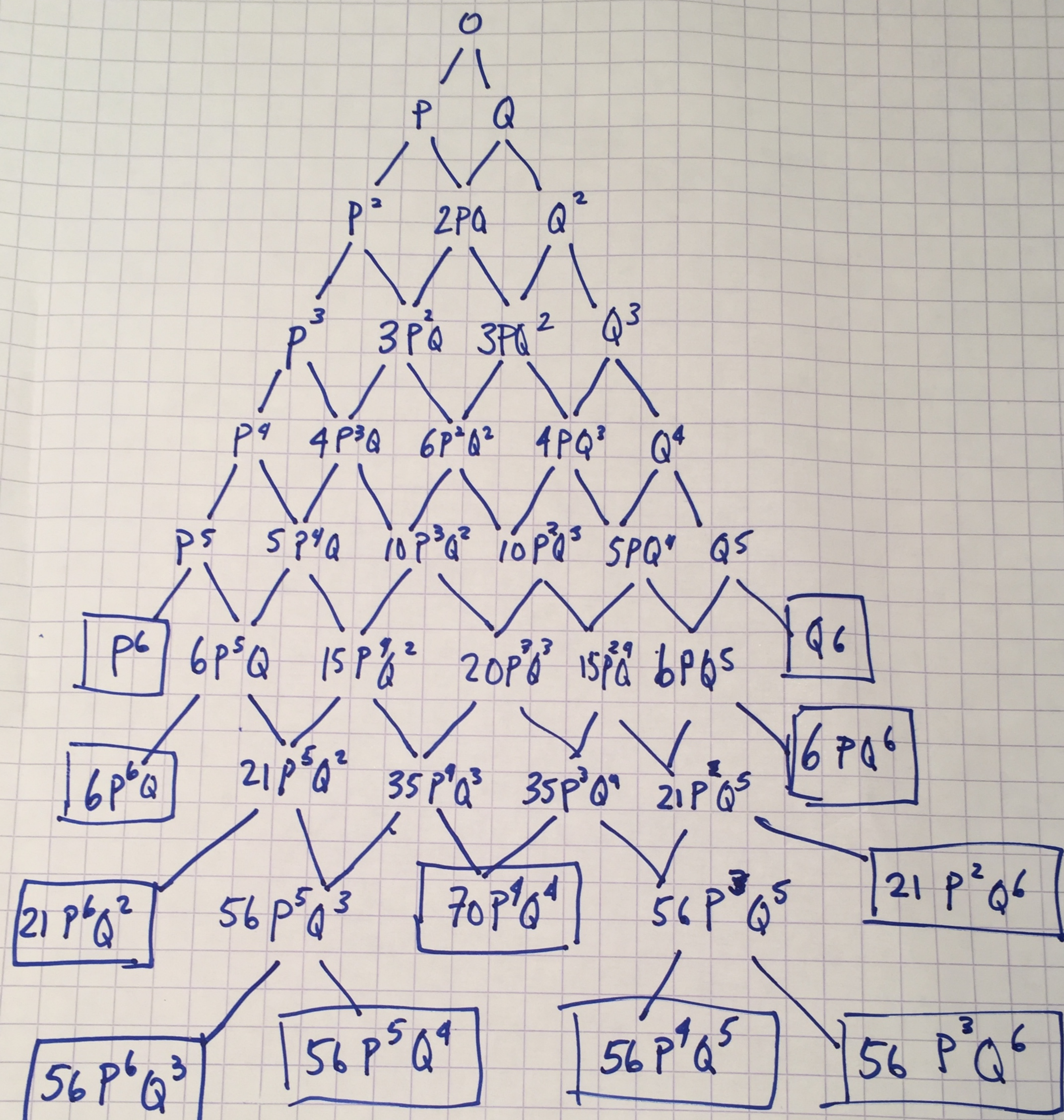


0
P Q
P^2 2PQ Q^2
P^3 $3P^2Q$ $3PQ^2$ Q^3
P^4 $4P^3Q$ $6P^2Q^2$ $4PQ^3$ Q^4
P^5 $5P^4Q$ $10P^3Q^2$ $10P^2Q^3$ $5PQ^4$ Q5
P^6 $6P^5Q$ $15P^4Q^2$ $20P^3Q^3$ $15P^2Q^4$ $6PQ^5$ Q6
$6P^6Q$ $21P^5Q^2$ $35P^4Q^3$ $35P^3Q^4$ $21P^2Q^5$ $6PQ^6$
$21P^6Q^2$ $56P^5Q^3$ $70P^4Q^4$ $56P^3Q^5$ $21P^2Q^6$
$56P^6Q^3$ $56P^5Q^4$ $56P^4Q^5$ $56P^3Q^6$

WISB134

Modellen & Simulatie

Lecture 15 - Resonantie en Synchronizatie

Universiteit Utrecht

Overzicht van ModSim

- Basisbegrippen dynamische modellen
  - Definities recursies, DVs, numerieke methoden
  - Oplossingen DVs
  - Convergentie numerieke methoden
- Dynamica
  - Scalaire dynamica
  - Dynamica op $\mathbf{R}^d$
  - Lineaire dynamica op $\mathbf{R}^2$
- Bijzondere gevallen
  - Lineaire kansmodellen (Markovketens)
  - ➡Niet-autonome systemen (Resonantie)
  - Diffusie

Vandaag

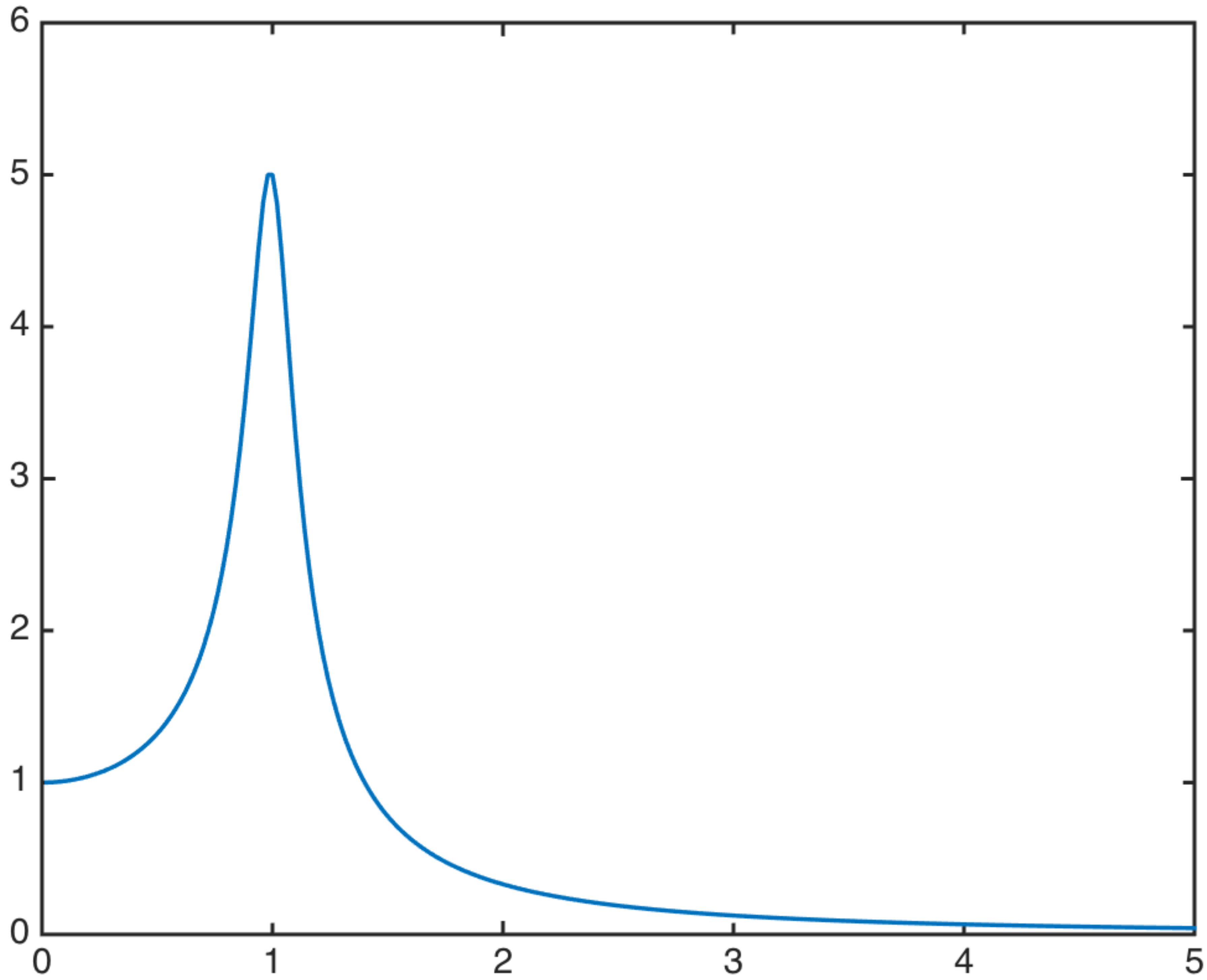
Resonantie en synchronisatie

- Niet-autonome systemen: homogene en particuliere oplossingen
- Resonantie in de veer-massa-demper systeem
- Resonantie in gekoppelde systemen
- Synchronizatie van oscillatoren

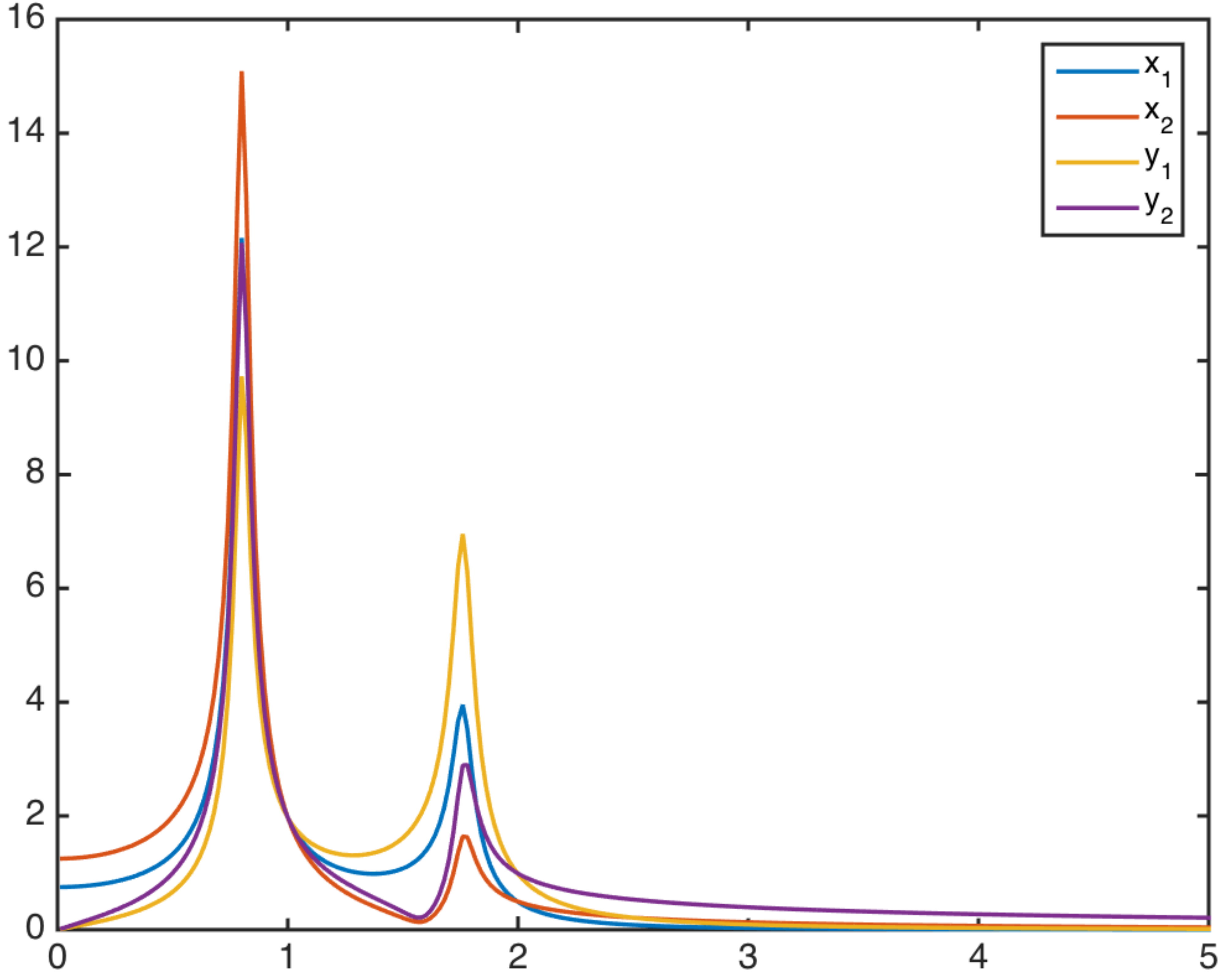
Party tricks

https://www.youtube.com/watch?v=BE827gwnnk4

Respons



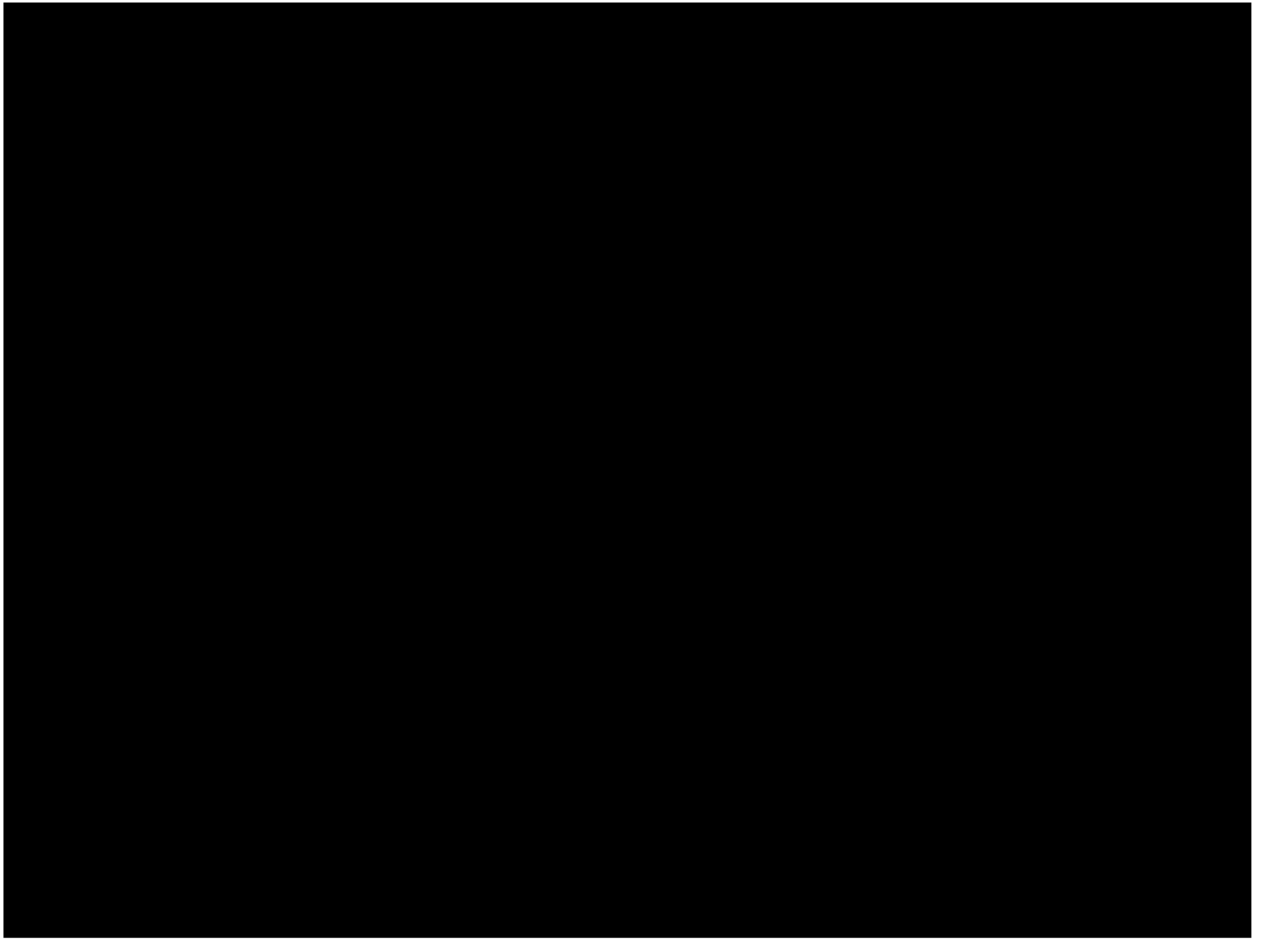
Respons



Het belang van input frequentie

https://www.youtube.com/v/I4FPK1oKddQ

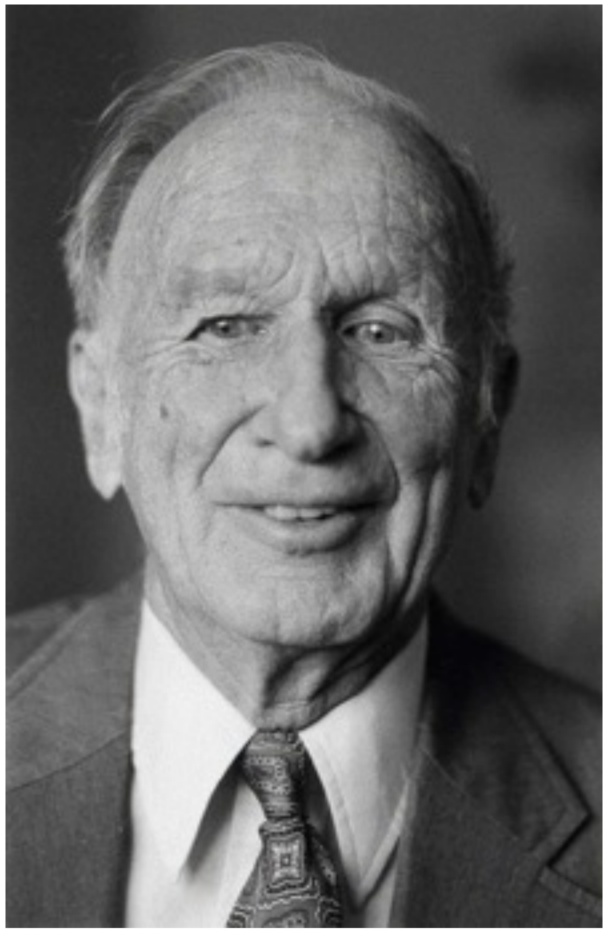
Synchronizatie van metronomen



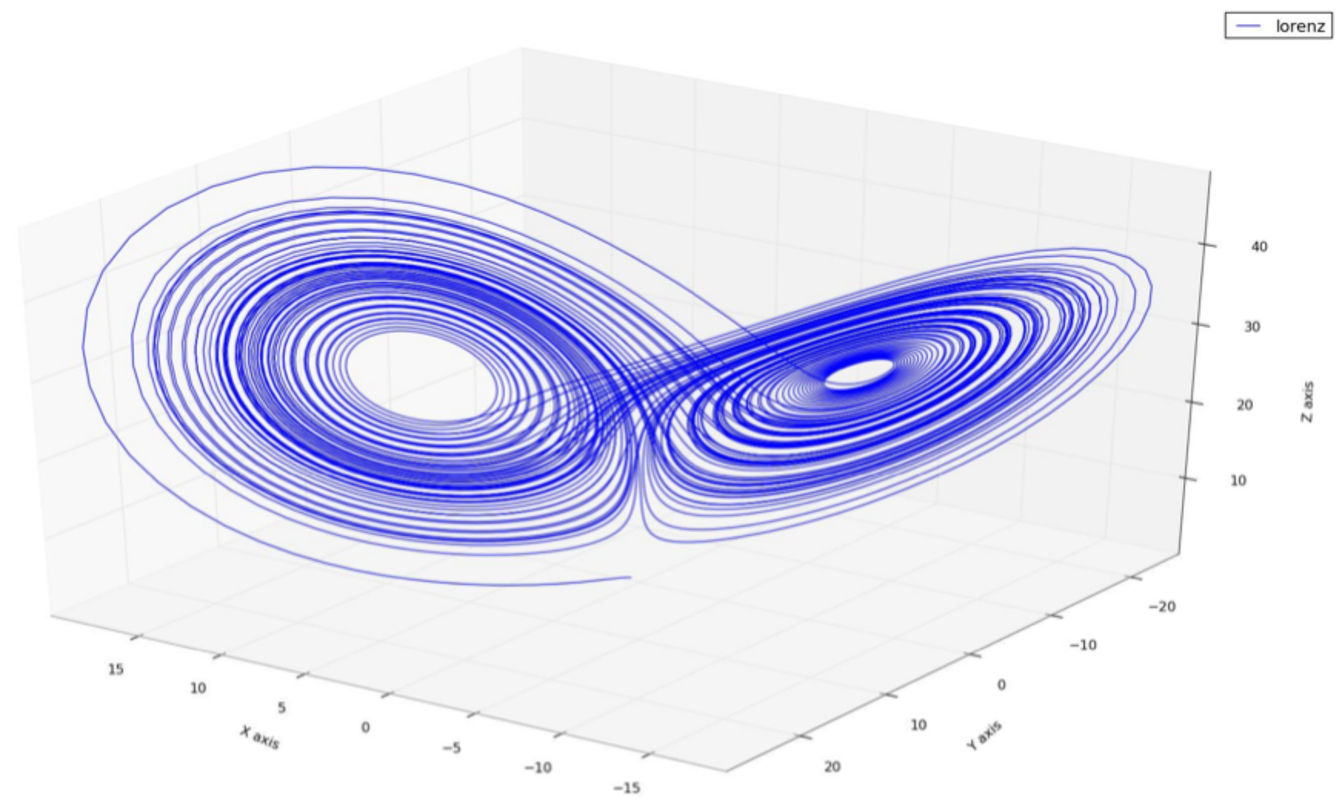
Synchronizatie van metronomen

https://www.youtube.com/v/Aaxw4zbULMs

Lorenz model (1963): het “eerste" chaotische systeem



E.N. Lorenz
1917 - 2008

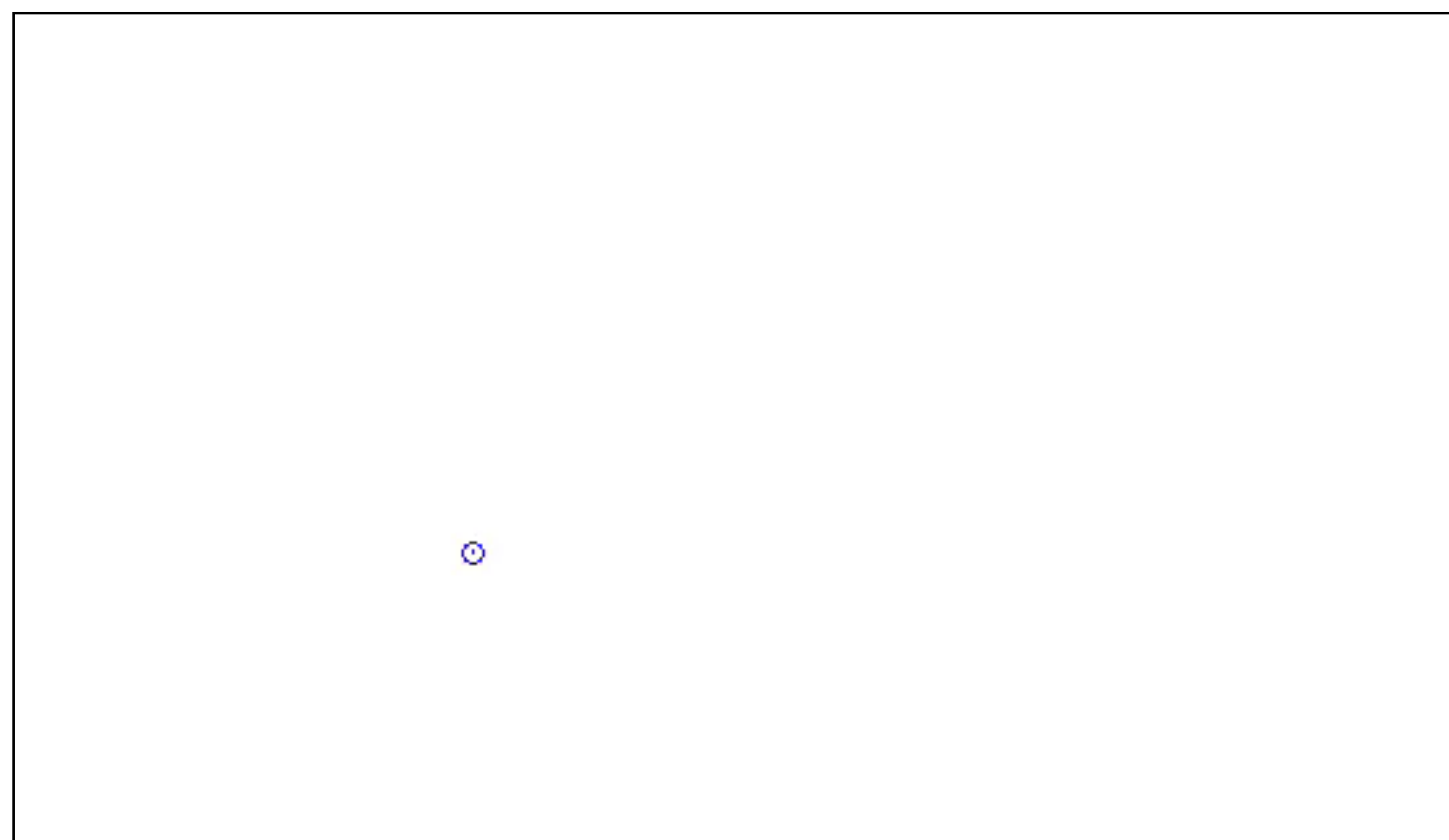


$$\frac{dX}{dt} = \sigma(Y - X)$$

$$\frac{dY}{dt} = X(\rho - Z) - Y$$

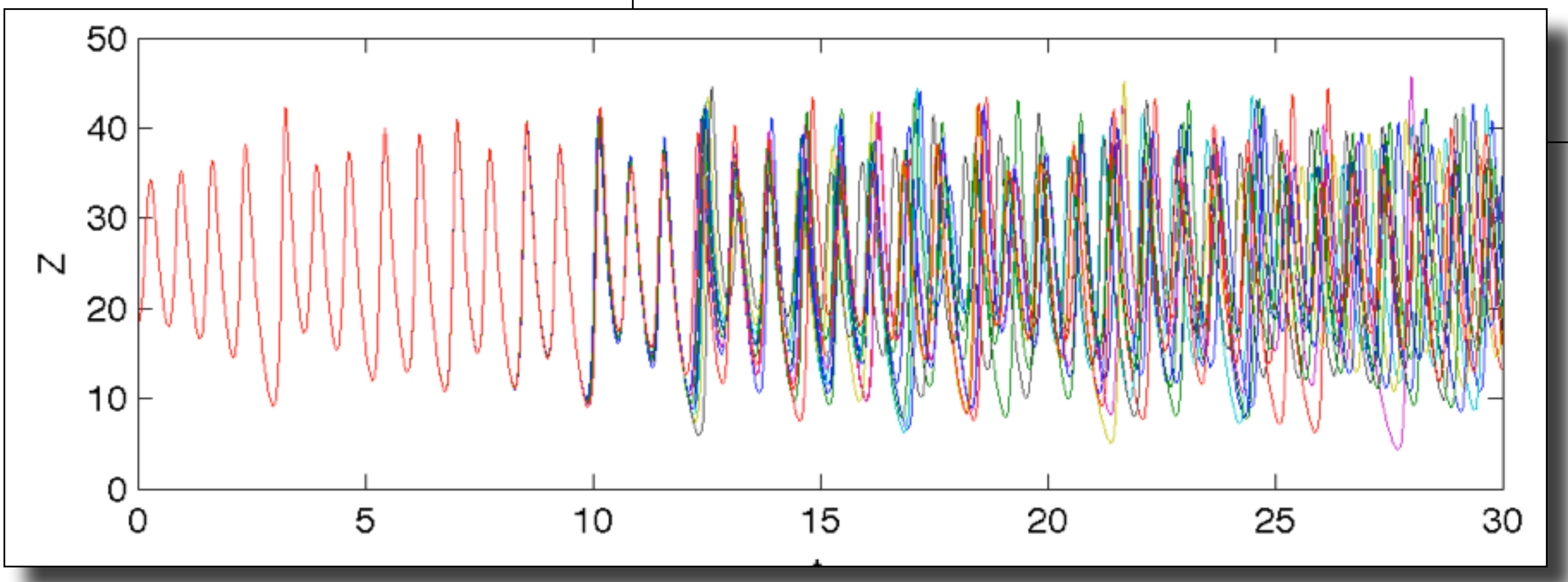
$$\frac{dZ}{dt} = XY - \beta Z$$

$$\sigma = 10, \quad \rho = 28, \quad \beta = 8/3$$



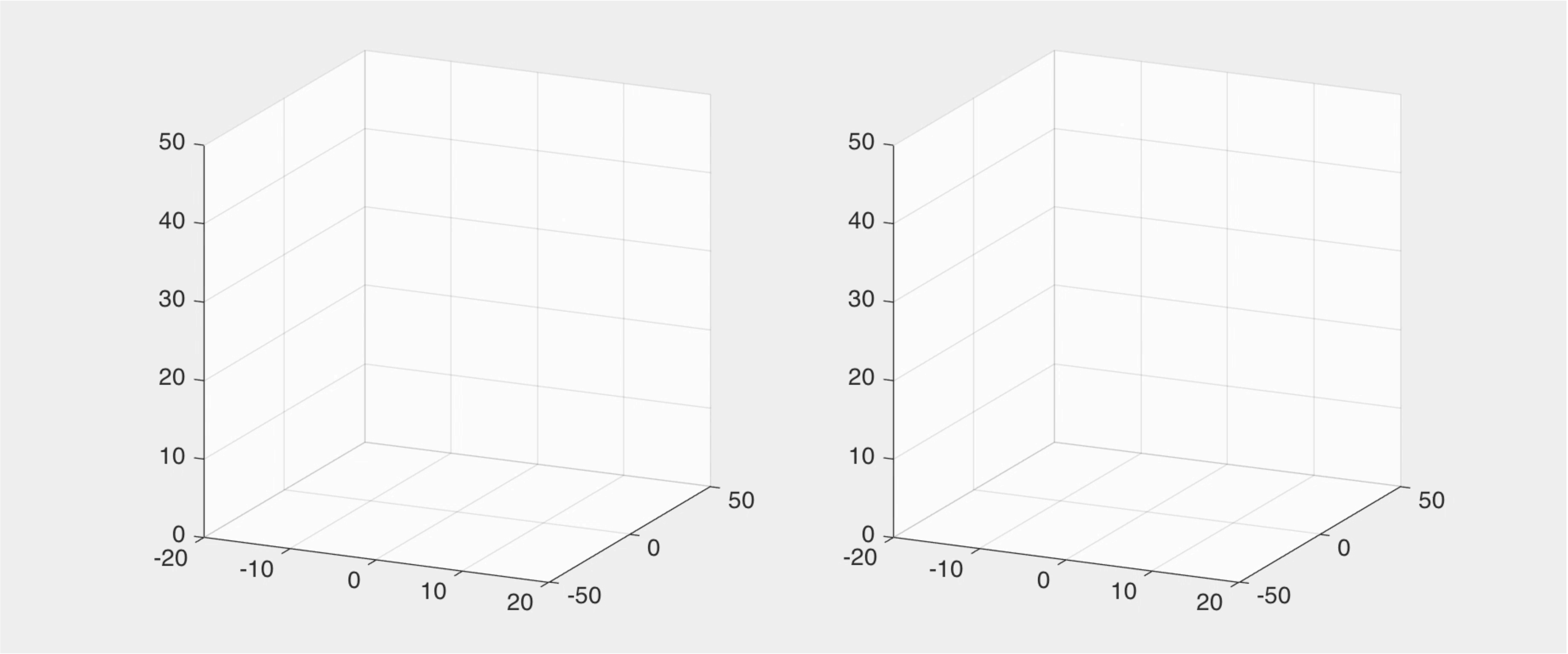
Chaotic behavior

10 Lorenz simulaties met een kleine verstoring van begin toestand.



Alleen de variabel Z als functie van de tijd.

X-Gesynchronizeerde Lorenz modellen



Bijzonder: *Een chaotisch systeem, maar door slechts een beetje waar te nemen, wordt het toestand steeds beter voorspelbaar.*

Pecora & Carroll 1990

The time series of the X variable of one Lorenz trajectory was used as a driving function in a second. The trajectories of the second system were observed to converge to those of the first.

$$\frac{dX}{dt} = \sigma(Y - X)$$
$$\frac{dY}{dt} = X(\rho - Z) - Y \qquad \frac{dy}{dt} = X(t)(\rho - z) - y$$
$$\frac{dZ}{dt} = XY - \beta Z \qquad \frac{dz}{dt} = X(t)y - \beta z$$

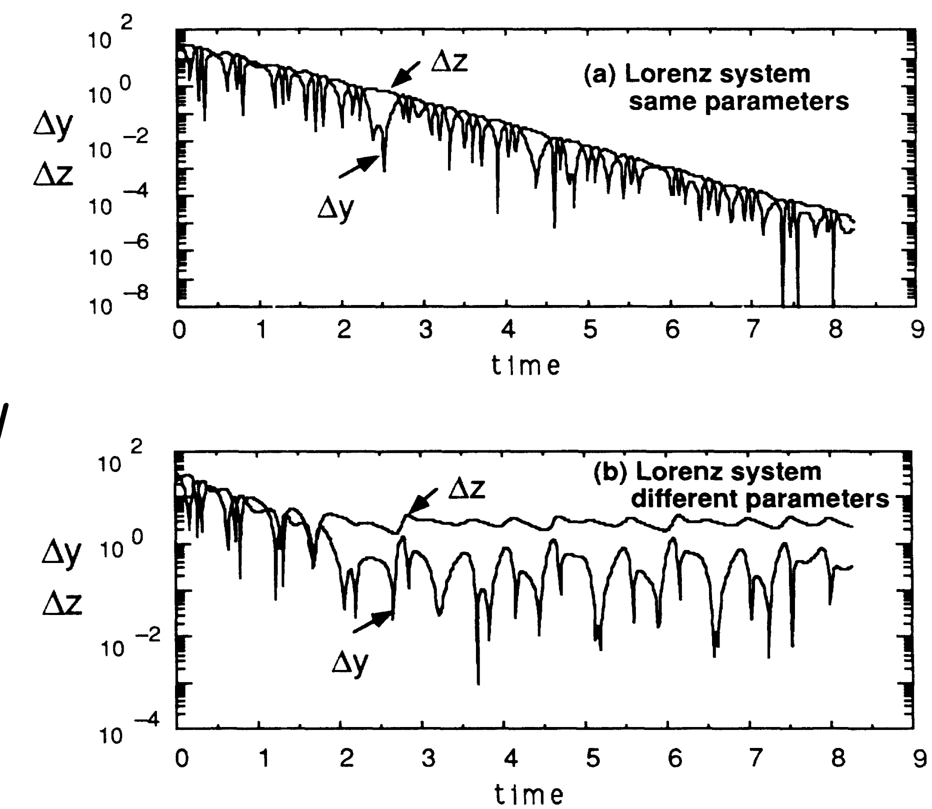


FIG. 2. The differences $y' - y$ and $z' - z$ between the response variables and their drive counterparts for the Lorenz system for (a) when parameters are the same for both systems and (b) when the parameters differ by 5%.

The same happened using y as a driver.
With z it didn't work.

Explanation: sub-Lyapunov exponents (negative for the first two components, positive for the last).

$$x_{n+1} = f(x_n)$$

$$\lambda(x_0) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln|f'(x_i)|$$