ModSim Project 1: Commodity pricing with adaptive expectations

Due: March 2, 2016, at 13.00

Remember to consult the course website for instructions on how to write your report. There is also an example of well-written report from a previous year.

Every year a farmer has to decide how much of each crop to plant based on his available resources and what he expects the prices to be. The choices made by all farmers determines the total supply of a given produce. This supply, coupled with the demand of buyers, ultimately determines the market price. But the farmer has to predict the price in order to decide on the right distribution of his resources.

Let p_n denote the (mean) *market price* of a commodity, say corn, in year n, and suppose that the demand is a linear function of price:

$$D(p) = a - d\,p, \tag{1}$$

where $a > 0$ and $d > 0$ are fixed positive constants.

It is common in economics to assume a *representative agent*, that is, a single "meta-farmer" whose choice represents the combined behavior of all farmers. By some means, which we will define later, the meta-farmer determines an *expected price* P_n for year n, and this price determines how much corn he plants and subsequently, the supply of corn on the market. The formula for supply is

$$S(P) = c + \arctan(\lambda(P - \bar{p})), \tag{2}$$

where c, λ and $\bar{p}$ are all positive constants.

Because corn is a perishable commodity, we assume that the price p_n adjusts such that the market *clears*, that is, all of the corn is sold in year n and supply equals demand:

$$D(p_n) = S(P_n). \tag{3}$$

To be clear, equation (3) determines the market price p_n as a function of the expected price P_n of the meta-farmer. Substituting (1) and (2) into (3) and solving for p_n we obtain a relation for the market price as a function of the expected price

$$p_n = \frac{1}{d}\left[a - c - \arctan(\lambda(P_n - \bar{p}))\right]. \tag{4}$$

The model is complete if we can now relate the meta-farmer's expected price P_n to the actual price p_n, and there are various economic theories about how our farmer does this.

For this project we study the *adaptive expectations* model:

$$P_n = (1 - w)P_{n-1} + w\, p_{n-1}, \tag{5}$$

where w is a weight factor, usually $0 \leq w \leq 1$. The idea here is that our meta-farmer adapts last year's expectation (P_{n-1}) toward last year's actual realized market price.

In a well written report (*see course website*!), handle the following questions. The parameters in the model are chosen as: $a = 4.1$, $d = 0.25$, $c = 1.5$, $\lambda = 4.8$ and $\bar{p} = 6$. The parameter w will take values in the interval $[0, 1]$. Example *Mathematica* codes for producing cobweb plots and bifurcation diagrams can be found in Section 12.6 of the book by Lynch (available online, see link from the course website).

(a) Combine (5) and (4) to construct a map $P_n = F(P_{n-1})$ that determines the expected price in year n as a function of the expected price in year $n-1$. Plot the function F for $w = 1$, $w = 0.5$ and $w = 0.3$. In each case, identify a bounded interval $\mathcal{D} = \{P \in \mathbf{R} \,|\, P_{\min} \leq P \leq P_{\max}\}$ that is invariant under F (i.e. such that $F : \mathcal{D} \to \mathcal{D}$).

(b) Derive the condition that defines a fixed point P^* of F. Does this condition depend on w? Derive a relation that must hold to ensure stability of the fixed point. Dos this condition depend on w? Compute the numerical value of the fixed point for $w = 0.3$. Estimate numerically the range of values w for which the fixed point is stable.

(c) Choose $w = 1$ (naive expectations). Assuming initial condition $P_0 = 6.2$, make a cobweb plot (graphical analysis method) of the iteration. Plot the composite function $G(P) = F(F(P))$. Which fixed points are stable? unstable? Describe the iterates P_n, P_{n+1}, P_{n+2}, etc. (under the map F) for large n. This phenomenon is referred to as the "hog cycle". Can you explain why such behavior is observed in real commodities markets?

A point of discussion in economics is whether seemingly random market fluctuations are caused purely by external forces, or if such behavior can follow from the internal market dynamics.

(e) Plot the market price over a long period (100 years) assuming the meta-farmer uses the adaptive expectations model (5) for weights $w = 1$, $w = 0.43$, $w = 0.3$, and $w = 0.5$. Discuss the behavior of the different solutions.

(f) Construct a bifurcation diagram as a function of w. What can you conclude about the existence of seemingly random fluctuations due purely to internal market dynamics?