# Conservative Dynamical Systems 2010/2011

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The homework consists of two exercises, to be handed in on Tuesday 19 October.

### 6.1 Dissipative subsystems and linearization

1. Show that every system of $n$ differential equations can be completed to a Hamiltonian system on $\mathbb{R}^{2n}$.

2. Show that the linearization of a Hamiltonian system is again Hamiltonian. How are the corresponding Hamiltonian functions related to each other?

### 6.2 Transformations in one degree of freedom

Let $H : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a given smooth function, with corresponding Hamiltonian vector field $X_H$. Here we use the standard symplectic structure on $\mathbb{R}^2$. Moreover, let $g : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a diffeomorphism. Consider both the function $K := H \circ g^{-1}$, together with the associated Hamiltonian vector field $X_K$, and the transformed vector field $g_\star(X_H)$, defined by $g_\star(X_H)(g(p)) := D_p g X_H(p)$. Show that

$$g_\star(X_H) = \det(Dg) X_K \quad .$$

*Hint:* exploit a coordinate free formulation of the fact that $X_H$ is the Hamiltonian vector field corresponding to $H$. Discuss the implication for the integral curves of $g_\star(X_H)$ and $X_K$. Also consider the time-parametrisation of these curves. What happens in the special case that $g$ is canonical?