# Dynamical Systems 2007

The last two exercises are homework, to be handed in on 26 March.

## 7.1 Reduced Euler top

Analyse the dynamics of the "reduced Euler top"

$$H(x,y,z) \;=\; \frac{x^2}{2a} \;+\; \frac{y^2}{2b} \;+\; \frac{z^2}{2c} \;, \quad 0 < a \le b \le c$$

on $S^2$ in the limiting cases $a \to b$ and $b \to c$.

## 7.2 A harmonic $n$-body problem

The particles $A_i$ with masses $m_i$ $(i = 1, 2, \ldots, n)$ move in three dimensional space. Any two distinct points $A_i$, $A_j$ attract each other by the force $F_{ij} = k^2 m_i m_j d_{ij}$, where $k > 0$ and $d_{ij}$ denotes the distance $\overline{A_i A_j}$. We suppose that the motions of $A_i$ and $A_j$ are not disturbed if they pass simultaneously through the same point. Determine the general motion of the particles.

## 7.3 Colombo's top

Analyse the dynamics of "Colombo's top" on $S^2$, the 2–parameter family with Hamiltonian functions

$$H_{\lambda,\mu}(x,y,z) \;=\; -\frac{1}{2}(z-\lambda)^2 \;+\; \mu\, y \;.$$

## 7.4 Steiner ellipse

A particle $P$ of unit mass moves in the plane of a given fixed triangle $A_1A_2A_3$. The force $F_i$ on $P$ is directed towards $A_i$ and is equal to $k\overline{PA_i}$ for $i = 1, 2, 3$, where $k$ is a positive constant, and $\overline{PA_i}$ denotes the distance between $P$ and $A_i$. Prove that there is a motion of $P$ the path of which coincides with the Steiner ellipse $S$ of $A_1A_2A_3$ (the ellipse $S$ passes through the vertices and the tangent at each vertex is parallel to the opposite side). Show moreover that $P$ covers the three arcs $A_1A_2$, $A_2A_3$ and $A_3A_1$ of $S$ in equal time.