Exercises 17. 10. 2012

1. Let $\dot{x} = f(x)$ be a differential equation on $\mathbb{R}^4$ with $f(0) = 0$ and linear part $A = Df(0)$ at this equilibrium. The eigenvalues of A are all on the imaginairy axis and read as $\pm$i and $\pm$2i. What is the normal form of f ? Determine the symmetry of the normal form.
2. Write the nonlinear oscillator $\ddot{x} + \mu\dot{x} - \dot{x}^3 + x = 0$ as a vector field und give phase portraits for well chosen values $\mu \in [-1, 1]$ of the parameter.
3. Show that the nonlinear oscillator of the previous exercise undergoes a Hopf bifurcation as μ passes through zero and determine whether this bifurcation is of supercritical or subcritical type.