BIFURCATIONS IN DYNAMICAL SYSTEMS: HW 2

We want to see why Theorem 2 partially proved in class implies Theorem 1. I.e why does the centre manifold theorem for maps imply the centre manifold theorem for flows. The proof is given in chapter 4 of [1]. However there are some claims in the proof that need some explanation. In particular:

(1) How exactly does the Jordan normal form of A allow us to see the existence of a $\tau > 0$ such that $T = e^{\tau A}$ satisfies the stated inequalities:
$$\|T_s\| < 1, \ \|T_s\| \|T_{cu}^{-1}\|^{k+1} < 1.$$
(2) How exactly can we deduce from equation (4.2) [1] that solutions to (4.3) exist for all time?
(3) **Bonus** Why does reversing time direction in the system (4.1) allow us to construct a centre-stable manifold instead of a centre-unstable one?

NB: You only need to do the first two of above three questions for homework, but if you feel like attempting all three there will be bonus points awarded for attempts on the third question.

References

[1] Bjorn Sandstede and Thunwa Theerakarn. Regularity of center manifolds via the graph transform. *Journal of Dynamics and Differential Equations*, 2015.