

# Matrices depending on parameters

Valesca Peereboom  
v.d.peereboom@students.uu.nl

Deadline: October 15, 2018.

For diagonalizable matrices the normal form can easily be written as a diagonal matrix of eigenvalues. However, not all matrices are diagonalizable and to find a similar normal form smoothly depending on parameters for every matrix, we used versal deformations.

We say that two matrices commute when  $SP = PS$  and the commutator  $[P, S] = SP - PS$  is in that case zero. Since the orbit of a matrix  $m$  is given by the set of matrices  $G(m) = \{gmg^{-1}\}$  for all  $g \in GL(n, \mathbb{C})$  which is the set of non singular  $n \times n$ - matrices, the tangent space of such a orbit can be written as  $TG(m) = \{[g, m]\}$  for all  $g \in GL(n, \mathbb{C})$ .

During these exercises we take a closer look at the Sylvester family and the versal deformations with the fewest number of parameters (also called miniversal deformations) discussed in class.

## Exercise 1.

Show by a direct computation of the commutators that the  $n \times n$  Sylvester family:

$$A(\alpha) = \begin{pmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & \cdot & \cdot & \cdot & \cdot \\ & & & 0 & 1 \\ \alpha_1 & \alpha_2 & & \dots & \alpha_n \end{pmatrix}$$

is transversal to the orbit of its  $A_0$  matrices and has the minimal number of parameters.

## Exercise 2.

a) Derive a miniversal deformation of the matrix:

$$H = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 11 & 6 & -4 & -4 \\ 22 & 15 & -8 & -9 \\ -3 & -2 & 1 & 2 \end{pmatrix}$$

(Hint: this matrix has eigenvalues 1 and -1, both of order 2)

**b)** Show that the miniversal deformation of matrix  $H$  is transversal to the orbit of its original matrix (that is  $A(0) = A_0$ )