• Given a space $X$ the Yoneda functor maps it to

$$Y(X) : \text{Top} \rightarrow \text{Sets}$$

$T \mapsto \text{Hom}(T, X)$.

• A groupoid is a category where all arrows are isomorphisms.

• Let $X$ be a set. The groupoid $I_X$ has $\text{Objects}(I_X) = X$ and only identity arrows.

• Let $G$ be a group. The groupoid $E_G$ has $\text{Objects}(E_G) = G$ and exactly one arrow between any two objects. Note that $E_G$ is equivalent to the trivial groupoid, and that it has a free action of $G$.

• The groupoid $B_G := E_G/G$ has only one object and $G$ many arrows.

• Let $G$ be a group acting on a set $X$. The quotient groupoid is given by $X//G := (X \times E_G)/G$. It has $\text{Objects}(X//G) = X$. Moreover, an arrow $x \rightarrow y$ is the same thing as a group element $g$ such that $gx = y$.

• A stack is a functor $F : \text{Top} \rightarrow \text{Gpds}$ such that for every space $T$ and every open cover $\{U_i\}$ of $T$, each time we are given:
  - objects $f_i$ in $F(U_i)$,
  - morphisms $\alpha_i : f_i \rightarrow f_j$ in $F(U_i \cap U_j)$,
  - making the diagram $\begin{array}{ccc}
  f_i & \xrightarrow{\alpha_{ij}} & f_j \\
  \downarrow & & \downarrow \\
  f_k & \xrightarrow{\alpha_{ik}} & f_k \\
  \end{array}$ commute in $F(U_i \cap U_j \cap U_k)$,
  then there exists an essentially unique choice of:
  - object $f$ in $F(T)$,
  - morphisms $\beta_i : f \rightarrow f_i$ in $F(U_i)$,
  - making the diagram $\begin{array}{ccc}
  f & \xrightarrow{\beta_i} & f_i \\
  \downarrow & & \downarrow \\
  f_j & \xrightarrow{\alpha_{ij}} & f_j \\
  \end{array}$ commute in $F(U_i \cap U_j)$.

• Example: The functor $Y(X)$ given by $T \mapsto \text{Hom}(T, X)$ is a stack.

• Given a functor $F : \text{Top} \rightarrow \text{Gpds}$, its stackification $F'$ is given by:
  - An object of $F'(T)$ is an open cover $\{U_i\}$ and a collection $f_i \in F(U_i)$, $\alpha_{ij} : f_i \rightarrow f_j$ as above.
  - A morphism $(U_i, f_j, \alpha_{ij}) \rightarrow (V_a, g_a, \beta_{ab})$ is a collection of morphisms $\gamma_{ia} : f_i \rightarrow g_a$ in $F(U_i \cap V_a)$ making the diagram $\begin{array}{ccc}
  f_i & \xrightarrow{\gamma_{ia}} & g_a \\
  \downarrow & & \downarrow \\
  f_j & \xrightarrow{\beta_{ab}} & g_b \\
  \end{array}$ commute in $F(U_i \cap U_j \cap V_a \cap V_b)$.

• Let $G$ be a topological group. Then $BG$ is the stackification of the functor $T \mapsto B\text{Hom}(T, G)$.

• Let $G$ be a topological group acting on a topological space $X$. Then the quotient stack $[X/G]$ is the stackification of the functor $T \mapsto \text{Hom}(T, X)//\text{Hom}(T, G)$. 