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HUYGENS' FOUNDATIONS OF PROBABILITY

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SUMMARIES

It is generally accepted that Huygens based probability on expectation. The term "expectation," however, stems from Van Schooten's Latin translation of Huygens' treatise. A literal translation of Huygens' Dutch text shows more clearly what Huygens actually meant and how he proceeded.

C'est un fait bien connu que Huygens fondait la probabilité sur l'espoir mathématique. Cependant ce terme ne provient pas de Huygens mais de son traducteur latin Van Schooten. Une traduction littéral du texte néérlandais de Huygens en anglais doit indiquer ce qui était l'intention de Huygens et de quelle manière il procédait.

Bekanntlich soll Huygens die Wahrscheinlichkeitsrechnung auf den Erwartungswert gegründet haben. Der Ausdruck "Erwartung" kommt aber von Huygens' Übersetzer ins Lateinische Van Schooten. Eine wörtliche Übersetzung von Huygens' holländischem Text ins Englische soll zeigen, was Huygens meinte, und wie er vorging.

During his stay in Paris in 1655 Huygens learned about Pascal's and Fermat's achievements in probability. Back in Holland he wrote a small treatise on probability--the first in history--*Van Rekeningh in Spelen van Geluck* (*Calculation in hazard games*). He sent the treatise to Van Schooten, who was glad to incorporate it into a work he was just preparing to be published in Latin and Dutch [Schooten 1657, 1660]. Van Schooten himself wrote the Latin version, *De ratociniis in Ludo aleae*, of Huygens' Dutch original. This Latin treatise was reprinted by James Bernoulli as an introduction to his posthumous *Ars Conjectandi* [Bernoulli 1713, 1975]. Modern versions of Huygens' treatise appeared in French as a part of the edition of Huygens' *Oeuvres* [Huygens 1914] and in German in the series *Ostwald's Klassiker* [Bernoulli 1899]. Important quotations from the Dutch text, translated into German, can be found in the modern edition of James Bernoulli's *Ars Conjectandi* [Bernoulli 1975].

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It is a well-known and generally accepted fact that Huygens founded probability on expectation, but it is my feeling that there is no clear idea about what "expectation" meant to Huygens and how he proceeded in details. To the contrary, I am afraid that some authors who have written about this matter have not properly grasped its essentials. I maintain that understanding the original Dutch text is indispensable. Van Schooten's translation is correct, although it is a fact that Huygens himself was not satisfied with it (letter to Sluse of 27 July 1657, in [Huygens 1914, II, 42]). The Latin equivalents which Van Schooten preferred when translating Huygens' terminology have been a source of misunderstanding and confusion ever since. Historically it was Van Schooten who introduced the term *expectatio*, though not in the sense of our "expectation," in probability. In order to make things clear from the beginning, I will anticipate one conclusion: in Van Schooten's version, *expectatio* means the pay-off table of a hazard game, whereas our "expectation" is covered by such terms as *aestimandam esse* (valuating) and *valor expectationis* (value of expectation). In other words, *expectatio* means the catalogue of what the player may expect when playing the game, whereas the cash value of this *expectatio* corresponds to what nowadays is called expectation.

Let me add a few remarks on the modern versions: the French translation in Huygens' *Works* [Huygens 1914] is excellent; the German translation suffers from the fact that the translator admittedly did not understand the Latin text (see Bernoulli [1713, 1899, I, 114]); the German translations of a few important quotations and the interpretation in Bernoulli [1975] are so convincing that the present exposition might be considered as redundant provided due attention were paid to this recent version. However, in order to enable readers not acquainted with the Dutch language to judge, for themselves, I will first translate a relevant fragment from Huygens' text as literally as I can into English, and afterward add some comments in a more modern terminology.

1. LITERAL TRANSLATIONS FROM HUYGENS' TEXT

> ... *I take it as a fundament* ... *that in gambling the chance* * [see Section 2.2 below] *that somebody has toward something is worth as much as that* [*with*] *which, having it, he can arrive at the same chance* * *by an equitable game, that is, where no loss is offered to anybody. For instance: if somebody without my knowledge hides 3 shillings in one hand and 7 shillings in the other and lets me choose which of either I want to get, I say this is worth the same to me as if I had 5 shillings for sure. Because, if I have 5 shillings,*

I can again arrive at having the same chance to get 3 or 7 shillings, and this by an equitable game; which will be shown afterward.

I. PROPOSITION. If I have the same chance to get a or b it is worth as much to me as $(a + b)/2$.

In order not only to prove but also to discover this rule, I put x *for what the chance* is worth to me. Hence having* x *I must be able to arrive at the same chance by an equitable game. Let it be the game which I play against another with stake* x, *where the other is also staking* x; *and let it be agreed that the one who wins shall give* a *to the one who loses. This game is equitable, and it appears that by this I have an equal chance to win* a, *that is, even if I lose the game, or* $2x - a$ *if I win, because then I get the stakes* 2x *from which I must give the other* a. *Suppose that* $2x - a$ *were as much as* b, *then I would have the same chance for* a *and* b. *So I put* $2x - a = b$, *and it follows that* $x = (a + b)/2$ *for the value of my chance. The proof of this is easy, because having* $(a + b)/2$, *I can venture against another who will also stake* $(a + b)/2$, *with the stipulation that the one who wins the game shall give* a *to the other. Therefore I will have an equal chance to get* a, *that is to say if I lose, or* b *if I win, because then I take* $a + b$, *which is the stake, and from this I give him* a.

In numbers: if I have the same chance to get 3 or 7, then by this proposition my chance is worth 5; and it is certain that having 5, I can arrive again at the same chance*. Because venturing the 5 against another who is staking 5 against it with the stipulation that the one who wins will give 3 to the other is an equitable game, and it appears that I have the same chance to get 3, that is, if I lose, or 7 if I win, because then I take 10 from which I give him 3.*

II. PROPOSITION. If I have an equal chance to get a or b or c, it is worth as much to me as though I had $(a + b + c)/3$.

In order to discover it again, let x *be put for the value of my chance as before. Then having* x, *I must be able to arrive at the same chance* by an equitable game. Let it be the game that I play against two others, while all three of us stake* x, *and let me agree with one of them that if he wins the game, he shall give me* b, *and that I shall give him* b *if I manage to win. With the other let me agree that if he wins the game he shall give me* c, *or I will give him* c *if I win it. It appears that* *this game is equitable. And therefore I will have an equal chance to get* b, *that is, if the first wins it, or* c *if the second wins it, or* $3x - b - c$ *if I win, because then I take the* 3x *that have been staked and from this give* b *to the one and* c *to the other. Provided* $3x - b - c$ *were equal to* a, *I would have equal chances for* a *or* b *or* c. *So I put* $3x - b - c = a$ *and* $x = (a + b + c)/3$ *results for the value of my chance. In the same way it is found that if I have an equal chance for* a *or* b *or* c *or* d, *this is worth as much as* $(a + b + c + d)/4$, *and so on.*

III. PROPOSITION. If the number of chances I have for *a* is *p*, and the number of chances I have *b* is *q*, then assuming that every chance can happen as easily, it is worth to me as much as $(pa + qb)/p + q$.

2. COMMENTS

2.1. In Proposition II is looks as if the same thing is proved twice. However, Huygens' procedure, as he explicitly says, is first to discover the unknown value of the chance and then to prove it--the same procedure followed in solving equations: first deriving the value of the unknown by transforming the equation, and afterward verifying it.

2.2 "Chance" in the preceding text is the translation of *kansse* at the places indicated by an asterisk, and of *kans* in the other places. In Proposition III "chances" is the translation of *kanssen,* which linguistically may be the plural of both of them.

2.3. "Chance" as occurring in my translation has a threefold meaning:

(a) It means the whole gambling situation, or more precisely the pay-off table of the game or as I put it elsewhere, the catalogue of what the player may expect when playing the game.

(b) It appears in the context "equal chance";

(c) It appears in the context "number of chances" (see Proposition III).

Huygens' problem is to determine the "value" of chance (a). Huygens seems to be inclined to use chance* in the sense of chance (a) and chance (without asterisk) in the sense of chance (b), though the use is not consequential. In no case is chance* ever used in the sense of chance (b).

Van Schooten translated chance (a) at its two first occurrences by *sors seu expectatio,* at all other places by *expectatio.* For chance (b) he has various equivalents: *aeque facile, pari facilitate, aequa sors, similis expectatio.* The number of chances (c) was translated by *numerus casuum* or by the number followed by *expectationes.*

2.4 Far fom begging the question or being tautological, or circular, or otherwise surreptitious, Huygens' reasoning is quite sophisticated. In modern language Huygens' definition may read as follows:

> *I take it as an axiom that in games of chance-- actually of a special kind-- the expectation of a pay-off table is the money I need to propose a game with the given pay-off table as a fair one.*

This definition is used to calculate the expectation of the pay-off table of a and b with equal chances as follows:

Staking $(a+b)/2$ while playing with another person, I can reconstruct the given pay-off table by making the (fair) stipulation that the winner pays the amount a to the loser.

This is not a trivial transformation. The original situation for which the expectation has to be calculated is that of an individual in a gamelike situation, yet with no adversary identified (say a lottery). It is reconstructed as an n persons' game (if n is the number of chances proposed) with equal stakes and the appropriate stipulations, which by virtue of their mutuality are fair. The value of the stakes under consideration is just the expectation.

2.5. "Equal chance," as used by Huygens, does not beg the question either. In Huygens' model, where the player is given the free choice to take one hand or the other, or, more generally, to decide himself which one of a number of possibilities he chooses, equal chance is validly defined by the complete ignorance and the free will of the player.

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