GAUGE FIELDS WITH UNIFIED WEAK, ELECTROMAGNETIC, AND STRONG INTERACTIONS

by

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1. INTRODUCTION

Only half a decade ago, quantum field theory was considered as just one of the many different approaches to particle physics, and there were many reasons not to take it too seriously. In the first place the only possible "elementary" particles were spin ½ bosons, spin ½ fermions, and photons. All other particles, in particular the ρ , the N^x , and a possible intermediate vector boson, had to be composite. To make such particles we need strong couplings, and that would lead us immediately outside the region where renormalized perturbation series make sense. And if we wanted to mimic the observed weak interactions using scalar fields, then we would need an improbable type of conspiracy between the coupling constants to get the V-A structure[1]. Finally, it seemed to be impossible to reproduce the observed simple behaviour of certain inclusive electron-scattering cross sections under scaling of the momenta involved, in terms of any of the existing renormalizable theories[2]. No wonder that people looked for different tools, like current algebra's, bootstrap theories and other nonperturbative approaches.

Theories with a non-Abelian, local gauge invariance, were known[3], and even considered interesting and suggestive as possible theories for weak interactions[4,5], but they made a very slow start in particle physics, because it seemed that they did not solve very much since unitarity and/or renormalizability were not understood and it remained impossible to do better than lowest order calculations.

When finally the Feynman rules for gauge theories were settled[6] and the renormalization procedure in the presence of spontaneous symmetry breakdown understood[7-19] it was immediately realized that there might exist a simple Gauge Model for all particles and all interactions in the world. The first who would find The Model would obtain a theory for all particles, and immortality. Thus the Great Model Rush began[29,35,36,44-53,56,58].

First, one looks at the leptons. The observed ones can easily be arranged in a symmetry pattern consistent with experiment[5]: SU(2) x U(1). But if we assume that other leptons exist which are so heavy that they have not yet been observed then there are many other possibilities. To settle the matter we have to look at the hadrons.

The observed hadron spectrum is so complicated with its octuplets, nonets and decuplets that it would have been a miracle if they would fit in a simple gauge theory like the leptons. They don't. To reproduce the nice

SU(3) x SU(3) structure one is forced to take the "quarks", the building blocks of the hadrons, as elementary fields. The existing hadrons are then all assumed to be composite. To bind those quarks together we need strong forces and here we are, back at our starting point. What have we won?

We have won quite a lot, because the tools we can use, renormalized gauge theories, are much more powerful than the old renormalizable theories. Not only do we have indications that they exhaust all possible renormalizable interactions20) but there is also a completely new property: the behaviour of some of these theories under scaling of all coordinates and momenta21). If you look at such a system of particles through a microscope, then what you see is a similar system of particles, but their interactions have reduced. The theory is "asymptotically free"21,22,23). The old theories always show a messy, strongly interacting soup when you look through the microscope24). If you assume that any theory should be defined by giving its behaviour at small distances, then the old theories would be very ill-defined, contrary to the gauge theories.

But when it comes to model building, then it is still awkward that the forces between the quarks are strong, because that makes gauge theories not very predictive and there are countless possibilities. I have seen theories with 3, 4, 6, 9, 12, 18 and more quarks. How should we choose among all these different group structures? Before answering the question let us first make up the balance. What we are certain of is:

1) Gauge theories are renormalizable, if firstly the local symmetry is broken spontaneously, and secondly the Bell-Jackiw anomalies are arranged to cancel.
2) Global symmetries may be broken explicitly, so we can always get rid of Goldstone bosons.
3) We can make asymptotically free theories for strong interactions.

Then the following statements are not absolute but have been learnt from general experience:

4) The Higgs mechanism is an expensive luxury: each time we introduce a Higgs field we have to accept many new free parameters in the system.
5) There are always more free parameters than there are masses in the theory, so we can never obtain a reliable mass relation for the elementary constituent particles. Of course, masses of composite systems are not free but can be calculated.
6) To break large groups like SU(4) or SU(3) x SU(3) by means of the Higgs mechanism is hopelessly complicated. Theories with a small gauge group like SU(2) x U(1) or large but unbroken groups are in a much better shape.

Of course, these are practical arguments, that distinguish useful from useless theories. But do they also distinguish good from false theories? Personally I tend to believe this. I find it very difficult to believe that nature would have created as many Higgs fields as are necessary to break the big symmetry groups. It is more natural to suppose that just one or two

Higgs fields are present and some remaining local symmetry groups are not broken at all.

2. STRONG INTERACTION THEORY

a. Towards permanent color binding

There is a general consensus on the idea that gauge vector particles corresponding to the color group SU(3)' can provide for the necessary binding force between quarks, which transform as triplet representations of this group. The states with lowest energy are all singlets. This theory explains the observed selection rules and the SU(6) properties of the hadrons. But now there are essentially two possibilities.

The first possibility is that SU(3)' is broken by the Higgs mechanism, so that the masses of all colored objects are large, but finite. The ψ particles can be incorporated in this scheme: they may be the first colored objects, as you heard in the sessions on color theories. Besides the disadvantages of such theories I mentioned before, it is also difficult to arrange suppression of higher order contributions to K^o-$\bar{K}^o$ mixing and $K_L \to \mu^+\mu^-$ decay[25] and the theories are not asymptotically free.

The alternative possibility is that SU(3)' is not broken at all. All colored objects like the quarks and the color vector bosons have strictly infinite masses[26,27]. I suspect that this situation can be obtained from the former one through a phase transition. Let me explain this.

In the Higgs broken color theories there exist "solitons", objects closely related to the magnetic monopole solutions[28] in the Georgi Glashow model[29]. Now let me continuously vary the parameter μ^2 in the Higgs potential

$$V = \tfrac{1}{2}\,\mu^2\phi^2 + \tfrac{1}{8}\,\lambda\,\phi^4 \qquad (1)$$

from negative to positive values (fig. 1).

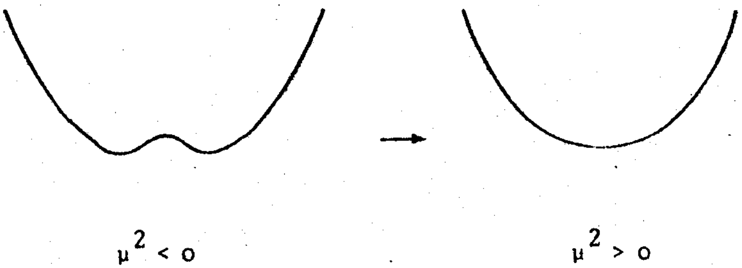

Fig. 1 The Higgs potential before and after the phase transition.

We keep λ fixed. Now the vacuum expectation value F_{Higgs} of the Higgs field ϕ is first roughly proportional to $|\mu|$ and so are the vector boson mass M_V and the soliton mass,

$$M_s \simeq \frac{4\pi M_V}{g^2} , \tag{2}$$

as indicated on the left hand side of fig. 2.

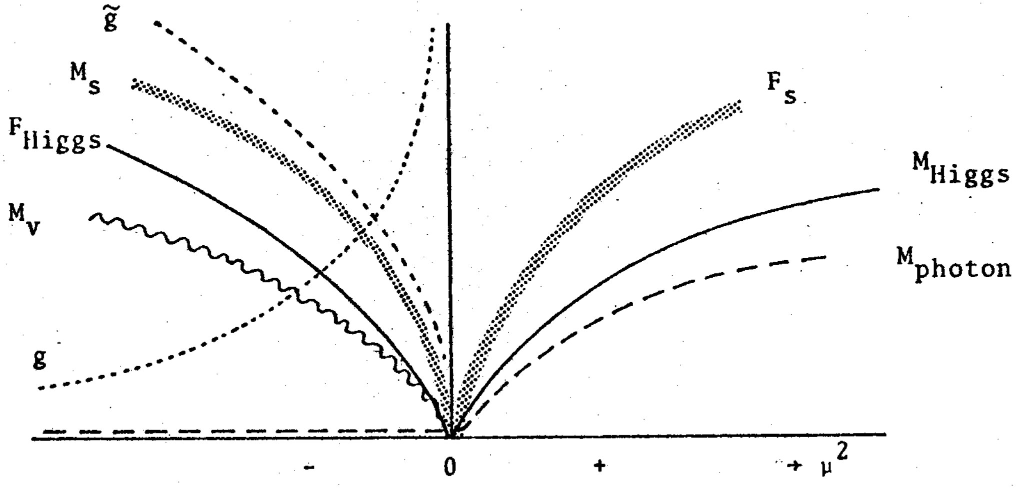

Fig. 2 The phase transition

M_{Higgs} = mass of unbroken Higgs field
F_{Higgs} = vac. exp. value of Higgs field
M_s = mass of soliton
F_s = vac. exp. value of soliton field
g = color (electric) coupling constant
$\tilde{g}$ = soliton coupling constant ($g\tilde{g} = 2\pi n$)

What I assume is that when μ^2 becomes positive, it is the soliton's turn to develop a non-zero vacuum expectation value. Since it carries color-magnetic charges, the vacuum will behave like a superconductor for color-magnetic charges. What does that mean? Remember that in ordinary electric superconductors magnetic charges are confined by magnetic vortex lines; as described by Nielsen and Olesen$^{30)}$. We now have the opposite: it are the color charges that are confined by "electric" flux tubes. So we think that after the phase transition all color non-singlets will be tied together by "strings" into groups that are color singlets. In this phase the Higgs scalars play no physical role whatsoever and we may disregard them now.

The great shortcoming of this theory is that it is intuitive and as yet no mathematical framework exists. But there are various reasons to take it seriously.

Theoretically:

i) we see it happen if we replace continuous space-time by a sufficiently coarse lattice[27)]: the action becomes that of the Nambu string.

ii) We see it happen in the only soluble asymptotically free gauge theories: Schwinger's model[31)] and, even better, in the SU(∞) gauge theory in two space-time dimensions[32)].

Experimentally:

a) the flux lines would behave like the dual string and thus explain the straight Regge-trajectories[33)].

b) This is the simplest and probably the only asymptotically free theory that explains Bjorken behaviour[22)].

c) The theory is closely related to the rather successful MIT bag model[34)].

In principle there exists an intermediate possibility: we propably have several phase-transitions when we go from unbroken SU(3) to for instance SU(2), U(1) and finally complete breaking. We will not consider the possibility that we are in SU(2) or U(1), but we must remember that from the low lying states it is difficult to deduce in what phase we really are.

If we are in the unbroken phase then $SU(3)^{\text{color}}$ must commute with weak and electromagnetic SU(2) x U(1), and with $SU(3)^{\text{flavor}}$[x)]. Consequently, we are obliged to introduce charm[35)], or even more new quarks perhaps[36)]. Now let us consider the ψ particles.

b. Charmonium

The most celebrated theory for ψ is that it is a bound state of a charmed quark and its antiparticle[37)]. Charmed quarks are assumed to be rather heavy. The size of the bound state wave function will therefore be small and we can look at the thing through a microscope (i.e. apply a scale transformation). Then we see rather small couplings, so we may use perturbation expansion to describe the system. The mathematics is doable here! At first approximation the gluon gauge field behaves exactly like Maxwell fields, even the SU(3) structure constants can be absorbed in the coupling constant. We can calculate the annihilation rate and level splittings exactly as in positronium. The annihilation rate of the positronium vector state is[38)]

$$\Gamma = \frac{2(\pi^2-9)}{9\pi} m_e \alpha^6 (1 + O(\alpha)) \tag{3}$$

For charmonium the formula would be

[x)] i.e. the familear broken symmetry group that transforms p, n and λ into each other. The word "flavor" has been proposed by Gell-Mann to denote both isospin and strangeness.

Table 1

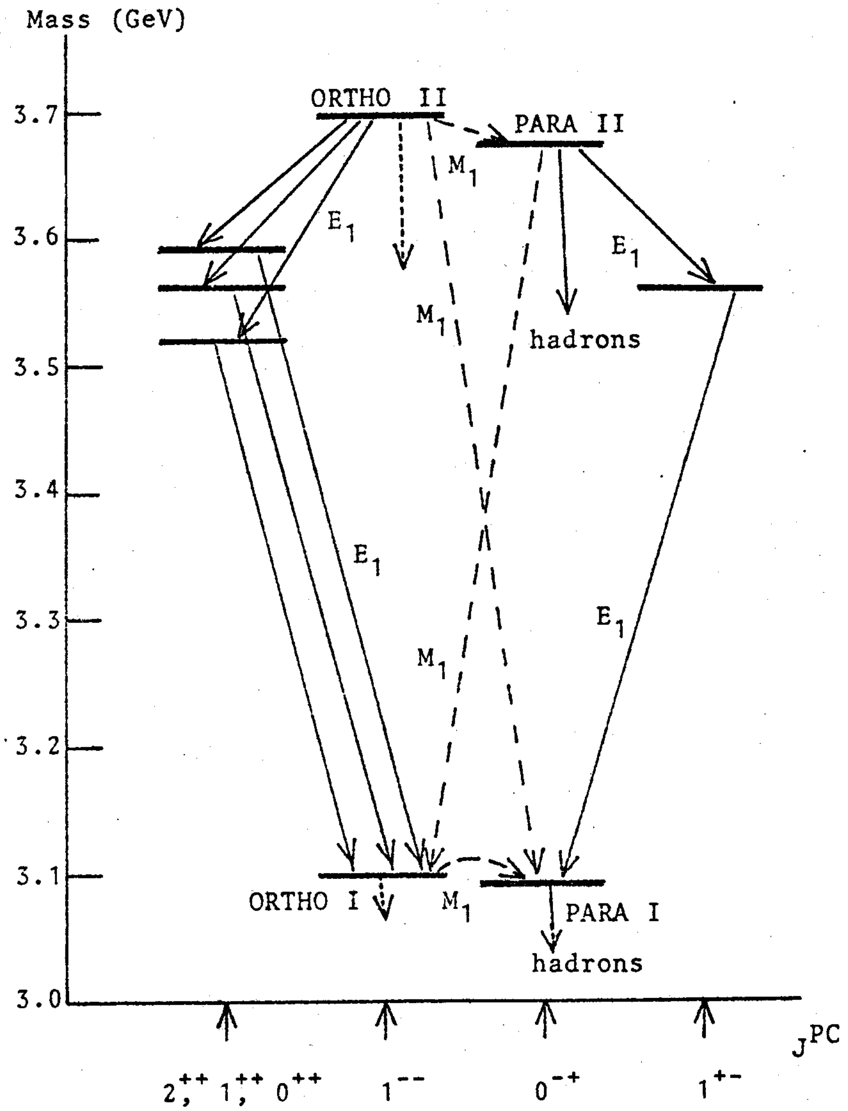

Table 2: SU(4) predictions

New Mesons Y Q I_3

charm	JP = 0^-	JP = 1^-	mass (GeV)
1	+ + 0	+ + 0	2
0	0	0	3.1
-1	0 - -	0 - -	2

New Baryons

charm	JP = $\frac{1}{2}^+$	JP = $\frac{3}{2}^+$	mass (GeV)
3		2+	5
2	2+ + +	2+ + +	3.7
1	2+ + 0 + + 0 + 0 0	2+ + 0 + 0 0	2.5

$$\Gamma = \frac{5(\pi^2-9)}{192\pi} m_{p'} \alpha_s^6 (1 + O(\alpha_s)) , \qquad (4)$$

where $\alpha_s = g_s^2/3\pi$ (the subscript s standing for "strong"). (5)

It fits well in the renormalization group theory if α_s is around 1/3 at 2 GeV. That it is raised to the sixth power explains the stability of ψ. Note that this is the rate with which the two quarks annihilate. It is independent (to first approximation) of the details of the hadronic final state[39].

Now this gauge theory for strong interactions gives precise predictions on the other charmonium states (table 1) and the charmed hadrons (table 2). They can be found in some nice papers by Appelquist, De Rújula, Politzer, Glashow and others[37].

It is very tempting to assume that this new quark is really the charmed one as predicted by weak interaction theories (see Sect. 3), but it could of course be an "unpredicted" quark.

If the predictions from this theory come out to be roughly correct then that would be a great success both for the asymptotically free gauge theory for strong interactions, and for the renormalization theories that predicted charm.

3. THE WEAK INTERACTIONS

a. SU(2) x U(1) theories

A simple SU(2) x U(1) pattern seems to be compatible with all experimental data on pure leptonic and semileptonic processes (neutral currents and charm), but with the pure hadronic weak interactions we still have a problem: there should be $\Delta I = 3/2$ and $\Delta I = 1/2$ transitions with similar strength and we only see $\Delta I = 1/2$. Also, these interactions seem to be somewhat stronger than other weak interaction processes. The traditional way to try to solve the problem is the possibility that $\Delta I = 1/2$ is "dynamically enhanced". Seen through our "microscope" at momenta of the order of the weak boson mass, the $\Delta I = 1/2$ and $\Delta I = 3/2$ parts of the weak interaction Hamiltonian may be equal in strength, but when we scale towards only 1 GeV, then $\Delta I = 1/2$ may be enhanced through its renormalization group equations. This mechanism really works, but can only give a factor between 6 and at most 14 in the amplitudes[40]. But there are many uncertainties, since we do not know exactly from where to where we should scale, and what the importance is of the higher order corrections. It has been argued that a similar mechanism might depress leptonic decay modes of charmed particles, thus giving them a better camouflage that prevents their detection[41] and the mechanism could influence parity and isospin violation[42].

An interesting alternative explanation of the $\Delta I = 1/2$ rule has recently been given by De Rújula, Georgi and Glashow[43]. In usual SU(2) x U(1) theories only the left handed parts of the spinors may be

SU(2) doublets, all right handed parts are singlets. These authors however take the right handed parts of p' and n to form a doublet. This adds to the hadronic current

$$\bar{p}'\,\gamma_\mu(1-\gamma_5)n\ , \tag{5}$$

which is only observable in purely hadronic events or charm decays. Note that there is no Cabibbo rotation. We get among others a term,

$$\cos\theta_c\,\bar{p}'\,\gamma_\mu(1+\gamma_5)\lambda \quad \bar{n}\,\gamma_\mu(1-\gamma_5)p'\ , \tag{6}$$

in the effective Hamiltonian. Note the absence of Cabibbo suppression by factors $\sin\theta_c$ and the $\Delta I = 1/2$ nature of this term.

The predictions on neutrino production of charmed particles, and the charm decay rates are radically changed by this theory. We can have

$$\nu p \to \mu^-\,\Sigma_c^{++}\ , \tag{7}$$

etc. with no $\sin^2\theta_c$ suppression, and pseudo $\Delta S = 2$ processes can take place like

$$\begin{aligned} &\nu p \to \mu^-\,p\,K^+\,D^o \\ &\qquad\qquad\quad \hookrightarrow \mu^-\,\bar{\nu}\,K^+ \end{aligned} \tag{8}$$

because of D^o-$\bar{D}^o$ mixing (as in $K_o\bar{K}_o$ mixing). Experts however are skeptical about the idea. The charmed quark in this current has to violate Zweig's rule and the charmed quark loop must couple to gauge gluons before the current can contribute.

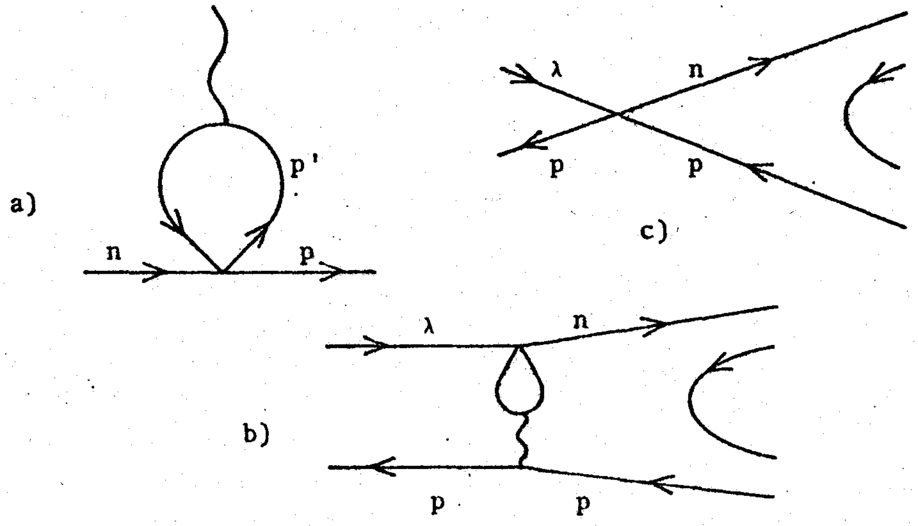

Fig. 3 a) The new strangeness changing processes
b) Diagram for the decay $K^- \to 2\pi$
c) Conventional picture of $K^- \to 2\pi$

The idea is very recent and I have not yet seen any detailed calculations. We will soon know what the contribution of such diagrams can be. Furthermore, the K_L - K_S mass difference, when naively calculated in this model seems to be too large compared with experiment.

b. Other and larger groups

SU(2) x U(1) theories do not really unify weak interactions and electromagnetism. The U(1) group may be considered as the fundamental electromagnetic group, and its photon is merely somewhat mixed with the neutral component of the weak SU(2) gauge field. True unification occurs only then when we have a single compact group.

1. Heavy leptons

The first example was originally invented to avoid neutral currents: the Georgi Glashow O(3) model[29]. Now we have a large class of models[44-52] based on SU(2) x U(1), O(3), O(4), O(4) x U(1), SU(3), SU(3) x U(1), SU(3) x SU(3) etc., all predicting new leptons. An extensive discussion of these models is given by Albright, Jarlskog and Tjia[53]. Table 3 gives the predicted leptons in these schemes. Albright, Jarlskog and Wolfenstein also analysed the possibilities to detect such objects by neutrino production[54].

Of course, we can always extend the lepton spectrum in other ways, for instance by addihg new representations[55], so that we get more members in the series $\begin{pmatrix} e \\ \nu_e \end{pmatrix}$, $\begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}$, $\begin{pmatrix} x \\ \nu_x \end{pmatrix}$, etc.

2. The Pati-Salam Model[56]

Theoreticians are eagerly awaiting the discovery of the first heavy lepton. But that may take quite a while and in the mean time we search for more guidelines to disclose the symmetry structure of our world. One such guideline is that eventually we do not expect that baryons and leptons are essentially different. That is, they might belong to just one big multiplet. This, and other ideas of symmetry and simplicity led Pati and Salam to formulate their most recent "completely unified model". Leptons and quarks from just one representation of SU(4) x SU(4), of which color SU(3) and weak SU(2) x U(1) are subgroups. If, as argued before, color is unbroken then we are free to mix the photon (also unbroken) with colored bosons, so there is no physical difference between the charge assignments

$$\begin{pmatrix} -1/3 & -1/3 & -1/3 & -1 \\ 2/3 & 2/3 & 2/3 & 0 \\ -1/3 & -1/3 & -1/3 & -1 \\ 2/3 & 2/3 & 2/3 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

because the photon may freely mix with components of the unbroken color

Table 3. Heavy leptons

Gauge group	Muonic leptons	Refs.
SU(2) x U(1)	μ^- ν_μ	Weinberg, Salam[4,5,17,44]
	ν_μ μ^- M^0	Bjorken and Llewellyn Smith no 3 [51] Bég and Zee[50]
	ν_μ μ^- M^+ M^0	Bjorken and Llewellyn Smith no 5 [51] Prentki and Zumino[52]
	μ^- ν_μ M^+	Prentki and Zumino[52]
	ν_μ μ^- M_0 M^+ M'_0	Bjorken and Llewellyn Smith no 4 [51]
0(3) = SU(2)	ν_μ μ^- M^+ M_0	Georgi and Glashow[29] Bjorken and Llewellyn Smith no 6 [51]
0(4) = SU(2) x SU(2)	ν_μ μ^- M^+ M_0	Pais[45]
	ν_μ μ^- M^0 M^+ M'_0	Cheng[45]
0(4) x U(1)	ν_μ μ^- M^+ M_0	Pais[46]
SU(3) x U(1)	μ^- ν_μ M^-	Schechter and Singer[47]
	M^0 μ^- ν_μ	Gupta and Mani[47]
	M'^0 M'^+ M^0 M^- M^+ M''^0 μ^- ν_μ	Albright, Jarlskog, Tjia[53]

Table 3, heavy leptons, continued

Gauge group	Leptons		Refs.
U(3)	left	right	Salam and Pati48)
	E^0 e^+ ν_μ μ^- M^0 M^+ $M^{0'}$ E^- $E^{0'}$	M^0 M^+ $\bar{\nu}_e$ E^- E^0 e^+ $E^{0'}$ μ^- $M^{0'}$	
	left	right	Salam and Pati48)
	M^0 μ^+ ν_e e^- E^0 E^+ $E^{0'}$ M^- M^0	E^0 E^+ $\bar{\nu}_\mu$ M^- M^0 μ^+ $M^{0'}$ E^- $E^{0'}$	
SU(3) x SU(3)	$\nu_e, \bar{\nu}_\mu$ e^- μ^+ L^0		Achiman Weinberg49)
	$\bar{\nu}_e$ e^+ L^0 L^- L^+ $L^{0'}$ μ^- ν_μ		Achiman49)

vector fields[x)].

4. THE HIGGS SCALARS

The Ugly Ducklings of all Unified Theories are the Higgs scalars. They usually bring along with them as many free parameters as there are masses in the theory or more. This makes the theories so flexible that tests become very difficult.

These scalars are needed for pure mathematical reasons: otherwise we cannot do perturbation expansions, and we have no other procedure at hand to do accurate calculations. But do we need them physically?[57)]

Various attempts have been made to answer this question negatively. First of all they are ugly, and physics must be clean. But that is purely emotional. Then: we do not observe scalars experimentally. But: there is so much that we do not observe: quarks, I.V.B.'s, etc.

Linde and Veltman raised the point that the scalars do something funny with gravity: their vacuum-expectation value gives the vacuum a very large energy-density. That should renormalize the cosmological constant. (Otherwise our universe would be as curved as the surface of an orange[58)]). On the other hand the net cosmological constant is very many orders of magnitude smaller than this. How will we ever be able to explain this miraculous cancellation? In this respect it is interesting to note an observation made by Zumino[59)]: in supersymmetric models there is no cosmological constant renormalization, even at the one-loop level.

Ross and Veltman then suggest that perhaps one should choose the scalars in such a way that the vacuum-energy density vanishes[60)]. In Weinberg's model that means that one should add an isospin 3/2 Higgs field. Such a Higgs field could reduce neutral current interactions, and that would be welcome to explain several experiments.

Personally I think that one has to consider the renormalization group to see what kind of scalars are possible. If we scale to small distances[106)] then the theory has nearly massless particles. Only a nearly exact symmetry principle can explain why their masses are so small at that scale. For fermions we have chiral symmetry for this. Scalar particles can be forced to be massless if they are the Goldstone bosons of some global symmetry. They then have the quantum numbers of the generators of that symmetry group. Since all global symmetry groups must commute with local gauge groups, it is difficult to get light scalar particles that are not gauge singlets.

[x)] If the integer-charge assignment is adopted however, then leptons couple directly to color gluons, and this would make interpretation of the experimentally observed ratio $R = \sigma(\text{hadrons})/\sigma(\mu^+\mu^-)$ in e^+e^- annihilation more complicated. So R must be computed from the non-integer charges. This 16-plet yields $R = 10/3$.

According to this argument it is impossible to have scalar Higgs particles, except when they are strongly interacting, since in that case we cannot scale very far because of non-asymptotic-freedom.

A notable exception may be constructions using supersymmetries (see Sect. 5).

In connection with Higgs' scalars I want to clear up a generally believed misconception. It is not true that theories with a Higgs phenomenon in general cannot be asymptotically free. For many simple Higgs theories one can obtain asymptotic freedom, provided that certain very special relations are satisfied, between coupling constants that would otherwise be arbitrary$^{62)}$. These theories do not have a stable fixed point at the origin but I cannot think of any physical reason to require that. Examples of such theories are supersymmetric theories, with possible supersymmetry breaking masses. Quark theories of this nature do not exist.

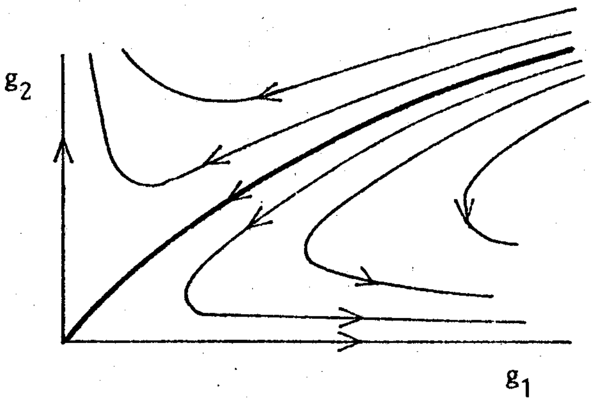

Fig. 4 Example of an unstable ultraviolet fixed point at the origin of parameter space. The arrows indicate the change of the coupling constants as momenta increase. The solid line is the collection of asymptotically free theories.

For weak interactions however I do not think that asymptotic freedom would be a good criterion, because any change of the theory beyond, say 10000 GeV would alter the relations between coupling constants completely.

5. SUPER-SYMMETRIC MODELS

Symmetry arguments can be deceptive. In the nineteenth century physicists argued that the Moon must be inhabited by animals, plants and people. This was based on theological and symmetry arguments (earth-moon-symmetry), similar to the ones we use today.

Keeping this warning in mind, let us now consider the supersymmetric models$^{63)}$. For supersymmetry we have a special session. But an interesting attempt by Fayet$^{64)}$ must be mentioned here, who constructs a Weinberg-like model with supersymmetry. There, Fermions and Bosons sit in the same multiplet. Thus the photon joins the elctron-neutrino. But which massless boson should join the muon-neutrino? This problem has not been solved, to my knowledge.

Even if supersymmetry would be ruled out by the overwhelming experimental evidence that fermion and boson masses are not the same, I think we do not have to drop the idea. Suppose that quantum field theory begins at the Planck length; we cannot go further than the Planck length as long as we do not know how to quantize General Relativity. And suppose that Quantum Field Theory is supersymmetric but the supersymmetry is very slightly broken (it is a global symmetry, so we may do that) and the breaking is described by a coefficient of the order of 10^{-40}. Suppose further that the representations happen to be such that there are no supersymmetric mass terms. Then at the mass scale of 1 GeV we would have all dimensionless couplings completely supersymmetric, but mass terms arise that break supersymmetry, because $1(\text{GeV})^2$ is 10^{-40} in our natural units defined by the Planck length. I would like to call this relaxed supersymmetry and it is conceivable that many interesting models of this nature could be build106).

6. PC-BREAKING

Theories without scalars are automatically PC invariant. To describe PC-breaking we can either introduce elementary scalar fields with PC = -1 and put PC-odd terms in the Lagrangian, or believe that all scalars are in principle composite. Then PC-breaking must be spontaneous as described by T.D. Lee65). Usually however the scalars are not specified, but only the currents66).

De Rújula, Georgi and Glashow observe that their chiral current can easily be modified to incorporate PC-breaking, a scheme already proposed by Mohapatra and Pati67) in 1972:

$$\Delta J = \bar{p}' \gamma_\mu (1-\gamma_5) (n \cos\phi + i\lambda \sin\phi) , \quad \phi << 1 . \tag{9}$$

7. QUANTUM GRAVITY

Quantum gravity is still not understood, but an interesting formal interpretation is given by Christodoulou68). He gives a completely new definition of time, in terms of the distance in "superspace" between two three-dimensional geometries. But his formalism is not yet in a shape that enables one to give interesting physical predictions, and he has not yet considered the problem of infinities.

Renormalizable interactions between gravity and matter have not yet been found69,70) but Behrends and Gastmans find suggestive cancellations in the gravitational corrections to anomalous magnetic moments of leptons71).

Numerous authors tried to put terms like $\sqrt{g}\, R^2$ or/and $\sqrt{g}\, R_{\mu\nu} R^{\mu\nu}$ in the Lagrangian, and thus (re)obtain renormalizability. It is about as clever as jumping to the moon through a telescope. Personally, I am convinced that if you want a finite theory of gravity, you have to put new physics in72). Very interesting in that respect are the attempts by Scherk and Schwarz to start with dual models.

8. DUAL MODELS

Though originally designed as models for strong interactions, the dual models have become interesting for other types of interactions as well. They are the only field theoretical scheme that starts from an infinite mass spectrum, forming Regge-trajectories with slope α'. In the limit $\alpha' \to 0$ they can mimic not only many renormalizable theories but also gravitation (always in combination with matter fields). If the tachyon-problem and the 26-dimension problem can be overcome then one might end up with a big renormalizable theory that unifies everything[73)].

Scherk and Schwarz point out that for $\alpha' \neq 0$ those models are equally or better convergent than renormalizable theories.

9. TWO-DIMENSIONAL FIELD THEORIES

Field theories in one space and one time dimension are valuable playgrounds for testing certain mathematical theorems that are supposed to hold also in four dimensions. There is no time now to discuss all the interesting developments of the last years[74)] but I do want to mention just three things.

In a recent beautiful paper S. Coleman[75)] explains the complete equivalency between two seemingly different structures: the massive Thirring model on the one hand, and the Sine-Gordon model on the other. The solitons (extended particle solutions) in one theory correspond to the fermions of the other. Thus we get one of the very few theories that can be expanded both at small and at large values of the coupling constant.

Solitons, and their quantization procedure have been studied in two and four dimensions by many groups[28,76,77)]. Even in theories with weak coupling constants, solitons interact strongly (they have very large cross sections!) so they could make interesting candidates for an alternative strong-interaction theory. No convincing soliton theory for strong interactions does yet exist, but several magnetic-monopole quark structures have been considered[30,77,78)].

Thirdly we mention the gauge theories in two dimensions with gauge group U(N) or SU(N). They are exactly soluble for either $N = 1$, $m_{\text{fermion}} = 0$, or for: $N \to \infty$, m_{fermion} arbitrary, in which case the 1/N expansion is possible. These theories exhibit most clearly the quark confinement effect, when color is unbroken[31,32)]. Quark confinement is almost trivial in two space-time dimensions because the Coulomb potential looks like fig. 5.

QED in two dimensions (Schwinger's model) is not a good confinement theory because the electrons manage to screen electric charges completely, so the dressed electron is free, except for one bound state. At $N \to \infty$, probably at all $N > 1$, we get an infinity of bound states, "mesons" that interact with strength proportional to 1/N. And they are on a nearly straight trajectory (actually a series of daughters because there is no angular momentum).

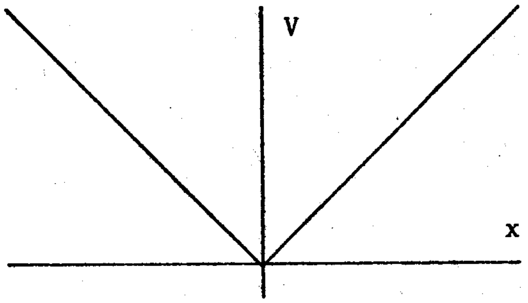

Fig. 5 The Coulomb potential in one spacelike dimension.

10. THE TWO ETA PROBLEMS

The latter one space one time dimensional model has lots of physically interesting properties. One is, that we can test ideas from current algebra. As in its four dimensional analogue, we have in the limit where the quark masses m_p , $m_n \rightarrow 0$ an exact SU(2) x SU(2) symmetry, and the pion mass goes to zero in the sence[79,107]

$$m_\pi^2 \rightarrow C\ (m_p + m_n)\ . \tag{10}$$

The proportionality constant contains the Regge slope. In two dimensions we have

$$C = g\sqrt{\pi} = 1/\sqrt{3\alpha'}\ . \tag{11}$$

So if m_π is small, then m_p and m_n must be very small, in the order of 10 MeV. Now we can consider two old problems associated with the eta-meson. The first is that our theory seems to have U(2) x U(2) symmetry, not only SU(2) x SU(2). Thus, there should also be an isospin-zero particle, η , degenerate with π_o , whose mass should vanish. Experimentally however, $m_\eta^2 >> m_\pi^2$. Also we do not understand why the π_o-η splitting goes according to $\pi_o = \bar{p}p - \bar{n}n$; $\eta = \bar{p}p + \bar{n}n$ despite of possible mass differences of p and n. This problem reappears every now and then in the literature. Fritzsch, Gell-Mann and Leutwyler[80] gave in 1972 a beautiful and simple solution: there is a Bell-Jackiw anomaly associated with the chiral U(1) subgroup:[81]

$$\partial_\mu\ j_5^\mu = 2m\, j^5 + C\ \epsilon_{\mu\nu\alpha\beta}\, Tr(G_{\mu\nu}\ G_{\alpha\beta})\ . \tag{12}$$

The symmetry is broken and the eta-mass is raised. But then came the big confusing counter argument: We can redefine the axial current so that it is conserved, by writing

$$\tilde{j}^{\mu}_{5} = j^{\mu}_{5} - 2C\,\varepsilon_{\mu\nu\alpha\beta}\,\mathrm{Tr}\,A_{\nu}(\partial_{\alpha}A_{\beta}-\partial_{\beta}A_{\alpha}+\frac{2}{3}g\,[A_{\alpha},A_{\beta}]) \quad , \qquad (13)$$

so there should be a massless eta after all. Considerable effort has been made the past years to show that this counter argument is wrong. There are four counter counter arguments, all based on the fact that $\tilde{j}^{\mu}_{5}$ is not gauge-invariant:

i) $\tilde{j}^{\mu}_{5}$ is not gauge invariant, therefore the massless eta carries color, so it will be confined and thus removed from the physical spectrum82).

ii) The gluon field that occurs explicitly in the "corrected" equation for the axial current is a long range field. The Goldstone theorem does not apply when long range interactions are present.

iii) In the Lorentz-gauge there is no explicit long range Coulomb force. But then there are negative metric states and this eta may be a ghost. It will be cancelled by other ghosts with wrong metric83).

iv) But the simplest argument is that we can calculate the eta mass exactly in two dimensions and see what happens. In two dimensions there is an anomaly only in the U(1) field107):

$$\partial_{\mu}\, j^{\mu}_{5} = 2m\, j^{5} + C\,\varepsilon_{\mu\nu}\, F_{\mu\nu} \quad . \qquad (14)$$

It is exactly this anomaly that raises the mass of the eta (it can be identified with Schwinger's photon), despite of the fact that we can find a new axial current:

$$\tilde{j}^{\mu}_{5} = j^{\mu}_{5} - 2C\,\varepsilon_{\mu\nu}\, A_{\nu} \quad , \qquad (15)$$

which is conservedx).

The second eta problem is the decay

$$\eta \rightarrow 3\pi \quad .$$

It breaks G-parity and thus isospin. If we assume this decay to be electromagnetic then current algebra shows us that it should be suppressed by

---

x) This argument is not quite correct, because in two space-time dimensions the Goldstone realization of a continuous symmetry is impossible84). However, if we let first $N \rightarrow \infty$, then $m \rightarrow o$ then we do get nevertheless the Goldstone mode. This is possible because the meson-meson interactions decrease like 1/N. Note that in any case there are no parity doublets for large N.

factors of m_π^2/m_η^2 , which it does not seem to be[85]. I think there is a very simple explanation in terms of the present theory: the proton- and neutron-quark masses are free parameters in our theory, not "determined" by electromagnetism. Their difference follows from known hadron mass splittings and tends to unmix $\bar{p}p$ and $\bar{n}n$, that is, mix π^0 and η . We get

$$\eta \to \pi^0 \to 3\pi$$

with the correct order of magnitude[86]. The essential difficulty with the current algebra argument was that all SU(2) breaking effects were assumed to be electromagnetic. That would be beautiful, we could do current algebra by replacing ∂_μ by $\partial_\mu + iqA_\mu$ everywhere[87].

In a gauge theory this is wrong. As I emphasized in the beginning, mass differences cannot be explained by electromagnetism alone, they are arbitrary parameters and must be fixed by experiment. We'll have to live with that. Only for composite systems mass differences can be calculated. This suggests of course that we must make a theory with composite quarks. We leave that for a next generation of the physicists.

Note that the breaking of SU(2) x SU(2) is governed by proton- and neutron quark masses alone. They are of order 10-20 MeV and differ by 5 MeV or so. So the breaking term of SU(2) x SU(2) also breaks SU(2) rather badly. That is why you may not use PCAC and isospin together to get factors of m_π^2/m_η^2 in the decay $\eta \to 3\pi$ [x].

11. MISCELLANEOUS

a. Perturbation expansion

The renormalizability of the perturbation theory for gauge fields being well settled, there are still new developments. The dimensional renormalization procedure is the solution to all existence and uniqueness problems for the necessary gauge-invariant counterterms.

But if one wishes to circumvent the continuation to non-integer number of dimensions then the combinatorics is very hard. The Abelian Higgs-Kibble model can now be treated completely to all orders within the Zimmermann normal product formalism[88] if there are no massless particles. Slavnov identities can be satisfied to all orders in this procedure also in the Non-Abelian Higgs-Kibble model if the group is semi simple (no invariant U(1) group) and if no massless particles are present. Unitarity and gauge invariance have explicitly been proven this way in a particular SU(2) model.

Massless particles are very complicated this way, but advances have been made by Lowenstein and Becchi[89] in certain examples of massless Yang-Mills fields, and Clark and Rouet[90] for the Georgi-Glashow model.

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[x] As yet we have no satisfactory explanation for a possible $\Delta I = 3$ component in $\eta \to 3\pi$.

It is noted by B. de Wit[91]) that there is a technical restriction on the allowable form of the gauge-fixing term, relevant for supersymmetric models. The gauge-fixing term cannot have a non-vanishing vacuum expectation value in lowest order, otherwise contradictions arise. Of course, the Slavnov identities make the vacuum expectation value of this term vanish automatically in the usual formulation.

b. The background field method

The algebra in gauge theories is often quite involved. For certain calculations it would be of great help if gauge-invariance could be maintained throughout the calculation. On the other hand we must choose a gauge condition, which by definition spoils gauge-invariance right from the beginning. The trick is now to use the so-called background gauge[92]): the fields are split into a c-number, called background field, and a q-number, called quantum field. Only the quantum part must be fixed by a gauge condition, whereas gauge-invariance for the c-numbers can be maintained. The method was very successful in the case of gravity[70,72]) and has also been applied to calculate anomalous dimensions of Wilson operators[93,94]). The method can be generalized for higher order irreducible graphs[105]).

c. Two-loop-beta

The Callan-Symanzik beta function has now been calculated up to two loops for several gauge theories[95,96]). We have

$$\beta(g) = A g^3 + B g^5 + O(g^7) .$$

Now A can have either sign, and can also be very close to zero. In general, B will not vanish (it can be either positive or negative). We can then get either an UV or an IR stable fixed point close to zero. In certain supersymmetric models, A vanishes. The question was whether perhaps $\beta(g)$ vanishes identically for such a model. Answer: no, because for these models B has now been calculated also and it is non-zero[96]).

d. The infrared problem

Massless Yang-Mills theories are very infrared divergent. As explained, we expect extremely complicated effects to occur, like flux tube formation and color confinement. A general argument is presented by Patrascioiu[97]) and Swieca[98]) that shows that if we have a local gauge-invariance and if we can have isolated regions in space (= particles) with non-vanishing total charge, then there must exist massless photons coupled to that charge. This is only proven for Abelian gauge symmetries, but if it would also hold for non-Abelian invariance, then the absence of massless colored photons must imply the absence of any colored particles (= color confinement).

e. Symmetry restoration at high temperatures

Just like a superconductor that becomes normal when the temperature is raised above a certain critical value, so can the vacuum of the Weinberg model become "normal" at a certain temperature$^{99)}$. The critical temperature is typically of the order of

$$kT \sim \frac{Mw}{e} \quad .$$

This assumes that Hagedorn's limit on high temperatures$^{100)}$ is invalid. Indeed it is invalid in the present quark theories, but the specific heat of the vacuum is very high because there are so many color components of fields. Observe that for SU(N) theories Hagedorn might be correct in the limit $N \to \infty$.

f. Symmetry restoration at high external fields

If we consider extremely strong magnetic fields then also the symmetry properties of the vacuum might change$^{101)}$. One can speculate on restoration of color symmetry, e-μ symmetry, Parity or CP restoration, and vanishing of Cabibbo's angle. In still larger fields formation of magnetic monopoles$^{28)}$ would make the vacuum unstable, as in strong electric fields.

g. Symmetry restoration at high densities

A very high fermion density means that $\bar{\psi}\psi$ and $\bar{\psi}\gamma_4\psi$ have a vacuum expectation value$^{102)}$. This also can have a symmetry restoring effect.

T.D. Lee, Margulies and Wick$^{103)}$ argue that chiral SU(2) x SU(2) might be restored at very high nuclear densities. Thus the mass of one nucleon would go to zero and perhaps very heavy stable nuclei could be formed. The most recent calculations show a very remarkable phase transition at no more than twice the normal nuclear density. Although the result is of course model dependent, this work seems to predict stable large nuclei with binding energy of 150 MeV/nucleon.

At still higher densities we can speculate on more transition points. Again we think that the quark picture is more suitable than Hagedorn's picture$^{104)}$.

12. CONCLUSIONS

a. Unifying everything

What I hoped to have made clear at this conference is that gauge fields are likely to describe all fundamental interactions including, in a sence, gravity. This is a breakthrough in particle physics and deserves to be called:"unification of all interactions".

b. Unifying nothing

But when we consider our present theory of strong interactions, the unbroken color version, then we see that it is unlikely to be really unified with weak and electromagnetic interactions, unless we go at ridiculously high energies, because the gauge coupling constants probably still differ considerably, and SU(3)^{color} commutes with SU(2) x U(1). Also if we look at weak and electromagnetic interactions, we see that true unification has not yet been reached. At small distances strong interactions become weak, weak interactions become strong and electromagnetic ones stay electromagnetic, but no unification yet.

Perhaps our knowledge of the particle spectrum is still far too incomplete to enable us to unify their interactions.

I have given air to my own feeling that we are going in the wrong direction by choosing larger and larger gauge groups.

Perhaps we can use the confinement mechanism again to build quarks and leptons from still more elementary building blocks (chirps, growls, etc.).

Instead of "unifying" all particles and forces, it is much more important to unify knowledge.

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$$c(\bar{p}p + \bar{n}n)$$

in the quark Lagrangian corresponds to a term

$$c'\sigma$$

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