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OBSTACLES ON THE WAY TOWARDS THE QUANTIZATION OF SPACE, TIME AND MATTER
— and possible resolutions —

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Abstract

Most attempts at obtaining theories that unify quantum mechanics with general relativity require violation of locality and/or causality to some degree. Here, we suggest that these problems actually are very deep and fundamental; hence they may require reconsideration of both quantum mechanics and general relativity at a very fundamental level. An approach called "deterministic quantization" is sketched.

Introduction.

The theory of Quantum mechanics and Einstein's theory of general relativity have been equally successful. Both are based on principles that are assumed to be *exactly* valid: quantum mechanics requires a hermitian hamiltonian to describe the evolution of vectors in a Hilbert space. Hermiticity is mandatory in order to ensure the conservation of probabilities, and giving up the probabilistic interpretation of the wave function would imply a big departure from the (highly successful) first principles of quantum mechanics. General relativity is based on invariance under general coordinate transformations. Any violation of the principle of coordinate invariance would imply the existence of some preferable set of coordinates of a kind never observed in Nature.

Thus, what these two theories have in common is that small deviations from their principal starting points cannot be tolerated since these would invalidate the underlying logic; the starting points must be exactly valid. The theories also have in common that they allow large varieties of *secondary 'laws of Nature'*: in quantum mechanics, we could call the Schrödinger equation the primary law; the secondary laws of Nature here are the ones that determine the interaction potentials and coupling strengths. In general relativity, Einstein's equation for the gravitational field is the primary equation, but the details of the matter field equations are secondary; they are not prescribed by the theory.

Because of this, one naturally suspects that sufficiently judicious choices for the secondary laws might enable one to join the two theories into one: a 'quantized theory of general relativity'*. It is here that one encounters obstacles that at first sight seem to be of a purely technical nature (Section 2), but after closer inspection — and decades of intensive research as well as myriads of ingenious approaches — they turn out to be uncannily stubborn. In Sect. 3, various of the ingenious proposals are briefly discussed, and in Sect. 4 some of the really persistent difficulties. A theoretical study of black holes leads to the so-called holographic principle (Scts. 5 and 6). Superstring theory (Sect. 7) claims some successes in reproducing the requirement of holography to its heaviest (black hole) states, at the cost of a very indirect physical interpretation of its foundations. This author tends to be more and more inclined towards the suspicion that the problems of quantum gravity are much more than purely technical ones; they touch upon very essential philosophical issues. The last sections are devoted to a new approach, in which a 'realist' attitude towards quantum mechanics is the central theme.

Canonical gravity.

In early approaches, quantum gravity is treated just as if it were a local gauge theory. Indeed, the first approaches towards quantizing local gauge theories were motivated by the desire to understand how to quantize gravity [1]. One starts with the Einstein-Hilbert action,

$$S = \int \mathrm{d}^4x\,\mathcal{L}(x)\,, \qquad \mathcal{L}(x) = \sqrt{g}\left(g^{\mu\nu}R_{\mu\nu} + \mathcal{L}^{\mathrm{matter}}\right)\,, \tag{2.1}$$

* In this paper we omit as much as possible capitalization, in order to avoid dramatization of the notions we wish to describe.

where $g^{\mu\nu}$ is the metric tensor (a physical degree of freedom), and $\mathcal{L}^{\rm matter}(x)$ represents additional matter fields (the 'secondary' component).

The action is invariant under local coordinate transformations,

$$x^\mu \to x'^\mu(x) = x^\mu + u^\mu(x)\,,$$

or, when $u^\mu(x)$ are infinitesimal,

$$\begin{aligned} g^{\mu\nu}(x) \;\to\; g'^{\mu\nu}(x) \;&=\; x^\mu_{,\alpha}\, x^\nu_{,\beta}\, g^{\alpha\beta}(x') = \\ g^{\mu\nu}(x) + u^\alpha\partial_\alpha g^{\mu\nu} - u^\mu_{,\alpha}\, g^{\alpha\nu} - u^\nu_{,\beta}\, g^{\mu\beta} \;&=\; g^{\mu\nu}(x) - D^\mu u^\nu(x) - D^\nu u^\mu(x)\,, \end{aligned} \tag{2.2}$$

where $x^\mu_{,\alpha}$ stands for $\partial_\alpha x^\mu/\partial x'^\alpha$, and $u^\mu_{,\alpha} = \partial_\alpha u^\mu = \partial_\alpha u^\mu/\partial x^\alpha$.

This invariance does not seem to be very different from the invariance of a (non-Abelian) gauge theory under local gauge transformations:

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + D_\mu\Lambda(x) = A_\mu + \partial_\mu\Lambda + ig[A_\mu,\, \Lambda]\,. \tag{2.3}$$

Therefore it was assumed that these theories can be handled in a similar fashion. Technically, the procedure seems to be straightforward and impeccable. In order to understand what the correct formalism is, one first imposes the *temporal gauge*, which is

$$A_0 = 0\;; \qquad g_{0\nu} = g_{\nu 0} = 0 \tag{2.4}$$

for the two theories, and this unambiguously fixes the *time derivative* of $\Lambda(x)$ and u^μ, respectively. Both theories then take the form of conventional (but infinite dimensional) anharmonic oscillators, so that unitarity and positivity of the energy are evident.

A minor but important complication in the temporal gauge is that invariance under gauge/coordinate transformations that are *local* in space, but *independent* of time implies the existence of a local conserved charge. We must impose the additional constraint that this extra charge (everywhere and at all times) vanishes. This amounts to Gauss' law and Einstein's equations.

Next, one wishes to assure invariance of the *quantized* theories under local coordinate transformations (otherwise, the gauge conditions (2.4) would violate Lorentz invariance). It was discovered that gauge invariance can be imposed by adding a *ghost field* to the system. The details of this procedure are not of much relevance for the present discussion; suffices to state the following facts:
— The ghost Lagrangians can be derived using the functional integral formulation of these quantum field theories [2];
— In the temporal gauges (2.4) the contributions of the ghosts vanish, so that the good properties of these systems of anharmonic oscillators are unaffected;
— The temporal gauge can be replaced by a *renormalizable* gauge (for the gauge theory), or a *less divergent* gauge (for gravity), and one can prove formally that the S-matrix amplitudes for transitions among physical particles are unaffected by the gauge change. It is in these new gauges that the ghost fields play important roles.

For non-Abelian gauge theories, this procedure assures renormalizability, and even asymptotic freedom, so there it can serve for the construction of models of elementary particle systems that can be compared with experiments, with the well-celebrated successes. In the case of gravity, one encounters a glitch: renormalizability is not achieved. The difficulty can be traced back to the fact that, in units where $\hbar = c = 1$, the coupling constant g in gauge theories is dimensionless, whereas the coupling parameter G_N of gravity — Newton's constant — has the dimensionality of $(\text{mass})^{-2}$, or the square of a length, the Planck length. At energies large compared to this mass, or at length scales short compared to the Planck length, the effects of the gravitational coupling explode.

This, however, may not prevent one from setting up a perturbative approximation scheme for quantum gravity. Non-renormalizability merely implies that, when one expands amplitudes in powers of G_N, more and more divergent integrations have to be performed. We have learned how to deal with such a situation in field theory. Assuming that the theory is not yet well understood at the tiniest distance scales, one may perform *subtractions*. One then obtains finite expressions, at the expense of having to introduce new, freely adjustable, constants of Nature for the higher-order expressions. The higher the powers of G_N one wants to include, the larger the number of freely adjustable, hence uncomputable subtraction constants will be needed. We wish to point out that, if one computes amplitudes up to some given order in G_N, very accurate approximations are obtained whereas only a few uncomputable numbers had to be introduced. Perturbative gravity seems to work 'reasonably well'. [2]

Next steps.

The situation described above is not at all new in particle physics. We have seen it all before. Fermi's theory of the weak interaction is an example at hand. Fermi's interaction constant, G_F, has the same dimensionality as Newton's constant G_N. His weak interaction theory, which was phenomenologically quite successful up to the '60s, requires the introduction of similar subtraction constants at higher orders. Here, it was seen how the situation could be improved. Fermi's theory is a low-energy/large distance approximation for an improved theory, the standard model, which is fully renormalizable and hence does not require any further unknown subtraction parameters.

Naturally, one tries to pull the same trick with gravity: find an improved short-distance theory, and the subtractions will take care of themselves. Various approaches have been tried:

1) *Rearranging diagrams.* At one time, it was thought that the theory could be improved by resumming the expansion in a carefully chosen order. If we first assemble all diagrams that contribute to the renormalized propagator, perhaps a more convergent effective propagator will be found (the "super propagator", Fig. 1) [3]. This is an example of a purely technical, mathematical refinement that is not based on any further physical insight in what happens at the tiniest distance scales. It was doomed to fail, and that is what it did.

2) Improving the *symmetry structure* of the theory. Remarkably, it was found that

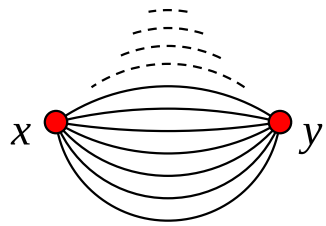

Fig. 1. The super propagator.

supersymmetry [4] could lead to significant further suppression of the infinities. Supersymmetry relates space-time translations to transformations between fermions and bosons. The infinity suppression is due to the fact that the infinities requiring subtraction all must possess the same symmetry, which restricts them a lot. The infinities are not totally suppressed, however, since potentially infinite terms with the same supersymmetry as the original action can be found. Eventually, one must conclude that supersymmetry is not a genuine cure against the infinities that are blocking the way towards a sound definition of our model.

3) Rearrange the *dynamical degrees of freedom* of gravity. This is a delicate and smart attempt. After fixing the gauge, the remaining degrees of freedom of gravity form a smaller space than the original space of functions $g_{\mu\nu}(\mathbf{x}, 0)$ and its first time derivative. One might as well concentrate just on the algebra of the canonical commutation rules [5]. Rovelli and Smolin [6] found that this algebra may be realized in a space containing only knots and links in three-space. The hamiltonian should be an operator acting on these knots and links. Distances, surfaces and volumes are then to be defined in terms of the complexity of the knots and links. This program did apparently run into difficulties, however. Different kinds of infinities were encountered when the need was felt to introduce points that connect different knots and links. Thus, not only the intertwinings but also the connections between curves are relevant. The difficulties with these 'braid theories' of space and time have not yet been overcome.

4) *String theory* [7]. Initially, string theory was introduced as an attempt to address the strong interactions. When fairly realistic analytic expressions for the amplitudes for meson-meson scattering had been proposed, it was discovered that these amplitudes naturally arise in the mathematics of 'ideal relativistic strings', breaking and joining at their end points. Later also closed strings were introduced, which interact by rearrangement when different stretches of strings meet at one point. The *action* of a string system can be seen to be nothing but the Lorentz-invariant surface of the 'world sheet', spanned by a string when it propagates in space-time. Quantizing such a system leads to elegant mathematics, which at first appeared to be quite useful for an alternative treatment for the strong forces felt by the hadrons. Ideally, one would like to regard string theory as an elegant mathematical procedure to partially sum the strong interaction gauge theory amplitudes. Although some quite interesting ideas were launched recently, not much real progress in this approach was made. One of the various difficulties is that string theory in four space-time dimensions appears to be inconsistent unless many internal degrees of freedom are added, and the theory appears to possess negative $(\text{mass})^2$ states. Not only does the more

accurate theory of QCD not allow for such tachyonic solutions, it also appears to lead to Regge trajectories (functions relating angular momentum J to the mass M^2) that are non-linear. This seems to be impossible to reproduce in string theories living in a flat space-time.

String theory was found to possess solutions that in many respects behave as gravitons. This is seen most clearly in the 'zero-slope limit'. Curiously, the symmetry that starts out as a reparametrization invariance on the string world sheet, then serves as a reparametrization symmetry that generates space-time coordinate changes of the form (2.2). It was concluded that, if string theory is used not to describe the strong force in the 1 TeV domain, but instead to refer to physics at the Planck scale, then the gravitational force will be a natural consequence of these theories. A daunting thought was that string theory appears to be essentially unique. Even the difference between open and closed string theories, and the difference between theories with different boundary conditions, were attributed to differences only in the choice of the vacuum state, that is, the choice of the starting point for some perturbative approximation scheme.

In many respects, string theory therefore appeared to be the ideal candidate for a fully quantized theory that includes the gravitational force. The 'infinities' of the canonical gravity theory are completely removed, many forms of matter are added in the form of higher excited string states, and there are no ambiguities left in the order-by-order calculations.

Yet there are problems with this theory as well. Basically it is — again — a perturbative expansion. This time, the expansion is a direct chain of Feynman-like diagrams. As before, we have to address the question of convergence of this expansion. At large distance scales, convergence will be reasonably rapid, but at scales beyond the Planck length there is no reason to expect any convergence at all. As the previous theories, this theory does not tell us exactly what is going on at energies large compared to those that correspond to the Planck mass. Secondly — and this is related to the first problem — the absence of ambiguities is formally true, but not in practice. In practice, one does not know which vacuum state to start off from; this is a question that can only be answered if one knows how to sum perturbation expansion, but, we do not. Ample experience in particle theory has taught us that the existence of a perturbation expansion is no guarantee whatsoever that a 'non-perturbative' version of the theory exists, or, if it exists, that it should be unique.[†]

5) In spite of the evidence for the contrary, the existence of a unique non-perturbative theory is being speculated upon. The theory, prematurely dubbed 'M-theory', is supposed to yield the various kinds of string theory, as well as supergravity, when it is expanded around different candidate states for the vacuum. Evidence in favor of this suspicion is a body of mathematical coincidences of symmetry structures and algebras. I am not

[†] Examples are the chiral effective renormalizable models of mesons. They all may be seen as perturbative approximations to QCD, but the details of QCD certainly do not follow from these models. Other examples are QED and other non-asymptotically free particle models, which cannot be extended beyond their Landau ghosts.

dismissing this favorable evidence lightly. Certainly explanations are needed for these numerous observations. It must be stressed, however, that a sound theory ought to be based on coherent constructions showing a definite causal order dictating unambiguously in which order sequences of equations must be solved so as to make irrefutable predictions. We have nothing of the sort at present. Neither string theories nor 'M-theory' are sufficiently coherent to allow us to make any prediction whatsoever concerning, say, features exhibited clearly by the standard model of the presently observed elementary particles.

Real Problems: (i) completeness.

The important question to ask here is whether any of the above mentioned attacks stand a real chance to be successful or whether there are deeper-lying problems that are not properly being assessed, so that, as a consequence, obstacles will continue to rise. It is also a dangerous question to ask, since it may sound like an invitation simply to dismiss previous successes, which is far from our intentions. Rather, the purpose of the question is a further widening of our view.

It is illustrative to study a toy model: N point particles gravitating in a 2 space-, 1 time-dimensional closed universe [8]. For simplicity one sets its cosmological constant equal to zero. Classically, this model is of a charming and exemplary simplicity. Since the gravitational field in 2+1 dimensions carries no local physical degrees of freedom, all its canonical degrees of freedom are the particle coordinates and momenta. Furthermore, there are constraints. Where space-time closes, no residual singularities should remain, which means that not only the total energy must match:

$$E^{\text{total}} = \int \sqrt{g}\,\mathcal{H}(\mathbf{x})\mathrm{d}^2\mathbf{x} = \int \sqrt{g}\,R(\mathbf{x})\mathrm{d}^2\mathbf{x} = 4\pi \tag{4.1}$$

(where R is the contracted Ricci curvature), but also the total momentum and angular momentum. If 2-space has an $S(2)$ topology, careful counting shows that, at any instant in time, the entire phase space of the system is $4N-11$ dimensional. A space-time with genus g has a $4N+12g-11$ dimensional phase space. The fact that each particle contributes with 4 units is easy to see: two coordinates and two momenta. The -11 may appear to be more mysterious. The number arises in the following way:

$$\begin{array}{rr} \text{choice of origin of 2-space:} & 2 \\ \text{choice of rotation angle:} & 1 \\ \text{choice of velocity of reference frame:} & 2 \\ \text{matching elements of Pioncaré group:} & 6 \\ \hline \text{total:} & 11 \end{array} \tag{4.2}$$

How can one quantize a system with an odd-dimensional classical phase space?

The answer to this is that matching the Poincaré group implies a constraint, Eq. (4.1), on the total energy. Its conjugated variable, overall time, is therefore unphysical.

Of course, the overall time coordinate is not a physical variable since we cannot allow for a clock, which would have to be living outside this universe. Removing the time variable replaces -11 by -12. We *could* allow for an external clock if the universe were not closed, but this would imply a constraint,

$$E^{\rm total} < 2\pi\,, \tag{4.3}$$

for the total energy. In that case, external observers could also fix the coordinate frame externally (that is, at a spot far from the interaction region), and no Poincaré matching would be necessary, so an open universe has a $4N$-dimensional phase space.

The limitation (4.3) causes severe difficulties for a consistent quantum formulation. The canonically associated variable, time, now appears not to be continuous but rather a discrete variable [9]. Indeed, if the Hamiltonian is an angle, then time is an integral multiple of a fundamental quantum of time‡. Consequently, not the hamiltonian but the evolution operator for one time-step, $U(\Delta t) = e^{-i\Delta t\, H}$, is the most fundamental object describing the evolution. The difficulty that we then have to face is that there is no way to define a ground state, or vacuum. Without a ground state we are unable to do thermodynamics, and we are deprived from our stability arguments: the distinction between stable, low mass particles and unstable massive ones can no longer be made, and it is impossible to define low-energy and high-energy domains of the theory. At first sight this may sound as an insignificant complication, but it is fundamental. We are not able to formulate the requirement that our theory should approach a conventional field theory at the large-distance scale.

An other difficulty is the question of formulating a complete set of states in Hilbert space. In the classical theory, the question does not arise. Here we can start with any state, and ask how it evolves. The evolution often turns out to be chaotic, either towards the future, or towards the past, or both. To even begin asking questions concerning a quantum model, we first must realize that there is no external time variable, so that we must employ a Heisenberg picture, where a state is considered to represent a universe at all times. In view of the chaotic nature of the evolution, it is prohibitively difficult to see whether two states are distinct or not, *i.e.*, to give a complete characterization of the set of all distinct states. But this is what we need in order to set up a Hilbert space description of the quantum theory. Limitations either of the form (4.1) or (4.3) on the hamiltonian give rise to severe complications.§

It may seem to be strange that, before even beginning to characterize the mathematical form of Hilbert space, we need to know the entire evolution of universes throughout time. Why did this not seem to be needed in the canonical formalism of Sect. 2? There, we made use of Cauchy surfaces. A Cauchy surface is an equal-time subspace of space-time, defined

‡ Time quantization is characteristic for 2+1 dimensional gravity, but probably not for 3+1 dimensions, where the hamiltonian is not bound by an upper limit [10].

§ But perhaps not yet all options for further analysis have been scrutinized; we might return to these questions in later publications.

after we imposed the gauge condition to identify the time coordinate. Constructing good Cauchy surfaces indeed had been the starting point of our construction of the classical 2+1 dimensional gravity theory. When particles with low masses move slowly, the definition of a Cauchy surface in their immediate neighborhood appears not to lead to any problems. The Cauchy surface is a locally flat two dimensional surface, but it has conical singularities where the particles are. Therefore, one is forced to use either curved coordinate frames, or flat coordinates with reparametrization ambiguities. In both cases it is hard to characterize a minimal parameter space, and its bounds, whenever the entire universe is considered. If we would limit ourselves to a 'small section' of the universe, then also we would have to limit the energy available there to be much smaller than 2π, so the constraint of Eqs. (4.1) or (4.3) is replaced by a tighter one, with its own unwelcome consequences.

Alternatively, one may consider the route offered by Wheeler and DeWitt [11]. They propose to replace the evolution equations, such as Schrödinger's equation, by a single functional integral, representing the "sum of histories". This considerably exacerbates the difficulty of interpreting the expressions obtained. Deprived of a Hilbert space of states, we can no longer determine probabilities for the future events, given a present configuration. Perhaps the equation can be called upon in calculating the 'cosmological S-matrix': the matrix that gives the 'final state of the universe', given its 'initial state'. But we do still need to know that the total probabilities are conserved, which means that this S-matrix needs to be unitary, and then we still need to know how to completely enumerate the states in Hilbert space, at the beginning and the end of the universe.

In our 2+1 dimensional model understanding completeness is a real problem. We have not even started trying to phrase the problem in 3+1 dimensions.

Real Problems: (*ii*) black holes.

Our 2+1 dimensional model is simpler than 3+1 dimensions in one fundamental respect: in the absence of a cosmological constant, there are no black holes.

In standard, unquantized general relativity in 3+1 dimensions, the existence of black holes [12] is undeniable[‖]. The equations show no obvious lower limit for the size of a black hole, although making very tiny black holes requires conditions that are not easy to realize in ordinary physical circumstances. In principle, black holes of any size can be made simply by colliding sufficiently energetic particles head-on. The center-of mass energy needs to be huge, but there appears to be no fundamental reason why such high energies should be forbidden. Hence, we see no reason why the existence of tiny black holes should be forbidden.

When quantum mechanics is switched on, one does encounter a natural lower limit for black holes: the Planck size and the Planck mass. Smaller 'black holes' would have to have a horizon with an even tinier radius, smaller than the Compton wavelength, and

‖ even though some still try to dispute the existence of astronomical-sized black holes. Perhaps they are unaware of the fact that no exotic conditions in matter are needed for the formation of a large black hole.

conflicts with the uncertainty relation would presumably invalidate the notion of a horizon altogether. The legitimate question to ask is: how does the 'spectrum' of black hole 'states' terminate near the Planck length, do the black holes 'merge' with ordinary particles there, or in any case: how do we formulate the laws of physics for these and similar objects whose masses are comparable to the Planck mass?

In an attempt to answer such questions, we return to black holes large compared to the Planck size, but still such that quantum effects might be relevant. At first sight it seems that the laws of general relativity make a lot of sense here, and they predict the properties of these black holes rather precisely. Assuming that at scales large compared to the Planck size some (effective) quantum field theory applies, one can calculate what will happen.

The outcome of the calculation, at first sight, appears to imply interesting and 'reasonable' physical properties of black holes. It is found that they emit particles with a thermal distribution. A black hole is found to have a temperature T_H that is uniquely determined by its mass, charge and angular momentum. For a chargeless, non-rotating black hole:

$$T_H = \frac{\hbar\, c^3}{8\pi k_B G_N M}\,, \tag{5.1}$$

where k_B is Boltzmann's constant and M is the Black hole mass. In the general case, in units where $k_B = c = \hbar = G_N = 1$:

$$T_H = \frac{r_+ - r_-}{4\pi\,(r_+^2 + a^2)}\,, \qquad \begin{matrix}\text{where}\\ \text{and}\end{matrix} \qquad \begin{matrix} J = M\,a\,, \\ r_\pm = M \pm \sqrt{M^2 - a^2 - Q^2}\,. \end{matrix} \tag{5.2}$$

Here, J is the angular momentum and Q is the electric charge.

This equation for the temperature turns black holes into physically quite plausible, thermodynamical objects¶. Taking the electric potential at the horizon, ϕ, and the angular velocity of the horizon, Ω, to be

$$\phi = \frac{Q\, r_+}{r_+^2 + a^2}\ ; \quad \Omega = \frac{a}{r_+^2 + a^2}\,, \tag{5.3}$$

we can write the equation for the entropy S as

$$T_H\, \mathrm{d}S = \mathrm{d}M - \Omega\, \mathrm{d}J - \phi\, \mathrm{d}Q\,, \tag{5.4}$$

which integrates into

$$S = \pi\big(r_+^2 + a^2\big) + C^{\underline{\mathrm{nst}}}\,. \tag{5.5}$$

Apart from the constant, this is exactly one quarter of the horizon area.

¶ The thermodynamical equations do imply a negative specific heat hor the non-rotating black hole, so that there cannot be true equilibrium. Departures from equilibrium due to this, however, would occur only on very large time scales; in practice this phenomenon is not expected to lead to any significant problem.

The constant does not follow from thermodynamical arguments unless one knows what happens when a black hole decays completely. However, it is easy to argue that, during the final stages of its evaporation, the black hole mass will approach the Planck mass, and its size the Planck size. Without a decent theory for Planck length physics we may never be able to estimate the constant.

According to standard arguments in statistical physics, the exponent of the entropy (5.5) — with constant — should correspond to the density of quantum levels of the black hole. Thus, for large black holes, the density of states is expected to grow like

$$\varrho \to C \cdot e^{\pi(r_+^2 + a^2)}\,, \tag{5.6}$$

which is exactly $C\cdot\exp\left({}^1\!/_4 \cdot \text{Area}\right)$. It is as if the degrees of freedom are equally distributed over some kind of lattice defined on the area of the horizon. One is tempted to infer from this that a dynamical theory of physical degrees of freedom at the horizon might describe the properties of a black hole.

Sometimes, S is written as

$$S \stackrel{?}{=} S_{\text{grav}} + S_{\text{matter}}\,, \tag{5.7}$$

where S_{matter} represents the particles close to the horizon, which are emitted as Hawking particles. The problem with this is however that any estimate of S_{matter} gives expressions which *diverge* badly at the horizon: $S_{\text{matter}} = \infty$! A brute force cut-off appears to be needed to define a finite S_{matter}. Setting $S_{\text{grav}} = 0$ requires a cut-off at a distance of the order of the Planck length from the horizon (as measured by local observers).

At first sight this seems to be a welcome result: no physical degrees of freedom should be admitted at scales shorter than the Planck scale. But then attempts to identify the surviving degrees of freedom leads to conceptual difficulties. The absense of degrees of freedom too close to the horizon leads to constraints that relate outgoing particles to the ingoing ones. Such constraints appear to be necessary if we wish to impose some sort of action-reaction principle for the black holes, as required when one expects a unitary evolution equation such as a Schrödinger equation for a black hole . But the existence of such constraints also appears to be at odds with the very essence of general relativity: the horizon cannot be more than a coordinate artifact, and ingoing particles should not be 'reflected' by a horizon at all!

Any decent theory of quantum gravity should allow us to anticipate what might happen in the strongest possible gravitational fields — those of black holes. The conundrum posed by the black hole statistics clearly should be dealt with. It can be summarised as follows: does the outgoing radiation come in quantum states that can be represented by a unitary (hence time reversible) transformation of the ingoing states? How can this unitary operation be derived from, or even reconciled with, the principle of general coordinate invariance?

Holography.

At face value, the general relativity principle might appear to imply that outgoing radiation cannot depend on ingoing matter, so that there is no unitary evolution at all [13]. This was indeed what was thought at first: black holes are a "corridor to another universe", and their evolution would only be unitary in a Hilbert space that also incorporates all possible states in that other universe. This would turn a black hole into a drain of 'quantum information'; pure states in Hilbert space would spontaneaously evolve into mixtures of states. In ordinary physics, this can happen only if detectors are made insensitive to some essential fragment of the data.

This behavior is quite strange and unprecedented in physics, certainly if, as one might suspect, black holes blend into the spectrum of ordinary particles at the Planck scale. It would be the kind of mixture of pure into mixed states that one might expect in theories where many of the physical parameters are not sharply defined, but known only as stochastic distributions. Thus, these tiny black holes would be controlled by physical parameters whose exact values are not dictated by theory, but, at least partly, by chance.

This is where our topic touches upon a philosophical question. Would it be acceptable if our theories would be fundamentally unable to yield sharp predictions for the physical characteristics of the entities that play a role at the Planck scale; must we be content with 'fuzzy' predictions? Up till now, the fuzziness produced by the quantum mechanical nature of atomic and sub-atomic physics could be limited to be precisely defined probabilistic distributions that never were worse than what one would have to expect anyhow since the initial states in an experiment can never be completely under control: in scattering experiments the *impact parameter* depends on haphazard distributions of particles within a particle beam, so that it must always be assumed to have some (fairly flat) probabilistic distribution. Therefore, the outcomes of experiments can never be sharply predicted, regardless whether one had a quantummechanical or a deterministic underlying theory. Up till now, however, all physical characteristics of (elementary or composite) objects were sharply defined or predicted by our theories.

A point made repeatedly by this author is that it is quite likely, at least philosophically more acceptable, that the quantum properties of black holes are indeed sharply defined by *some* theory. It would be premature to assert that this would be at odds with general relativity. That *would* involve assumptions concerning behavior of matter near the Planck scale, and such assumptions nay be suspected to be wrong. Ingoing particles that encounter outgoing ones at a Planckian distance away from the horizon do indeed influence them, while passing through. If not the ordinary standard model interactions perturb the outgoing particles, then certainly the gravitational force, due to graviton exchange, will do the job. But the job done by gravitons is difficult to compute: it diverges.

It was attempted to make the next step: compute such effects. To some extent we succeeded in obtaining a unitary scattering matrix for black holes, but its Hilbert space still contained more states than allowed by the value (5.6) as dictated by the entropy. The only way to obtain the correct density of states appears to be by assuming that there really are no more states to be discussed than just that number. By itself, this appears to be an interesting and physically meaningful piece of information: the number of mutually orthonormal states to be employed in the description of the horizon of a black hole is limited by Eq. (5.6). But its consequences are far-reaching: these states seem to be distributed at the horizon, which is a two-dimensional plane. Yet the states one started off with, using general coordinate transformations to describe the properties of a black hole once the properties of the vacuum world experienced by a local observer near the horizon are understood, appear to be distributed in a three-dimensional space!

This led us to formulate [14] the so-called 'holographic principle':

> The complete set of degrees of freedom for all particles populating a certain region in space and time, can be represented as if they were all situated on the *boundary* of this space-time. Roughly, there is one Boolean degree of freedom for every $4\ln 2$ Planck lengths squared.

This complete rearrangement of the physical degrees of freedom in the theory of quantized particles in the Planck regime, has far reaching implications for this theory. It invalidates the unsual distinction between *intensive* and *extensive* variables. Usually, extensive variables such as total mass, charge and energy may be seen as integrals of the corresponding densities over three-space. This will no longer be true; most integrations will be over some surface instead. When we arrived at the holographic principle, we took this surface to be the horizon of a black hole, but for a local observer this surface would be indistinguishable from any other surface. Thus, one must conclude that the physical degrees of freedom may be projected onto any (infinite) surface at any time in three-space.

Superstring theory.

New developments led to a picture of what a non perturbative version of superstring theory might be like. New components in the non-perturbative versions of string theory are membranes and their multidimensional counterparts, D-branes.

A particularly important issue of investigation is now the spectrum of the degrees of freedom of the new theories. M-theory is said to be far from understood, but one can consider the number of modes that D-branes can unfold in if we consider a black hole background metric. Unfortunately, this black hole has to be in, or at least not far away from, the BPS limit, the extremal limit, in which the horizon does not have the same structure as in the Schwarzschild black hole. In this limit, however, counting does suggest the area law (5.6) for these degrees of freedom. And it is here that a somewhat modified version of the holographic principle appears to apply. The theory defined on the lower-dimensional surface is however not a lattice theory but rather some conformally invariant field theory. These results are considered to be very promising successes for string theory. At least for this author, they came unexpected: these theories could not be disqualified on the basis of impossible spectral structures in the vicinity of black hole horizons.

The highly structured topological features of string- and M-theory, the fact that they appear to include gravitational forces, and their successes in generating some of the desired spectra of states for black holes, are often considered to be striking evidence that they should, some way or other, be at the very basis of an all-comprising theory, a completely finite and consistent description of all particles and forces in Nature. If this were true, this all-comprising theory would be

i fundamentally quantum mechanical. This means that our questions concerning quantum cosmology remain unanswered: is there a Schrödinger equation for the entire universe and how do we interpret 'probabilities' for its evolution, if there is fundamentally no place for 'external' observers?

ii There would not be a fundamental space-time metric. The notions of space and time themselves would be ambiguous, as we see most clearly in the matrix theory, where the coordinates do not commute. This appears to imply that there is no such thing as locality. Even the notion of causality appears to be difficult to maintain in such a theory *. These features are often presented as necessary and unavoidable complications for any quantum gravity theory, but on the long run they may well backfire: in absence of strict locality or causality, the complete systematic rules for implementing the "laws of physics" for such theories may remain obscure, and application of these theories may end up requiring artistic rather than scientific skills.

iii Degrees of freedom for this theory would not be localized in 3-space but in some arbitrariliy chosen 2-space. The possibility to transform from one 2-space to any other 2-space should imply a tremendously large symmetry group for this theory.

These features are so far removed from daily life experiences that it is hard to imagine how these theories can be linked to the real world. A further difficulty that will have to be addressed by the proponents of this theory is the construction of the 'correct' ground state, or vacuum. At present, this is not known, and many 'vacua' are equally possible. Will there be a way to select the 'true' vacuum from first principles, or do we need to determine it experimentally? In the latter case it would be misleading to say that string theory has no free parameters; it has at least as many free parameters as is necessary for fixing the vacuum state.

We should hasten to add that numerous other problems in our present view of the universe and its fundamental laws have also not yet been addressed by string- and related theories. Most notably, there are the *hierarchy* problem and the *cosmological constant* problem. The hierarchy problem is the simple fact that there are various enormous differences in scales, such as the wide separations between the neutrino masses, the lepton and quark masses, the weak scale and the Planck scale. These wide separations in scales should have a natural explanation, and this may become a considerable problem in theories such as string theory, which purportedly do not possess any dimensionless free parameters. The cosmological constant in the real world seems to be extremely accurately tuned to zero, whereas the only known mechanism that might be related, supersymmetry, is

* The issue of causality can be disputed. It is stressed that causality still holds in all those corners of the M-theory where it coincides with one of the perturbatively understood theories. Naturally, one expects a causal unified theory. On the other hand we note that the local and causal structures of each of the limiting theories appears to be so different that one may fear a non-local and non-causal construction whenever one tries to combine them.

strongly violated. How can a crippled symmetry produce a cancellation over 120 orders of magnitude?

A plea for deterministic quantization [15].

Being dissatisfied with the state of affairs sketched above, the author has searched for an approach where the mysteries of quantum mechanics itself are dealt with together with our problems in understanding quantum gravity. Noting that familiar notions of space, time, probability, causality and locality are likely to be severely affected by whatever theory there will be that is powerful enough to clear the obstacles against quantum gravity, we suspect that it will also shed new light on the older difficulties. The usual no-go theorems telling us that hidden variables are irreconcilable with locality, appear to start with fairly conventional pictures of particle systems, detectors, space and time. Usually, it is taken for granted that events at one place in the universe can be described independently from what happens elsewhere. Perhaps one has to search for descriptions where the situation is more complex. Maybe, it needs not be half as complex as superstring theory itself.

The conventional Copenhagen interpretation of quantum mechanics suffices to answer all practical questions concerning conventional experiments with quantum mechanics, and the outcome of experiments such as that of Aspect et al can be precisely predicted by conventional quantum mechanics. This is used by some to state that no additional interpretation prescriptions for quantum mechanics are necessary. Yet we insist that the axioms for any 'complete' quantum theory for the entire cosmos would present us with as yet unresolved paradoxes. The point we wish to make, however, is that there is no inevitable contradiction between quantum mechanics on the one hand and a "deterministic" world view on the other. If quantum mechanics can indeed be reconciled with determinism at the Planck scale, this could possibly solve many of our present problems. Since we are talking of many degrees of freedom that are operative at the Planck scale, the phrase "hidden variables" is appropriate, although the variables are not truly hidden; they are nothing but the highest energy excitations of otherwise conventional degrees of freedom, and just difficult to detect by conventional means.

Let us first take a big step backwards. The basic philosophy behind our version of hidden variables — and without doubt it must also be at the basis of many other related approaches — is that there are assumed to be deterministic fundamental laws of physics, relevant only for the very tiniest scales of distance and time, and that the pervading solution of these equations is not a 'stationary' vacuum, but rather a highly chaotic one. Our universe is filled with what at first sight looks like white noise, but where this state is called 'vacuum' by us, there are actually all sorts of subtle short and long distance correlations. The one-, two- or many particle states are also noisy solutions, but they are characterized by slightly different correlation patterns. The laws determining these fluctuations may be entirely deterministic, but the only regularities that we can pick up at our macroscopic, or atomic, distance scales, are statistical at best. It is these regularities, essentially just minor correlations in the chaos, that we physicists have learned to model as precisely as we can. We are unable to predict anything with certainty, as too many of the fluctuating parameters could not be brought under control. Our predictions therefore more often than not contain statistical elements. This effective description is what we presently call quantum mechanics.

As stated earlier, the 'hidden variables' are nothing but the numerous fluctuating degrees of freedom at the tiniest distance scales. These variables just represent the heaviest elementary fields in the system; they are not truly hidden, but merely very difficult to observe if one's accelerator cannot reach the required energy (read: the Planck energy). One might suspect that the laws leading to these chaotic solutions could well be local; this would surely suffice to enable the emergence of chaotic solutions, as can easily be verified in simple computer animations of cellular automata.

However, plausible as this scenario may seem at first sight, it is often criticized precisely because it does not seem to explain how such a quantum mechanics can violate Bell's inequalities. Will we need fundamental 'action at a distance' after all? There is not the slightest doubt that

(i) Quantum mechanics violates Bell's inequalities, and that

(ii) Experimental observations confirm the quantummechanical predictions.

In several cases, the violation of Bell's inequalities was verified experimentally. Any hidden variable theory that cannot accommodate these facts must be discarded. The remainder of this paper will describe the present author's approach in more detail. Although it is not clear how violations of Bell's inequalities can come about in this theory, it is also difficult to prove that they cannot be violated. This is because the Copenhagen interpretation of the wave function is kept totally intact. We have *both* quantum mechanics and determinism. This means that we have deterministic microscopic laws, but we use the entire machinery of Hilbert space techniques to represent the probabilities that may occur.

It is duly stressed that we have been unable to come forward with any satisfactory model that shows in detail how the mechanism that we have in mind should work. Rather than attributing this failure to the Bell inequalities so as to dismiss the entire approach, we make the following important remarks: It may very well be that we are overlooking some essential complications. It could be that some kind of non-locality will be needed in the end. Perhaps there is an exotic boundary condition that has to be implemented. But it could also be that the missing ingredient is actually nothing more than some powerful symmetry that relates 'beables' to 'changeables' (notions that we shall define shortly, Sect. 9). If this is the case for many of the ordinary symmetries of Nature, such as rotation and translation invariance, then that could be the beginning of an explanation why our world appears to us to be quantum mechanical. A model where this happens could be constructed: all harmonic oscillators can be described this way, but as yet we were not able to introduce interactions, in the form of anharmionic terms, unless they are nonlocal.

Thus, if the price to be paid for quantummechanical behavior in a deterministic theory is just some delicate mathematical symmetry or else perhaps a mild form of non-locality, then it is worth-while to pursue this option further. For this reason, we choose as our strategy to ignore the Bell inequalities and to focus on other aspects of the problem. Later, we shall encounter another price that we would be willing to pay: information loss.

Ontological states.

The central assumption in deterministic quantization is the existence of ontological states[†]. We assume that the universe evolves from ontological states to ontological states, according to a fixed law of evolution without quantummechanical mixing. Thus, if we are certain which state we have at one time, the laws will give with complete certainty only one state at all later times. To specify a state, we may either use discrete or continuous variables, or both. There are now two different possibilities to consider: the evolution could either be *time reversible* or *irreversible in time*. In the latter case, several different ontological states at time t_1 could evolve in one and the same ontological state at time t_2.

Let us first consider the time reversible case. It is then perfectly legal to define a Hilbert space by identifying the ontological states with the elements of a basis for this Hilbert space, the *primordial* or *ontological* basis. The evolution over some time interval Δt is then described by a unitary operator $U(\Delta t)$, which, in the ontological basis, takes the form of a pure permutation matrix, or kernel, having only zeros and either ones or Dirac delta functions as entries. We can now use the same operator $U(\Delta t)$ to describe the evolution of arbitrary vectors in this Hilbert space, where we can interpret the absolute squares of the coefficients as probabilities. Conservation of probabilities is automatically guaranteed by the unitarity of $U(\Delta t)$. Quantum interference does not occur, and hence, if the coefficients are complex, the phase angles have no direct interpretation at all. If, however, we use any other basis for this Hilbert space then the phase angles will be relevant. The fact that we do observe quantum interference in the world of atoms and molecules implies that the basis used is not an ontological basis. The ontological basis of our universe would require all Planckian degrees of freedom for its description, and is therefore not known.

In the ontological basis, we can define the ontological observables, also called *beables*. They are defined to be observables that refer directly to properties of the ontological states, so that in the ontological basis they are diagonal. This implies that all beables, at all times, commute with one another. In ordinary quantum field theory, no such operators are known, though in some very special simple models they can be constructed. [15]

All operators that are not diagonal in the ontological basis will be referred to as *changeables*. They replace an ontological state by another state. The evolution of changeables is described by the same unitary operators $U(\Delta t)$ as the evolution of the beables. We shall assume that present day physicists are unable to distinguish beables from changeables. Indeed, as stated earlier, there might exist symmetry relations between beables and changeables.

Since $U(\Delta t)$ is unitary, it can be diagonalised. One then obtains a new basis where it is easy to identify the hamiltonian. All states obey the Schrödinger equation. We deduce that the Schrödinger equation holds for time-reversible deterministic systems just as much as for conventional purely quantummechanical systems.

[†] In previous work, the phrase "primordial states" was used for what we now decided to call 'ontological states'.

There appears to be, however, one very important distinction between conventional quantum systems and this apparently 'quantum' description of deterministic systems. The hamiltonians derived for deterministic systems nearly never appear to have a ground state. If time is discrete, the hamiltonian is periodic; its eigenvalues are defined *modulo* an integral multiple of $2\pi/\delta t$, where δt is the smallest allowed time interval. In that case, there seems to be no way in which we can identify any of its eigenstates as representing the state with lowest energy. If time is continuous, one usually starts with a deterministic evolution equation of the form

$$\frac{\mathrm{d}q^i(t)}{\mathrm{d}t} = f^i\big(\vec{q}(t)\big)\,. \tag{9.1}$$

Defining the Hilbert space operators $p_i(t) = -i\hbar\partial/\partial q^i(t)$, we find that the hamiltonian must be

$$H = \sum_i p_i\, f^i\big(\vec{q}(t)\big)\,, \tag{9.2}$$

which clearly has no lower bound.

Attempts to remedy this situation mostly amount to finding constraints such that only the non negative values of the hamiltonian eigenvalues are physically allowed. In principle, there is nothing against imposing such constraints, and so it seems that we obtain interesting ontological theories that generate 'quantummechanical behavior'. The difficulty comes when we try to reconcile such constraints with locality.

Is this conflict with locality due to the fact that we are trying to build a theory that violates Bell's inequalities? Could it be that the violation of locality only takes place at Planckian distance scales and can therefore be made acceptable — after all, string theories also tend to generate nonlocality? Or should we attempt to construct more sophisticated models? There is one important avenue not yet discussed.

Information loss [15].

In deterministic models it may well happen that different ontological states, after a certain number of time steps, evolve into the same ontological state. In that case, the construction described above of an evolution operator is unsatisfactory, since the result would not be a unitary operator. In that case, the ontological states cannot be identified with the basis elements of our Hilbert space. The absolute values squared of the coefficients of a wave function would then not correctly describe the probabilities.

Instead, we can still use a Hilbert space procedure, provided that each element of the basis be identified with an *equivalence class*. Two ontological states are in the same equivalence class if, some time in the future, they evolve into one and the same state. This definition is not without danger; in principle, all states could one day turn out to be equivalent, so that there is only one equivalence class. This could be the case in a shrinking universe. In an expanding universe, like ours, this danger is probably not there. We assume that the definition converges reasonably well so that the equivalence classes are sufficiently well distinguishable in practice. In any case, with this definition, the evolution is again unitary by construction. We can proceed as in the previous section. What have we won?

First, we should emphasize that the resulting quantum theory is again unitary, and its hamiltonian is hermitean. We do not anticipate that there would be transitions from quantummechanically pure states into mixed quantum states. At first sight, therefore, our 'quantum theory' will look as before, and also we expect to encounter the same difficulty concerning the positivity of the hamiltonian and the absence of a good candidate of the vacuum.

At various levels of sophistication, however, information loss may make a lot of difference. At a rather naive level, we note that information loss will make some ontological states much more abundant, or probable, than others. Some states will have much more possible precursors than others. This is a welcome feature if we wish to explain vacuum correlations. In a theory with exact time reversibility, one would expect all states to be equally probable, so that no nontrivial correlations are to be expected, whereas in the real world nontrivial vacuum correlations play an essential role.

Secondly, information loss is difficult to avoid in the description of classical (unquantized) black holes. Attempts at writing a strictly time-reversible generally relativistic model usually fail when the possibility of gravitational collapse is taken into account. The formation of trapped regions can often not be made undone. Allowing for information loss simply implies that we decide not to worry about this. The laws are sufficiently chaotic so that even if information disappears into black holes, enough chaotic behavior is generated to compensate for this.

Thirdly, information loss may give a natural explanation for the holographic principle. Up till now, it had been virtually impossible to think of natural laws of evolution that would appear to be local in three-dimensional space, yet require a basis of states that can be mapped onto a two-dimensional surface. But with information loss, it is quite conceivable that the data in a three-dimensional volume dissipate away, so that only the information left on the boundary, a two-surface, suffice to describe the equivalence class. Although this is by no means obvious, we do suspect that this might be the case in a reasonable class of interesting models. Note that the holographic principle implies equivalence classes to be very *large*, and that the non-equivalent ontological states generate a much smaller Hilbert space than in models without information loss. Thus, even if information loss might be achieved by very tiny corrections in the deterministic equations of motion, its effects will be huge.

Fourth, information loss may imply that many of the candidate ontological states will be illegal, since they will have no precursor at all. In an effective quantum description, this will lead to constraints. At first attempts, these constraint do not resemble the ones that we require if we want the hamiltonian to have a lower bound, but in more sophisticated approaches, they do!

Fifth, information loss implies that many periodic solutions become unstable. They tend towards discrete subsets, the stable *attractors*, separated by forbidden regions where the orbits repel. Thus one naturally obtains 'quantization' of orbits, much like what we find in quantized harmonic oscillators. The quantum nature of many phenomena in physics, usually explained by conventional quantum mechanics, is here seen in a different light.

Sixth, there are the Bell inequalities. Our definition of quantum states hinges on the definition of equivalence classes. These, in turn, can only be defined if we know how a given system evolves in the future. Thus, an accurate description of a given setup in terms of quantum states requires some knowledge of the future. It is exactly this kind of predeterminism that seems to be needed for the interpretation of an Einstein-Rosen-Podolsky experiment in terms of an ontological theory. We know very well that the experiment itself never allows the experimenter to glance into the future, but if we try to interpret what happens, some anticipation of the future experiment seems to be needed. We suspect that this observation might provide for the kind of loophole needed to avoid the conclusion that hidden variable theories of this sort require non-locality.

Dissipation at the Planck scale. Conclusions.

Difficulties with the conventional approach to quantum gravity only become apparent at the Planck scale, 10^{-33} cm, as is most clearly seen when we try to reconcile the laws of quantum mechanics with gravitational collapse. This does not mean that the Planck scale has to be the ultimate distance scale in physics; it could be that the ultimate distance scale is much closer to the conventional distance scales of physics, say a fraction of a TeV^{-1}, but at present this is not considered to be very likely. As a working hypothesis, we take the Planck scale to be the ultimate scale. It is here that we expect a detailed description of the ontological states and the occurrence of information loss.

A nice feature of theories with information loss, or dissipation of information, is that one could keep a strict continuum of space, time and fields in space-time, yet have only a discrete set of equivalence classes (we could have added this as a seventh feature in the previous section). Imagine a Navier-Stokes fluid with viscosity. The dimensionality of viscosity, in a fluid with a given density, is $\mathrm{cm}^2/\mathrm{sec}$. In a relativistic theory, where centimeters and seconds are linked by the speed of light, the dimensionality of viscosity will be just that of a length. Let us take this to be the Planck length. Then, at distance scales shorter than the Planck length, viscosity, hence dissipation of information, will dominate. No ontological data will survive at that scale, so that we can understand why quantum phenomena never require scales shorter than that.

At distance scales large compared to the Planck scale, dissipation of information is negligible, and turbulence sets in. Of course, the details of the microscopic laws are expected to be very different from conventional Navier-Stokes fluids; in particular there is no form of energy that continually decreases, so that it would bring all initial configurations at rest eventually. Our system at the Planck scale is expected to stay in a chaotic mode, but it is attracted towards limited sets of possible stable attractors. Simple computer models of cellular automata can be used to illustrate this expected behavior.

If indeed our system remains deterministic and continuous up to and beyond the Planck scale, then this is evidently a way to reassure locality, hence local Lorentz invariance and causality. It would be a conceptually quite attractive way to resolve the apparent paradoxes met in attempting to reconcile what is presently understood to be quantum mechanics with general relativity.

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