

QUARKS AND GAUGE FIELDS

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A B S T R A C T

Professor D. Gross' communication has
not reached us.

Quark confinement in four space-time dimensions can be obtained in a simple model described by a local but not renormalizable Lagrangian. The interactions can be made such that the electric components of a gauge field are squeezed into vortex lines so that there are stringlike forces between quarks. The Lagrangian can perhaps be looked upon as an "effective" Lagrangian of the real world, and the model may give some insight in the dynamics of quark confinement.

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1) INTRODUCTION

To obtain a detailed understanding of the mechanism that keeps the quarks permanently bound is one of today's major problems in particle physics. It would be most satisfactory if the feature could be explained in terms of infra-red effects in a renormalizable field theory. And then we are naturally led to a non-Abelian gauge theory without Higgs-Kibble mechanism, for three reasons :

- a) The theory contains massless fields interacting with each other and is therefore infra-red divergent. The absence of mass follows from the symmetry and needs not be put in by hand.
- b) The theory is infra-red unstable and therefore the long-distance behaviour is not described by the classical limit of small coupling, and there need not be physical massless particles.
- c) The theory contains vector fields. These could form vortex lines which behave like strings.

It is this latter point to which we want to focus attention. Vortex line solutions are known to exist in an Abelian theory with Higgs mechanism ¹⁾. However, in that case, it is the magnetic field lines which are trapped in a vortex. If quarks are to sit at the end points of such a vortex, so that they will be permanently bound, then they have to be magnetic monopoles ²⁾. The quantum rules necessary to exclude exotic states are then not very elegant.

We think that it should be the electric (i.e., time-space) components of $F_{\mu\nu}$ that are trapped in a vortex. In that case the triality zero selection rule comes out more naturally, as we shall see. We asked ourselves whether electric vortices can occur in a classical Lagrangian field theory. The answer is yes, but the Lagrangian is not renormalizable. It deviates from a renormalizable one only for small values of certain fields. Since these fields have the dimension of a mass, this is a modification in the infra-red region. It is not inconceivable that higher order quantum corrections from zero mass particles give rise effectively to such modifications. This is why we call our Lagrangian an effective Lagrangian.

2) CONSTRUCTION OF THE LAGRANGIAN (ABELIAN CASE)

We can follow the guide of the renormalization group. We want to describe an asymptotically free, infra-red unstable theory. Let us assume that for momenta going to zero, the effective coupling goes to infinity :

$$\begin{aligned} &\text{if } k \rightarrow 0, \\ &\text{then } g_{\text{eff}} \rightarrow \infty. \end{aligned} \tag{2.1}$$

This we now take as an input. So from now on we can forget that the theory was non-Abelian, and first construct an effective Lagrangian with this property for Abelian gauge fields.

All charged particles will be represented by an external source function $J(x)$. Consider the Lagrangian

$$\mathcal{L} = -\frac{1}{4} Z F_{\mu\nu} F_{\mu\nu} - J_{\mu} A_{\mu}, \tag{2.2}$$

where

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu},$$

and Z is just a constant. Of course one could scale Z out of the kinetic term :

$$\begin{aligned} A_{\mu} &\rightarrow Z^{-1/2} A_{\mu}, \\ \mathcal{L} &\rightarrow -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} - Z^{-1/2} J_{\mu} A_{\mu}. \end{aligned} \tag{2.3}$$

So we see that the interactions are proportional to Z^{-1} . Therefore we want :

$$\begin{aligned} &\text{if } k \rightarrow 0, \\ &\text{then } Z \rightarrow 0 \end{aligned} \tag{2.4}$$

Now if we took Z to be momentum dependent then we would have a non-local Lagrangian, and (in the non-Abelian case) gauge invariance would be destroyed. The trick is to let Z depend on an auxiliary scalar field φ with the dimension of a mass :

$$\begin{aligned} &\text{if } \varphi \rightarrow 0, \\ &\text{then } Z(\varphi) \rightarrow 0. \end{aligned} \tag{2.5}$$

Thus we are led to consider the following Lagrangian,

$$\mathcal{L}(A, \varphi) = -\frac{1}{4} Z(\varphi) F_{\mu\nu} F_{\mu\nu} - \frac{1}{2} (\partial_\mu \varphi)^2 - V(\varphi) - \mathcal{J}_\mu A_\mu \quad (2.6)$$

By definition we shall require

$$\begin{aligned} V(\varphi) &> 0 \quad \text{if } \varphi \neq 0, \\ V(0) &= 0. \end{aligned} \quad (2.7)$$

In practice one can take

$$V(\varphi) = \frac{1}{2} m^2 \varphi^2 + \frac{\lambda}{24} \varphi^4 \quad (2.8)$$

(there is no reason to require symmetry breaking here). And further

$$\begin{aligned} Z(\varphi) &\rightarrow 1 \quad \text{for large } |\varphi|, \\ Z(\varphi) &\rightarrow 0 \quad \text{for } \varphi \rightarrow 0. \end{aligned} \quad (2.9)$$

The exact form of Z for small values of φ shall be left open for the time being. For simplicity we take Z and V both to be increasing functions of φ^2 .

3) WHAT HAPPENS TO COULOMB'S LAW

From the Lagrangian (2.6) we derive the Lagrange equation,

$$\partial_\mu E_{\mu\nu} = \mathcal{J}_\nu \quad (3.1)$$

where $E_{\mu\nu}$ is the induction field,

$$E_{\mu\nu} = Z(\varphi) F_{\mu\nu} \quad (3.2)$$

It is convenient to take $E_{\mu\nu}$ as a separate variable and write

$$\begin{aligned} \mathcal{L}(E, A, \varphi) &= \\ &= \frac{1}{4Z(\varphi)} E_{\mu\nu} E_{\mu\nu} - \frac{1}{2} E_{\mu\nu} F_{\mu\nu} - \frac{1}{2} (\partial_\mu \varphi)^2 - V(\varphi) - \mathcal{J}_\mu A_\mu \quad (3.3) \\ &= \frac{1}{4Z(\varphi)} E_{\mu\nu} E_{\mu\nu} - \frac{1}{2} (\partial_\mu \varphi)^2 - V(\varphi) + A_\nu (\partial_\mu E_{\mu\nu} - \mathcal{J}_\nu) \\ &\quad + \text{total derivative.} \end{aligned}$$

Variation with respect to $E_{\mu\nu}$ gives (3.2) and the original Lagrangian (2.6). Now the vector potential acts as a Lagrange multiplier.

Suppose now that the charged particles are at rest and we are interested in the stationary solution :

$$\begin{aligned} \mathcal{J}_i &= 0, \quad i, j = 1, 2, 3. \\ \mathcal{J}_4(\vec{x}) &= \rho(\vec{x}), \\ E_{ij} &= 0. \\ E_{i4} &= iE_i. \end{aligned} \quad (3.4)$$

Then the Lagrangian equals minus the energy density, which is

$$\begin{aligned} \mathcal{H} &= \frac{E_i^2}{2Z(\varphi)} + \frac{1}{2} (\partial_i \varphi)^2 + V(\varphi); \\ \partial_i E_i &= \rho(\vec{x}). \end{aligned} \quad (3.5)$$

The field configuration is found by requiring the total energy to be a minimum under the additional condition $\partial_i E_i = \rho$.

Since we want to find the Coulomb force between two charges far apart, we are mainly interested in fields \vec{E} that are nearly constant as a function of space also. In that case the derivative term for the φ field is unimportant. The field φ is then determined by the requirement that

$$\frac{1}{2} \frac{E^2}{Z(\varphi)} + V(\varphi) \quad (3.6)$$

be minimal, so finally all that is relevant is the relation between Z and V . Suppose that for $\varphi \rightarrow 0$,

$$Z = C \cdot V^\alpha ; \quad C > 0, \quad \alpha > 0. \quad (3.7)$$

(so if we take $V = \frac{1}{2}m^2\varphi^2 + O(\varphi^4)$ then $Z(\varphi) \rightarrow C(\frac{1}{2}m^2\varphi^2)^\alpha$)

(3.6) is minimal if

$$[V(\varphi)]^{\alpha+1} = \frac{\alpha}{2C} E^2. \quad (3.8)$$

So the energy density for small values of E is

$$\mathcal{H}(E) = \left(1 + \frac{1}{\alpha}\right) \left(\frac{\alpha E^2}{2C}\right)^{\frac{1}{1+\alpha}} = C' |E|^{\frac{2}{1+\alpha}}. \quad (3.9)$$

For large values of E we have

$$\mathcal{H}(E) = \frac{1}{2} E^2, \quad (3.10)$$

so we have roughly,

$$\mathcal{H}(E) = C' |E|^{\frac{2}{1+\alpha}} + \frac{1}{2} |E|^2 \quad (3.11)$$

Now consider a flux line going from particle A to particle B carrying a small flux Φ [Fig. 1. Remember that due to (3.1) we have flux conservation]. Let it have a cross-section Ω . The electric induction is then

$$E = \Phi / \Omega, \quad (3.12)$$

and the energy U per unit of length,

$$U = \Omega \mathcal{H} = \Omega \left(C' \left| \frac{\Phi}{\Omega} \right|^{\frac{2}{1+\alpha}} + \frac{1}{2} \left| \frac{\Phi}{\Omega} \right|^2 \right). \quad (3.13)$$

In Fig. 2 the function $U(\Omega)$ is sketched in three cases: $\alpha > 1$, $\alpha = 1$ and $\alpha < 1$.

In the case $\alpha < 1$ the flux lines tend to take as large a volume as possible and Ω will be of the order r^2 where r is the distance between the particles. We get a power law for the potential $V(r)$:

$$V(r) \propto r U(r^2) \propto r^{\frac{3\alpha-1}{1+\alpha}} \quad (\alpha < 1). \quad (3.14)$$

leading to quark confinement if $\alpha > 1/3$.

If $\alpha = 1$ the energy of a flux line will become independent of its width and just proportional to its length. We get

$$V(r) \propto r \quad (\alpha = 1). \quad (3.15)$$

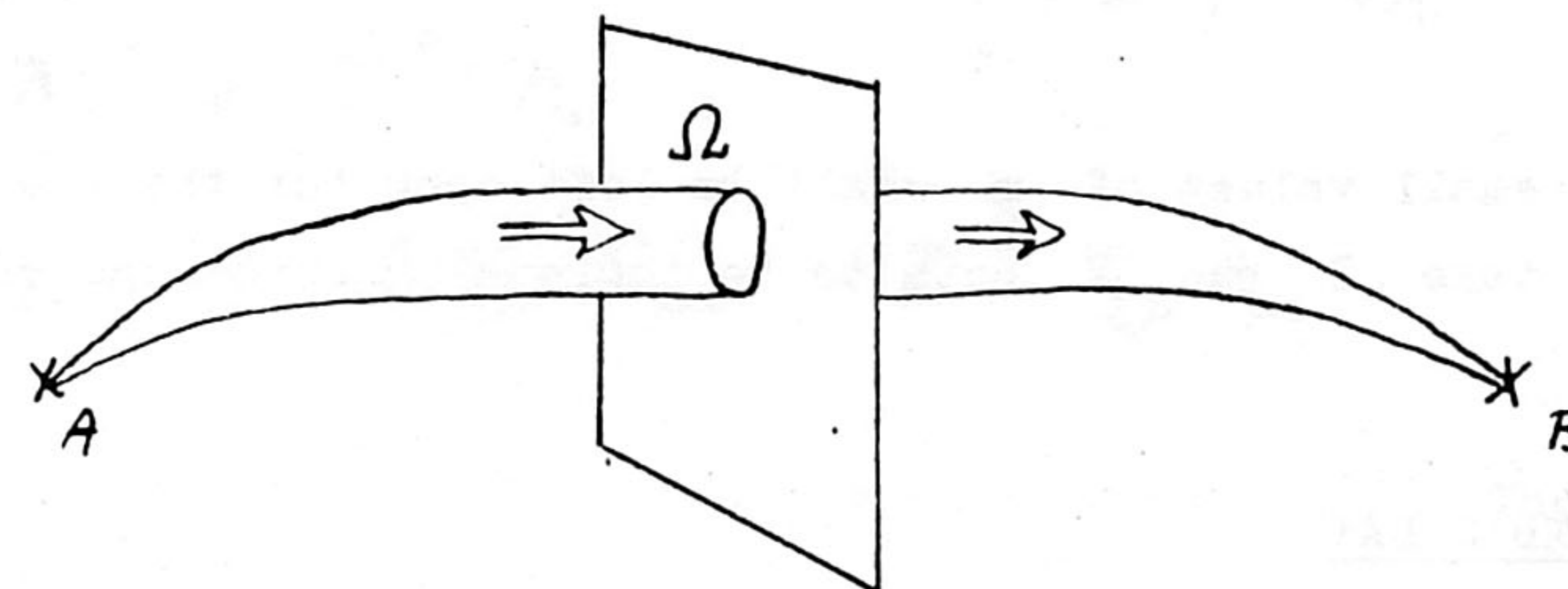


Fig. 1

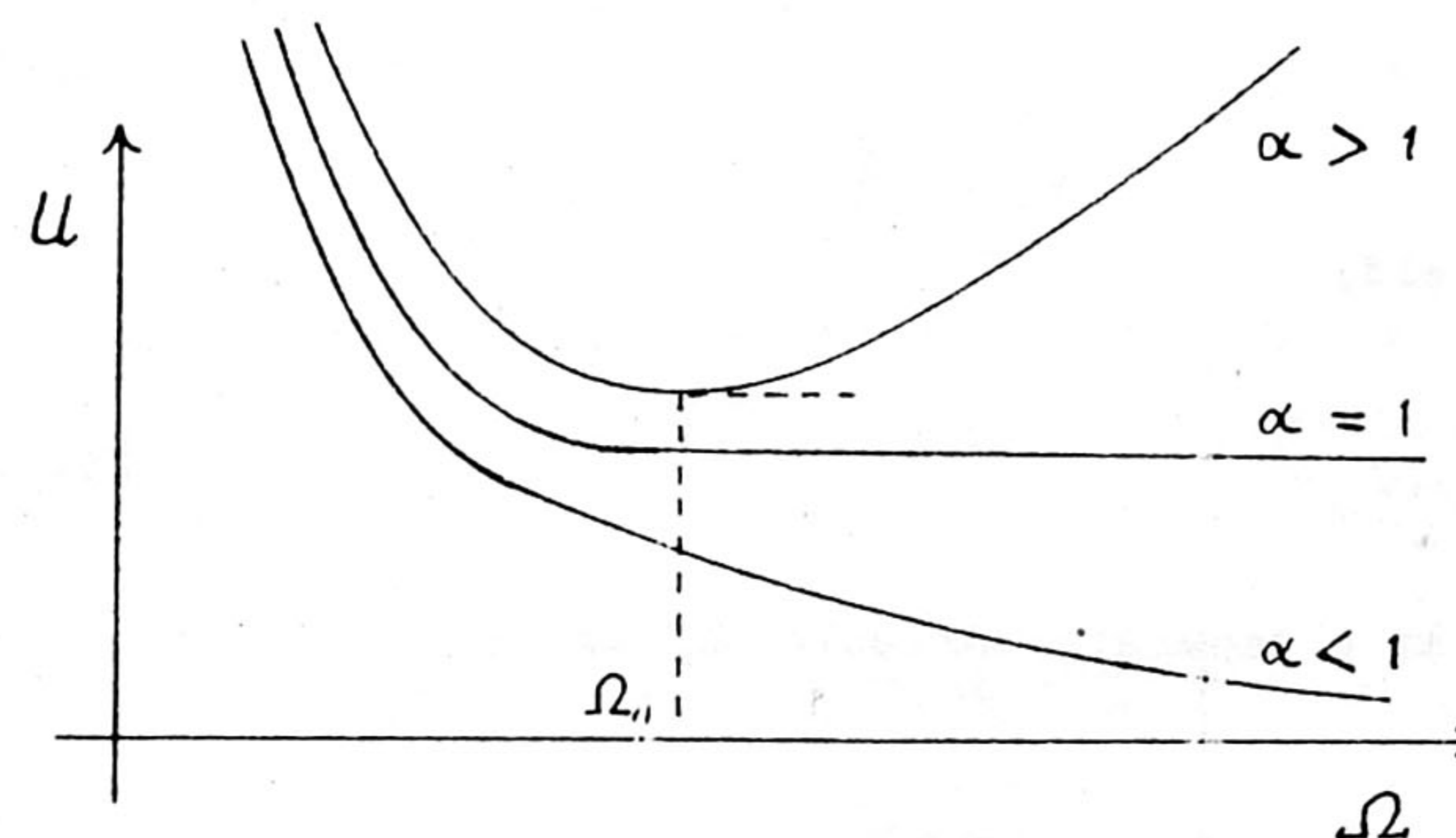


Fig. 2

If $\alpha > 1$ then the flux lines will not spread further than the cross-section Ω_0 where the energy is minimal (Fig. 2). The whole field E will be confined in a tube with a definite width and arbitrary length. These are the electric vortices meant in the beginning. They will behave exactly as the dual string. So here also

$$V(r) \propto r \quad (\alpha > 1) \quad (3.16)$$

The success of the dual models in describing the phenomenology of the strong interactions indicates that most likely $\alpha > 1$.

Let us now turn back to the term $(\partial_1 \varphi)^2$ in Eq. (3.5). It does not change our arguments. But in the case $\alpha > 1$ it is the only term that raises the energy if we try to split the tube. If it would be absent then the field would drop sharply to zero at the edge of a tube. So its task is to make the vortex soft at the edge, and continuous inside.

We observe from the solution that, in fact, a symmetry breaking mechanism takes place: the gauge field term in the Hamiltonian (3.5) forces φ to become non-zero if a vector field is present. In other regions $\varphi = 0$, and there the gauge field cannot penetrate. Thus our theory resembles the bag theory³⁾: inside we have $\varphi \neq 0$, gauge fields present, and outside $\varphi \rightarrow 0$. But we have a soft bag: everything is continuous. The hard bag³⁾ would correspond to neglecting the kinetic term for φ and taking $Z(V)$ to be a step function ($\alpha = \infty$). Charged particles are automatically confined to the bag because of the field they drag along.

4) THE NON-ABELIAN CASE

In the non-Abelian case the induction field $E_{\mu\nu}^a$ satisfies the equation

$$D_{\mu}^{ab} E_{\mu\nu}^b = j_{\nu}^a \quad (4.1)$$

where D_{μ}^{ab} is the covariant derivative. So flux is no longer conserved, and we cannot extend our classical solution to this case: there are no classical, stable vortex lines. Nevertheless we believe that also this theory will have vortex lines if $\alpha > 1$. The argument goes as follows.

Let us take one isospin direction, say the 8 direction in $SU(3)$ space, and consider the electric field $\partial_{\mu} A_{\nu}^8 - \partial_{\nu} A_{\mu}^8$ separately. All other fields, i.e., the charged Fermi particles and the charged gauge vector particles, are described by the source J_{μ} . The difficulty we had in the beginning can now be formulated as follows: given a pair of charged particles with an electric vortex line in between. Then the charged gauge bosons can be created in pairs and thus neutralize the electric field, and the vortex will fall into pieces.

However, the quarks have charges $1/3$, $1/3$ and $-2/3$ with respect to this colour electric field. The gauge bosons, on the other hand, have charges 0, and ± 1 only. In Fig. 3a we depicted what we expect to happen if a pair of gauge particles tends to neutralize a vortex between a single quark-antiquark pair. In Fig. 3b we see that three vortex lines can be eliminated if they are parallel. Only colourless, triality zero states have no vortex lines emerging.

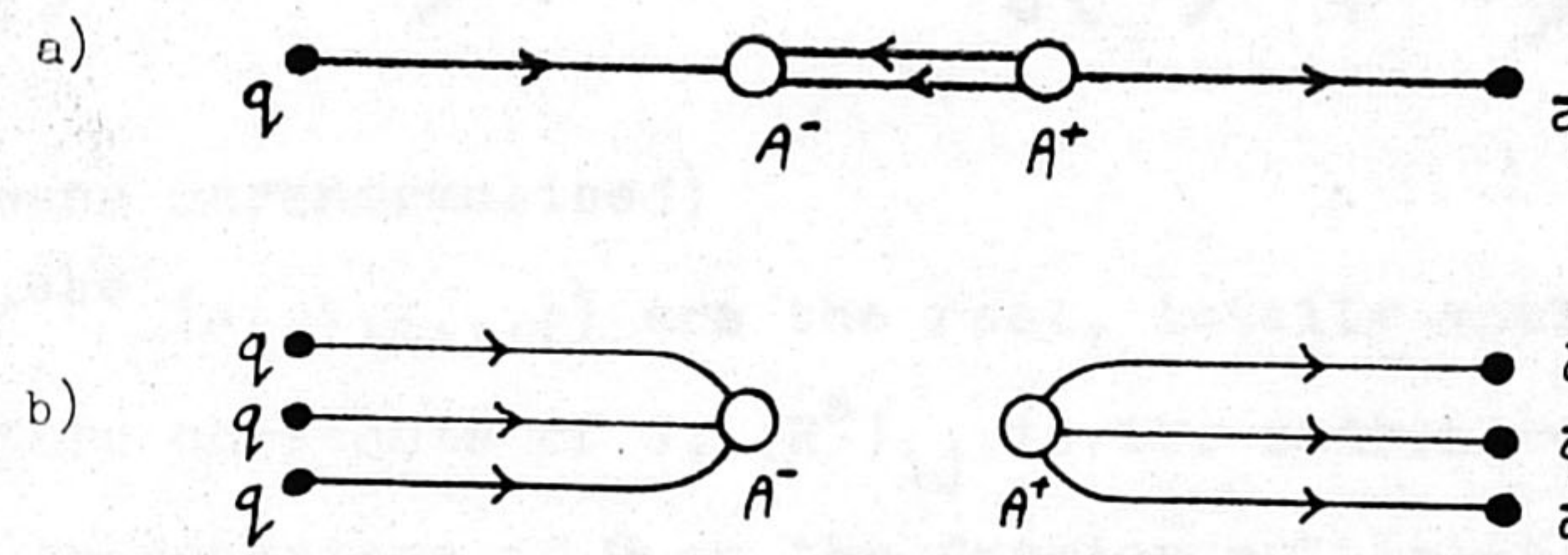


Fig. 3

After completion of these notes the author became aware of the work of Kogut and Susskind⁴⁾, who describe essentially the same model.

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Two Loop Diagrams in Yang Mills Theory

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ABSTRACT

A calculation of the renormalisation constants of the Yang-Mills field to $O(g^4)$ is presented. The function $\beta(g)$ is hence evaluated to $O(g^5)$ and possible implications for gauge theories of the strong interactions discussed.

