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Quantum Black Holes, Firewalls and the Topology of Space-Time

A pedagogical treatment of a new approach

version #3 January 22, 2017
version #4 February 9, 2018
version #5 March 8, 2018

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Introduction

This is the ‘Lecture Notes’ version, and an extension, of my latest papers on black hole microstates. These notes highlight the essentials, such that they should be easier to read and understand. My plan is to keep this updated. If new insights arise, they will be added.

In version v3, a chapter, with figures, on “virtual black holes and space-time foam”, was added at the end. In version v4 further remarks were added, as well as several slides to explain better how some apparent contradictions are to be understood, describing in more detail the generic quantum state of a black hole, and how these evolve.

Version v5: More extended introduction, clarifications, responses to criticism, and finally some new features of the current algebra, in particular the relation between mode numbers such as ℓ and m , and the angular momentum.

These notes must be seen as a personal view of the author. Most importantly, we try to avoid any wild assumptions, as we usually see in the literature. Our claim is that everything here is essentially derived from basic laws that are well-established. Not even String Theory is used (this might change in the future).

These notes are not intended for use in talks (too much information on single “slides”) with apologies for not always following the same notation as used elsewhere.

Black holes are an essential feature of the gravitational force. Without a thorough understanding of their properties one cannot understand quantum gravity. See: G. 't Hooft, [arxiv:1612.08640 \[gr-qc\]](https://arxiv.org/abs/1612.08640) + [references there](#).

String theory could have been a good tool as well, and I don't go as far as saying that string theory would be wrong, but this lecturer does not understand string theory well enough.

The assumption that “particles are pieces of string” is just an assumption. I think we do not need to make such assumptions. If string theory is right, we will be able to derive it from other first principles.

Anyway, I here describe a path towards understanding without dubious assumptions at all. I even avoid AdS/GFT arguments. Just derivations from known principles, such as Quantum Mechanics and General Relativity.

Note, that these theories are also assumptions, but they are much more compelling. Dropping or loosening these assumptions is much more difficult than keeping them as long as we can. No theory is sacred, and some sense of taste for what to keep and what not, is a useful attire for a theoretical physicist.

We claim that topics such as microstates can be understood very well in our theory. The total number of orthonormal microstates was *not* (yet?) derived, but we expect this to be possible in due time.

The picture of quantum black holes given in several publications suggests that it is a chaotic system characterised by a large number of states that are all basically indistinguishable. Ordinary physics seems to be inapplicable since quantum gravity effects in black holes are claimed to be non-perturbative. Thus one of the few approaches adhered to is AdS/CFT, a mapping of a gravitational world in an anti-de Sitter space (AdS) onto a strongly interacting conformal quantum field theory (CFT). To me this appears to be a mapping of one poorly understood system onto another.

Most importantly, in these notes it will be found that quantum gravity in the bulk region up to very near the event horizons can be handled by standard perturbation expansion.

The point is this: all out-going particles will have energies comparable to the inverse of the black hole mass in Planck units. We can keep all in-going particles in that domain as well. Then all perturbations on the black hole's space-time metric will be small.

Only at the horizon itself, one has to take into account the *gravitational back reaction*. Surprisingly perhaps, this entirely non-perturbative feature can be handled, since it happens to be described by linear equations. This brings order in the chaos.

In particular, we emphasise that one can use ordinary coordinates in the bulk region, while at the (past as well as future) event horizons something else happens.

What used to be thought of as chaos is now brought to order. Our system is comparable to a large number of quantum particles in a multi-dimensional rectangular box, with non-rational ratios in the lengths of the edges and the momenta of the particles. If viewed as a gas, it looks chaotic. But it is not. The particles do not interact, so all of their motions, in all directions, are decoupled. Every single particle, in every of the principal directions in the box, is described by a simple, one dimensional (quantum) equation of motion, which is trivial to solve by an undergraduate student. The black hole is just like this.

Due to the fact that we can expand the distribution of in- and out-going matter in *spherical harmonics* we obtain equations that show that all these different spherical harmonic modes completely decouple. Their equations factorise, just as the particles in our toy example, or the quantum modes in the hydrogen atom.

The gravitational back reaction also factorises. The great advantage of this feature is that now we can easily solve the mathematical equations and see what happens. At first sight, it seems that what we do is plainly wrong, and it was criticised as such in my email exchanges with colleagues. But the equations are so simple that one can investigate all alternatives.

It is essential to have Cauchy surfaces, and examine how the data on these surfaces evolve with time. What is the time coordinate? How can we ensure that the Cauchy data evolve through unitary equations? The Cauchy data should not get lost between the crevices of the horizons, which can easily happen while you think you are doing things correctly. So, the spherical wave expansion takes the place of the particle experiments when physicists were working hard to construct what is now known as the Standard Model.

We have no experiments now, so we have to be especially careful and use devices such as spherical wave expansions or whatever else we can put our hands on, to get a clearer picture and to simplify our subject.

To my opinion, such advices is shoud be followed meticulously. See here what you get.

This author however, is still working mostly alone, and yes, I can make mistakes. If you, reader, think I made mistake(s) please do not hesitate to contact g.thooft@uu.nl

The tortoise coordinates

Our prototype is the Schwarzschild black hole. No serious complications are expected when generalized to Kerr-Newman or such. The *extreme* black hole would not serve our purposes, because it is a limiting case, and its horizon is fundamentally different from the more generic black holes.

Consider the Schwarzschild metric:

$$ds^2 = \frac{1}{1 - \frac{2GM}{r}} dr^2 - \left(1 - \frac{2GM}{r}\right) dt^2 + r^2 d\Omega^2 ; \quad \begin{cases} \Omega & \equiv & (\theta, \varphi) , \\ d\Omega & \equiv & (d\theta, \sin\theta d\varphi) . \end{cases}$$

Go to Kruskal-Szekeres coordinates x, y , defined by

$$\begin{aligned} xy &= \left(\frac{r}{2GM} - 1\right) e^{r/2GM} ; \\ y/x &= e^{t/2GM} . \end{aligned}$$

$$ds^2 = \frac{32(GM)^3}{r} e^{-r/2GM} dx dy + r^2 d\Omega^2 .$$

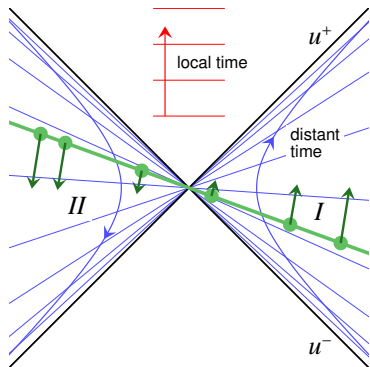
At $r = 2GM$, we have $x = 0$: past event horizon, and
 $y = 0$: future event horizon.

We concentrate on the region close to the horizon: $r \approx 2GM$. There:

$$x = \frac{\sqrt{e/2}}{2GM} u^+ ; \quad y = \frac{\sqrt{e/2}}{2GM} u^- ; \quad 2GM \equiv R$$

$$ds^2 \rightarrow 2du^+ du^- + R^2 d\Omega^2 .$$

$$\text{time } t/4GM = \tau$$



$$u^-(\tau) = u^-(0)e^\tau$$

$$u^+(\tau) = u^+(0)e^{-\tau}$$

As time goes forwards, u^+ approaches the horizon asymptotically;

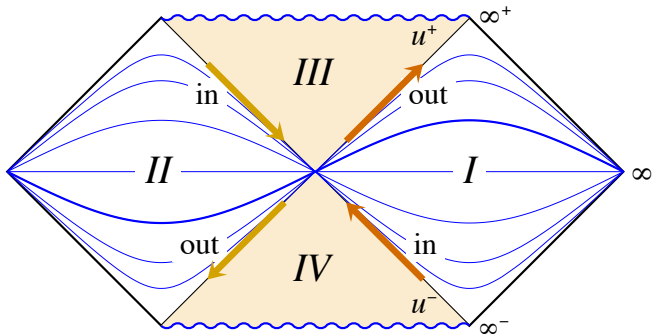
as time goes backwards, u^- approaches the past horizon asymptotically (tortoises).

Particles going in generate wave functions on u^+ , particles going out start as wave functions on u^- .

If we redefine $u^+ \rightarrow f(u^+)$, $u^- \rightarrow g(u^-)$, then the metric keeps the form $ds^2 = 2A(u)du^+du^- + r^2(u)d\Omega^2$. \rightarrow *Map u^\pm on compact domains:*

The Penrose diagram.

For pure Schwarzschild (without matter either responsible for the formation of a black hole, or representing its final decay):



Now, we begin to deviate from standard practice:

This is the “eternal black hole” (it never formed, it will never decay).

We keep it. Why?

We put the “microstates” on a Cauchy surface. As yet, we are only interested in how they evolve for a short amount of time. For the quantum properties of the BH, the distant past, and the distant future, should be irrelevant. They are always irrelevant in quantum physics. Late in the future, or long ago in the past, the states we are looking at now, will be highly entangled states. Not suitable for generating a well-defined, curved space-time. Leave all those particles out.

This picture, however, is to be considered as an intermediate state of our discussion. We use it as a starting point to describe small variations in the black hole state during small periods in time. Later, we will address the black hole’s long-term time dependence more accurately.

Hard and soft particles

First, consider the particles near the horizon(s).

Mass shell: $2p^+p^- + \tilde{p}^2 + \mu^2 = 0$.

Here, \tilde{p} is the transverse part of the momentum, $\mu = \text{mass}$;

$|\tilde{p}| \approx L/R$ and μ are basically constant.

But $p^-(\tau) = p^-(0)e^\tau$, $p^+(\tau) = p^+(0)e^{-\tau}$ (See slide 8)

Define **soft** particles: $|\tilde{p}|, \mu \ll M_{\text{Planck}}$ Negligible effect on space-time curvature.

Define **hard** particles as particles that do cause appreciable space-time curvature.

As $\tau \rightarrow \infty$, $p^+ \rightarrow 0$, $p^- \rightarrow \infty$: all in-particles will become **hard**;

As $\tau \rightarrow -\infty$, $p^- \rightarrow 0$, $p^+ \rightarrow \infty$: all out-particles originally were **hard**.

$|\tilde{p}|$ and μ stay small.

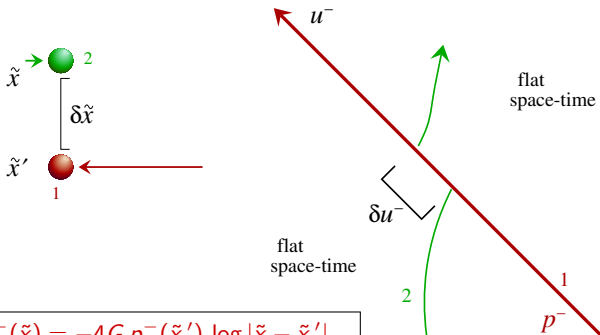
To understand what happens with the evolution at longer time intervals, we have to understand what the hard in- and out- particles do.

As $\tau \gg 1$, the in-particles become hard. Their interactions with other in-particles are negligible (they basically move in parallel orbits), but they do interact with the out-particles. The interaction through QFT forces stay weak, but the gravitational forces make that (early) in-particles interact strongly with (late) out-particles.

The gravitational force between hard and soft particles cannot be ignored; but can be calculated:

The gravitational backreaction:

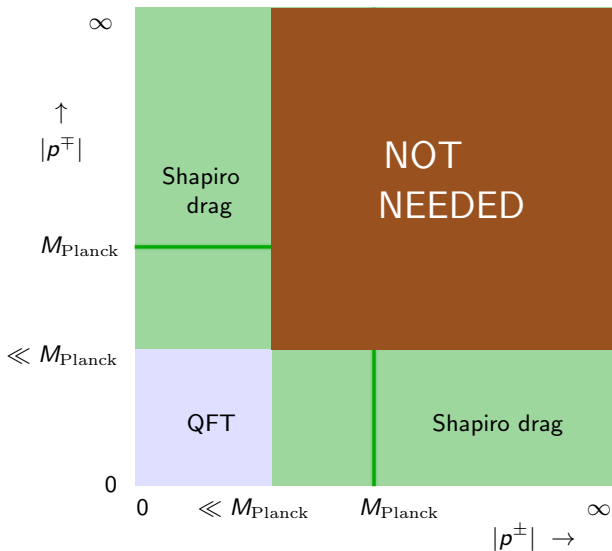
Calculate the *Shapiro time delay* caused by the grav. field of a fast moving particle:
 simply Lorentz boost the field of a particle at rest:



$$\delta u^-(\tilde{x}) = -4G p^-(\tilde{x}') \log |\tilde{x} - \tilde{x}'| .$$

P.C. Aichelburg and R.U. Sexl, *J. Gen. Rel. Grav.* **2** (1971) 303,
 W.B. Bonnor, *Commun. Math. Phys.* **13** (1969) 163,
 T. Dray and G. 't Hooft, *Nucl. Phys.* **B253** (1985) 173.

What we need to know is the interactions between in- and out-particles.



The fact that the mutual interactions between hard particles, at the Planck mass or beyond, will not be needed, is a very important aspect of this work. As you will see.

We start with only soft particles on a Cauchy surface of the Penrose diagram. These will define all quantum microstates of the black hole *at a given time*.

Now, the question is how do these evolve with time.

The soft particles won't stay soft; their longitudinal momenta will quickly explode. To see what happens, calculate δu^- (the Shapiro shift).

An in-particle with momentum p^- at solid angle $\Omega' = (\theta', \varphi')$ causes a shift δu^- at solid angle $\Omega = (\theta, \varphi)$:

$$\delta u^-(\Omega) = 8\pi G f(\Omega, \Omega') p^- ; \quad (1 - \Delta_W) f(\Omega, \Omega') = \delta^2(\Omega, \Omega') .$$

If there are many in-particles:

$$p^-(\Omega) = \sum_i p_i^- \delta^2(\Omega, \Omega_i)$$
$$\delta u^-(\Omega) = 8\pi G \int d^2\Omega' f(\Omega, \Omega') p^-(\Omega') .$$

Our “reference” black hole will have a few soft particles seen by a *local* observer. This quantum state will be close to the Hartle Hawking state (see slide 39).

The *distant* observer will see unending streams of in- and out-particles with given positions u^+ or u^- . Suppose $p^-(\Omega)$ represents *all* in-particles needed to describe any black hole in a given quantum state.

Later, we will see how imploding matter may be described starting with momentum distributions p^- at the distant past.

Then the out-particles will be at positions $u^-(\Omega)$ given by

$$u_{\text{out}}^-(\Omega) = 8\pi G \int d^2\Omega' f(\Omega, \Omega') p_{\text{in}}^-(\Omega') .$$

Notice that *very early* in-going particles have huge values for p^- . Therefore they generate out-particles with huge values of u^- – these must have separated from the black hole long ago! We return to this subject later.

Spherical wave expansion:

$$u^\pm(\Omega) = \sum_{\ell, m} u_{\ell m} Y_{\ell m}(\Omega) , \quad p^\pm(\Omega) = \sum_{\ell, m} p_{\ell m}^\pm Y_{\ell m}(\Omega) ;$$

$$[u^\pm(\Omega), p^\mp(\Omega')] = i\delta^2(\Omega, \Omega') , \quad [u_{\ell m}^\pm, p_{\ell' m'}^\mp] = i\delta_{\ell\ell'}\delta_{mm'} ; \quad (1)$$

$$u_{\text{out}}^- = \frac{8\pi G}{\ell^2 + \ell + 1} p_{\text{in}}^- , \quad u_{\text{in}}^+ = -\frac{8\pi G}{\ell^2 + \ell + 1} p_{\text{out}}^+ ,$$

$p_{\ell m}^\pm$ = total momentum in of in^{out} -particles in (ℓ, m) -wave ,

$u_{\ell m}^\pm$ = (ℓ, m) -component of c.m. position of in^{out} -particles .

Because we have linear equations, all different ℓ, m waves decouple, and for one (ℓ, m) -mode we have just the variables u^\pm and p^\pm . They represent only one independent coordinate u^+ , with $p^- = -i\partial/\partial u^+$.

The dynamics completely factorises in (ℓ, m) spherical harmonics – probably this remains true in the harmonics of a Kerr black hole, but that generalization is not considered here.

3 more steps to be taken:

1. The momentum density $p^\pm(\Omega)$ for every quantum state in a QFT is well-defined.

However, our evolution law will only be unitary if we can find the QFT quantum state back from the momentum density. That's difficult.

Use vertex insertions as in string theory?

2. What is the physical interpretation of region II ?

We demand *locality*. This means that commutators of space-like separated operators should vanish. Then, since in-going signals in region I produce signals in the out-going objects in region II, we must guarantee that I and II are not space-like separated.

Postulate that region II refers to the same black hole as region I, but *not at the same solid angle* $\Omega = (\theta, \varphi)$. Only one possibility:

The antipodal identification: $\Omega \rightarrow \tilde{\Omega} = (\pi - \theta, \varphi + \pi)$

3. The “firewalls”. Soft particles become hard particles. Must be ‘removed’.

Obvious suggestion: all information carried by the in- particles is now present in the out-particles. The in-particles are redundant (“quantum clones”). Leave the hard ones out. Hilbert space is completely specified by the coordinates $u^-(\Omega)$ of the out-particles. Note: as soon as $|u^-| > L_{\text{Planck}}$, these out-particles are soft.

This is the “firewall transformation”; it removes firewalls.

The basic, explicit, calculation

The algebra (1), slide 16, generates the scattering matrix, by giving us the *boundary condition* that replaces $|\text{in}\rangle$ -states by $|\text{out}\rangle$ -states. This boundary condition *replaces the old brick wall model* and it is embarrassing easy to derive

(The catch will be exposed later) .

All of this is NOT a model, or a theory, or an assumption . . .

Apart from the most basic assumption of unitary evolution,
this is nothing more than applying GR and quantum mechanics !

At some points in the following derivations, it may seem that choices were made as to how to continue, such as the eternal BH Penrose diagram and the antipodal identification, and some of my choices are being criticised as they may seem illogical. I emphasize however that they will all be vindicated by the final result, a unitary evolution law. There is no other way to obtain that. None of the other approaches in the literature that I have seen produce such explicit results.

Much of the work described here has its roots in my work of 1986 and the early 1990s, particularly the pioneering ideas collected in the paper on the "Scattering Matrix Ansatz". The fact that I had to discover the new alleys described here myself, suggests that those early papers may not have received enough attention. This may also hold for the new papers.

Let there be two operators, u and p , obeying the commutator equation

$$[u, p] = i, \quad \text{so that} \quad \langle u|p\rangle = \frac{1}{\sqrt{2\pi}} e^{ipu}.$$

and a wave function $|\psi\rangle$, defined by $\psi(u) \equiv \langle u|\psi\rangle$. Its Fourier transform is

$$\hat{\psi}(p) \equiv \langle p|\psi\rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} du e^{-ipu} \psi(u).$$

Now introduce **tortoise coordinates**, and split both u and p in a **positive part** and a **negative part**:

$$u \equiv \sigma_u e^{\varrho_u}, \quad p = \sigma_p e^{\varrho_p}; \quad \sigma_u = \pm 1, \quad \sigma_p = \pm 1, \quad \text{and}$$

$$\tilde{\psi}_{\sigma_u}(\varrho_u) \equiv e^{\frac{1}{2}\varrho_u} \psi(\sigma_u e^{\varrho_u}), \quad \tilde{\psi}_{\sigma_p}(\varrho_p) \equiv e^{\frac{1}{2}\varrho_p} \hat{\psi}(\sigma_p e^{\varrho_p});$$

normalisation requires:

$$|\psi|^2 = \sum_{\sigma_u=\pm} \int_{-\infty}^{\infty} d\varrho_u |\tilde{\psi}_{\sigma_u}(\varrho_u)|^2 = \sum_{\sigma_p=\pm} \int_{-\infty}^{\infty} d\varrho_p |\tilde{\psi}_{\sigma_p}(\varrho_p)|^2. \tag{2}$$

What is the Fourier transform in these tortoise coordinates?

$$\tilde{\psi}_{\sigma_p}(\varrho_p) = \sum_{\sigma_u=\pm 1} \int_{-\infty}^{\infty} d\varrho_u K_{\sigma_u\sigma_p}(\varrho_u + \varrho_p) \tilde{\psi}_{\sigma_u}(\varrho_u) ,$$

$$\text{with } K_{\sigma}(\varrho) \equiv \frac{1}{\sqrt{2\pi}} e^{\frac{1}{2}\varrho} e^{-i\sigma} e^{\varrho} .$$

Notice the symmetry under $\varrho_u \rightarrow \varrho_u + \lambda$, $\varrho_p \rightarrow \varrho_p - \lambda$, which is simply the symmetry $u \rightarrow u e^{\lambda}$, $p \rightarrow p e^{-\lambda}$, a property of the Fourier transform, and an invariance of our algebra.

We now use this symmetry to write plane waves:

$$\tilde{\psi}_{\sigma_u}(\varrho_u) \equiv \check{\psi}_{\sigma_u}(\kappa) e^{-i\kappa\varrho_u} \quad \text{and} \quad \tilde{\psi}_{\sigma_p}(\varrho_p) \equiv \check{\psi}_{\sigma_p}(\kappa) e^{i\kappa\varrho_p} \quad \text{with}$$

$$\check{\psi}_{\sigma_p}(\kappa) = \sum_{\sigma_u=\pm 1} F_{\sigma_u\sigma_p}(\kappa) \check{\psi}_{\sigma_u}(\kappa) ; \quad F_{\sigma}(\kappa) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} K_{\sigma}(\varrho) e^{-i\kappa\varrho} d\varrho .$$

Thus, we see left-going waves produce right-going waves. One finds (just do the integral):

$$F_\sigma(\kappa) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{dy}{y} y^{\frac{1}{2}-i\kappa} e^{-i\sigma y} = \frac{1}{\sqrt{2\pi}} \Gamma\left(\frac{1}{2} - i\kappa\right) e^{-\frac{i\sigma\pi}{4} - \frac{\pi}{2}\kappa\sigma} .$$

Matrix $\begin{pmatrix} F_+ & F_- \\ F_- & F_+ \end{pmatrix}$ is unitary: $F_+ F_-^* = -F_- F_+^*$ and $|F_+|^2 + |F_-|^2 = 1$.

Look at how our soft particle wave functions evolve with time τ , slide # 8 or 11.

Their Hamiltonian is the dilaton operator. κ is the energy:

$$H = -\frac{1}{2}(u^+ p^- + p^- u^+) = \frac{1}{2}(u^- p^+ + p^+ u^-) =$$

$$i \frac{\partial}{\partial \varrho_{u^+}} = -i \frac{\partial}{\partial \varrho_{u^-}} = -i \frac{\partial}{\partial \varrho_{p^-}} = i \frac{\partial}{\partial \varrho_{p^+}} = \kappa ,$$

(if we apply the previous slide to the coordinates u^+ and p^-).

The scattering matrix

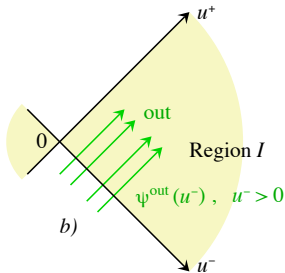
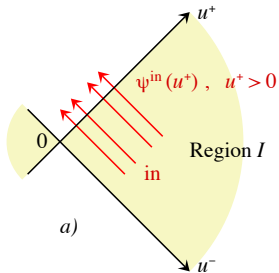
Add the scale factor $\frac{8\pi G}{\ell^2 + \ell + 1}$, to get, if $u^\pm = \sigma_\pm e^{\rho^\pm}$,

$$\psi_{\sigma_+}^{\text{in}} e^{-i\kappa\rho^+} \rightarrow \psi_{\sigma_-}^{\text{out}} e^{i\kappa\rho^-}, \quad (3)$$

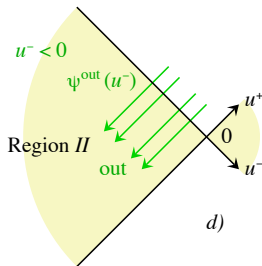
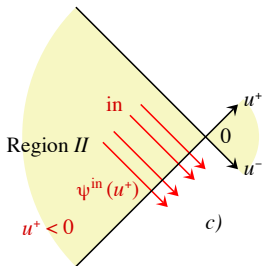
$$\psi_{\sigma_-}^{\text{out}} = \sum_{\sigma_+} F_{\sigma_+\sigma_-}(\kappa) e^{-i\kappa \log(8\pi G/(\ell^2 + \ell + 1))} \psi_{\sigma_+}^{\text{in}}$$

These equations generate the contributions to the scattering matrix from all (ℓ, m) sectors of the system, where $|m| \leq \ell$. At every (ℓ, m) , we have a contribution to the position operators $u^\pm(\theta, \varphi)$ and momentum operators $p^\pm(\theta, \varphi)$ proportional to the partial wave function $Y_{\ell m}(\theta, \varphi)$. The signs of $u^\pm(\theta, \varphi)$ tell us whether we are in region *I* or region *II*. The signs of $p^\pm(\theta, \varphi)$ tell us whether we added or subtracted a particle from region *I* or region *II*.

- a) Wave functions $\psi(u^+)$ of the in-particles live in region I, therefore $u^+ > 0$.
 b) Out-particles in region I have $\psi(u^-)$ with $u^- > 0$.



The physical picture



- c, d) In region II, the in-particles have $u^+ < 0$ and the out-particles $u^- < 0$.

Note that the in-particles will never get the opportunity to become truly hard particles.

Eq. (3) is to be seen as a “soft wall”-boundary condition near the origin of the Penrose diagram. Wave functions going in are reflected as wave functions going out. These again emerge as soft particles.

Thus, there is no firewall, ever.

In the previous slide, the total of the in-particles in regions *I* and *II* are transformed (basically just a Fourier transform) into out-particles in the same two regions.

Note that the regions *III* and *IV* in the Penrose diagram (see slide 9) never play much of a role, even if an observer falling in region *III* would want to assure us that (s)he is still alive.

These regions are best to be seen as lying somewhere on the time-line *where time t is somewhere beyond infinity* (thus a mere repetition of the degrees of freedom we have seen before)

The catch?

Our “boundary condition at the origin” is in terms of momentum distributions $p^\pm(\theta, \varphi)$ and center-of-mass positions $u^\pm(\theta, \varphi)$ only.

But we would need the quantum wave functions as elements of Fock space (which would be specified by positions or momenta, but also by other quantum numbers!)

Such mapping should be unitary in Hilbert space. We are not certain that this can be done, but it is natural to look at how it is done in string theory. Our momentum distributions are like vertex insertions, although they are on the horizon instead of the string world sheet.

Another catch: it seems as if we get too many microstates now. Presumably we are describing black holes with variable sizes. There may well be a cut-off at some large ℓ values depending on M . Further to be investigated.

The antipodal identification

Regions I and II of the Penrose diagram are exact copies of one another. Often, it was thought that region II describes something like the 'inside' of a black hole. That cannot be right, since region II , like region I , has asymptotic regions. Hawking suggested that region II might be some other black hole, in an other universe, or far away in our universe. However, our 2×2 scattering matrix implies that the two regions are in contact with each other quantum mechanically. In ordinary branches of physics, such long-distance communication never takes place, and I don't think theories with such features make any sense.

It is far more natural to assume that region II describes the same black hole as region I . It must then represent some other part of the same black hole. Which other part? The local geometry stays the same, while the square of this $O(3)$ operator must be the identity.

There is exactly one possibility: This is the $O(3)$ operator $-\mathbb{I}$, which is:
the antipodal mapping.

So this is *not* an element of $SO(3)$, which will give us constraints on the CP symmetry structure of the Standard Model interactions (I thank J. Maldacena for a discussion of this point).

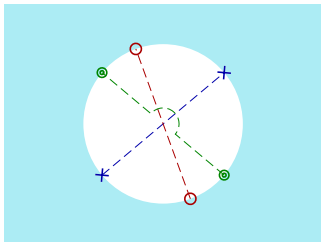
Antipodal *identification* only holds for the central point (origin) of the Penrose diagram. Regions *I* and *II* are different regions of the universe. But by relating region *II* to region *I* by demanding that the angular coordinates are antipodes, means that now the mapping from Schwarzschild coordinates to Kruskal Szekeres coordinates is one-to-one. This now turns out to be an essential property of our coordinate transformations. Thus, we arrive at a new restriction for all general coordinate transformations:

In applying general coordinate transformations for quantized fields on a curved space-time background, to use them as a valid model for a physical quantum system, one must demand that the following constraint hold: the mapping must be one-to-one and differentiable.

The emergence of a non-trivial topology needs not be completely absurd, as long as no signals can be sent around. We think that this is the case at hand here. It is the absence of singularities in the physical domain of space-time that we must demand.

Black emptiness: blue regions are the accessible part of space-time; dotted lines indicate identification.

The white sphere within is *not* part of space-time. Call it a 'vacuole'.



At given time t , the black hole is a 3-dimensional vacuole. The entire life cycle of a black hole is a vacuole in 4-d Minkowski space-time.

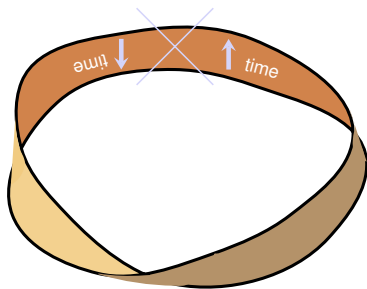
Space coordinates change sign at the identified points

– *and also time changes sign*

(Note: time stands still at the horizon itself).

Not that all u^\pm and p^\pm coordinates are odd when switching between antipodes. Therefore, only odd ℓ contribute in the spherical harmonic expansion.

A timelike Möbius strip



Draw a spacelike closed curve:

Begin on the horizon at a point

$$r_0 = 2GM, \quad t_0 = 0, \quad (\theta_0, \varphi_0) .$$

Move to larger r values, then
travel to the antipode:

$$r_0 = 2GM, \quad t_0 = 0, \quad (\pi - \theta_0, \varphi_0 + \pi) .$$

You arrived at the same point,
so the (space-like) curve is closed.

Now look at the environment $\{dx\}$ of this curve. Continuously transport dx around the curve. The identification at the horizon demands

$$dx \leftrightarrow -dx, \quad dt \leftrightarrow -dt, .$$

So this is a Möbius strip, in particular in the time direction.

Note that it makes a *CPT* inversion when going around the loop.

There are no direct contradictions, but take in mind that
the local Hamiltonian density switches sign as well.

This is not true for the *total* Hamiltonian adopted by distant observers. locally, near the horizon, this is the *dilaton operator*. That operator leaves regions *I* and *II* invariant, and does not flip sign along the loop. Also, the boundary condition, our “scattering matrix”, leaves this Hamiltonian invariant.

The *S*-matrix commutes with the Hamiltonian.

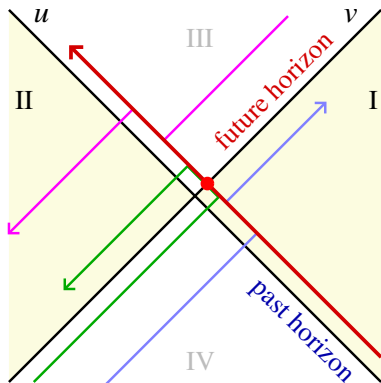
Since the antipodal identification switches either *P* or *CP*, direct *CP* violation cannot be accommodated for. We conclude that the antipodal identification will require that *P* or *CP* in the Standard Model must be *spontaneously* broken.

There are numerous treatises in the literature claiming solutions to the black hole information paradox, and about as many publications that dismiss these claims.

This author dismisses all claims, from both sides, that either ignore the gravitational back reaction of quantised excitations, or ignore the antipodal identification of points on the horizon – meaning that the horizon is a *projective 2-sphere*.

Critical questions.

The effect of the gravitational footprint of an in-going particle (red line to upper left) onto the out-going particles (other coloured lines) is correctly accommodated for in this work. But there is an oddity, see our picture here of the allowed regions *I* and *II* and the unphysical regions *III* and *IV*.



A bizarre sign switch

The gravitational footprint moves the *pure states* of all out-particles combined in the light cone direction by an amount $\delta u^-(\theta, \varphi)$ in the same direction as the in-momentum $\delta p^-(\theta', \varphi')$.

This is a unitary transformation in the space of the combined wave functions only if all out-particles move by the same amount in the same direction, as is drawn here.

But this led to an apparently valid criticism:

This configuration does not appear to correspond to a valid solution of Einstein's equations for the local observer. What are we doing wrong?

It appears that this corresponds to a solution of Einstein's equations only if the momentum δp^- of the hard, in-particle flips its sign while passing through the future event horizon. At the same time, of course, the arrow of time switches sign.

Moreover, *those out-particles that are dragged right across the horizon (green line), do not seem to follow a geodesic at all, as regarded by the local observer.* Yet, of course, with our definition of the arrow of time, we have no choice, the particle has to continue its way as shown. What should a local observer say?

For the local observer, the diagram here does not show a classical solution, but it shows the action of an operator. The operator creates an extra in-particle (red line) that, in all states in regions I and II , shifts the geodesics of all out-particles by the same amount, in the same direction.

But if we represent the result of this operator as what happens later in time, then the fact that time runs backwards in region II leads to the impression that a discontinuity occurs on the other (future) horizon.

Operators should not be regarded as effects that agree with evolution equations; they represent what happens to a system if we make a change on the Cauchy surface, at some moment in time.

The change brought about here, has the effect of a positive δp^- particle in region I and at the same time the effect of a negative δp^- particle (an annihilated particle) in region II . Note that the operators u^\pm and p^\pm on the horizons all switch sign when passing from region I to II , since they are each other's antipodes, on the horizon.

The BMS group.

In a development parallel to our work here, it has been argued that “conservation of information” for black holes can be understood by using the BMS group. It was claimed that this group generates an infinite class of conserved charges that must be held responsible for safeguarding the information processed by the black hole. These charges are associated to supertranslations, and as such must take the form of (θ, φ) -dependent momenta in the light cone direction. To the present author, the physical interpretation of these arguments is less transparent. The arguments seem to be very formal, and thus suspect. I would like to see how the BMS group generates a unitary S -matrix.

The black hole mass

Other question often asked: how does the mass of the black hole respond on the absorbed and emitted particles?

This is actually very easy. The energy (the sum of the numbers κ at every value of (ℓ, m)), is exactly conserved. To see how this works, arrange all in- and out- particles into three classes:

- (i) The particles coming in when still far away from the future horizon, which are the firewall transforms of the out-particles that are still very close to the past event horizon, ready to emerge at some later era,
- (ii) The soft particles in the parts of regions *I* and *II* that are in the neighborhood, but not too close the any of the horizons, and
- (iii) The particles moving out at a safe distance from the black hole, they are the firewall transforms of the particles that have arrived at a very close distance from the future event horizon.

The total energy, black hole plus these particles, is strictly conserved. But the particles of group (i) (the very late particles) and group (iii) (the very early particles) are so far away that we should not count those as part of the black hole mass/energy. The particles of group (ii) however, are so close to the horizon that these must be considered as part of the black hole.

Perhaps we have to postulate that the entire black hole mass consists of the contributions of these particles, but this we leave for later considerations.

Clearly, this mass is time dependent, as the distinction of a particle's classes is time dependent.

Note however, that a black hole is always surrounded by the cloud of entangled Hartle-Hawking particles.

The particles discussed above are the ones observed by *local observers near the horizon*.

The complete set of black hole states

When a local observer sees a vacuum, it means that the black hole is in the *Hartle-Hawking state*,

$$|\Omega\rangle = C \sum_{E,n} e^{-\frac{1}{2}\beta E} |E, n\rangle_I |E, n\rangle_{II} .$$

It is seen as a *vacuum* $|\Omega\rangle$ for the local observer, but contains states $|E, n\rangle$ in regions *I* and *II* for the global observer. C is a normalization constant, and n represents all quantum numbers other than the energies E of the particle states – as seen by the global observer.

This expression is obtained when we define creation operators $a^\dagger_{E,n}$ and annihilation operators $a_{E,n}$ as Fourier transforms of the time dependence of the local field variables $\phi(\vec{x}, t)$. General coordinate transformations are assumed to act locally on these fields $\phi(\vec{x}, t)$. The operators a^\dagger are the positive frequency Fourier coefficients and a are the negative frequency coefficients. Since time means something different for the different observers, the transformation from global observables to local ones involves a Bogolyubov transformation between a and a^\dagger , and since the vacuum state $|\Omega\rangle$ is assumed to be the vacuum for the local observer, so that for him, $a_{\kappa,n}|\Omega\rangle = 0$, this is not the vacuum $|\Omega\rangle_{\text{glob}}$ for the global observer. The straightforward calculation gives the above expression. $E = \sum_{\ell,m} \kappa_{\ell,m}$; $\beta = 1/kT_{\text{Hawking}} = 1/2\pi$ in the units defined by our rescaled time parameter τ .

In previous slide, $|\Omega\rangle$ describes the in- and out-particles for the distant observer being in an entangled state (the antipodes are entangled). The observer cannot change that state for the out-particles, but can *choose* the in-particles any way he likes, both in region *I* and in regio *II*. But since our *S*-matrix relates the out-particles to the earlier in-particles, eventually the observer can choose his state any way he likes. Thus we can consider states $|E_I, n_I\rangle|E_{II}, n_{II}\rangle$ where the parameters can be anything.

The generic quantum state of a black hole can be considered in a basis where the states are not entangled.

Important point: as time τ (for distant observer) proceeds towards ∞ , the out-particles move further out, causing no further difficulties, but the in-particles generate more and more momentum, so the firewall transformation must be used to replace them by out-particles. Where exactly will the point be that this transformation replaces states by softer states, and which states will be left?

Consider a given (ℓ, m) spherical mode. Let $\lambda^2 = \frac{8\pi G}{\ell^2 + \ell + 1}$. Then choose new coordinates $[x, p] = i$ and

$$p \equiv \lambda p_{\text{in}}^- \quad \text{and} \quad x \equiv u^+ / \lambda, \quad \text{so that} \quad \lambda p_{\text{out}}^+ = -x \quad \text{and} \quad u_{\text{out}}^- / \lambda = p$$

We have $[x, p] = i$, that is, QM in one dimension. Now search for new variables such that, in these new variables, we just have a simple bounce near the horizon. For example, we can express the wave functions in the form of Gaussians. The Fourier transforms of Gaussians are again Gaussians. Narrow Gaussians in x space are wide Gaussians in p space and vice versa. So we plan to flip from x space to p space such that the widths of the Gaussians in all u space are the largest. Thus our wave functions are made as soft as possible. Goes as follows. Consider a symmetric wave function. In p space:

$$\phi(p) = \phi(-p) \quad \text{and} \quad \psi(x) = \psi(-x)$$

(antisymmetric wave functions can be handled similarly). Write this wave function as a superposition of complex Gaussians:

$$\phi(p) = \int_0^\infty d\alpha \alpha^{-\frac{3}{4}} \varrho(\alpha) e^{\frac{1}{2}i\alpha p^2} \quad \text{then if}$$

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty dp \phi(p) e^{ipx}, \quad \text{one derives}$$

$$\psi(x) = \int_0^\infty d\beta \beta^{-\frac{3}{4}} \varrho(1/\beta) \frac{1+i}{\sqrt{2}} e^{-\frac{1}{2}i\beta x^2}, \quad \text{and a solution is given by:}$$

$$\varrho(\alpha) = \frac{\alpha^{3/4}}{2\pi} \int_0^\infty p dp \phi(p) e^{-\frac{1}{2}i\alpha p^2} = \frac{\alpha^{-3/4}(1-i)}{2\pi\sqrt{2}} \int_0^\infty x dx \psi(x) e^{ix^2/2\alpha}$$

By scaling our x and p variables with the quantity λ , we achieved that the grav. backreaction just transforms p variables into x variables and back. Using the expressions of the previous slide, we can now simply keep the contributions from the variables α only when $\alpha \geq 1$ and $\beta \leq 1$. The firewall transformation is performed when $\alpha = \beta = 1$.

Note that our Hamiltonian is the dilation operator, so that not only x and p simply scale as time proceeds, but α and β do the same thing.

Thus, all our wave functions are defined at $\alpha > 1$ for the p variables so that p remains small while $\beta < 1$ ensures that contributions from small x are suppressed.

Our particles stay soft.

This constitutes the complete "Fock space" for the black hole.

Note that, in this scheme, we can describe a time dependent black hole. Our only limitation today is that we only allow relatively light particles enter and leave from the black hole.

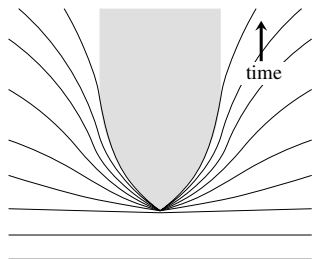
Note, that we have not been able to reproduce exactly the total number of microstates. How exactly to reproduce the entropy this way is not (yet?) understood. We do however have Hawking radiation at the intensity expected for the usual Hawking expression, and this should suffice to solve this problem. But this has not (yet?) been done.

The moment of collapse and the moment of the final explosion

How does the antipodal identification get started when matter collapses into a black hole?

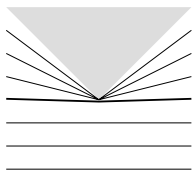
How does it end after the hole ended its life through evaporation?

This is a hard problem because at those moments the background metric is non stationary. Energy and momentum are ill defined.

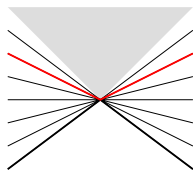


1) Realistic coordinates, with equal-time lines

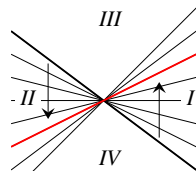
Penrose diagrams:



2) situation at $t = 0$



3) situation at $t \gg 0$ showing Cauchy surfaces



4) antipodal identification

To obtain straight Cauchy surfaces, we need to invert time in region II, the antipode.

and now the grav. shift can be accounted for

Previous slide:

We describe here a metric with time-dependent source. the above was a suggestion for taking the back reaction into account, also during collapse. From picture (4) onwards, we propose to work just as in the “eternal” case.

When inspected in picture (1), we see that

The antipodal identification also implies an inversion in the time direction.

The entire inside region (grey) is removed,

so that only pure particle states can arise, also with gravitational back reaction. As in the eternal case, we replace the energetic in-particles at the past horizon by their footprints: the soft out-particles at the future event horizon.

The time-reverse of this, the final evaporation of the black hole, must be handled in exactly the same way.

More on the horizon algebra.

In my publications between ~ 1985 and ~ 1996 , it was emphasised that the amplitudes produced by the mechanism of the gravitational backreaction, are strongly related to those of string theory. Perhaps it is string theory, although it seems more like a string theory with purely imaginary α' . The mathematical similarity originates simply from the fact that, as in string theory, we end up with algebraic relations of particle coordinates on a two-dimensional surface.

The string world sheet is replaced by the point of intersection of the future and the past event horizons. In- and out-going particles resemble the vertex insertions on a closed-string world sheet. Thus, one might expect an algebra akin to the Virasoro algebra. Yet I want to derive the black hole horizon algebra with utmost care, since the physical circumstances are quite different. If there is a relation, we need to find it, but also focus on differences.

In what follows we start producing some commutation rules, but calculations were only just begun.

For the time being, replace the spherical horizon(s) by an infinite, flat Rindler space-time. Replace the spherical harmonics $Y_{\ell m}(\Omega)$ by plane waves $e^{i\tilde{k}\cdot\tilde{x}}$, where the flat, transverse coordinates \tilde{x} replace $\Omega = (\theta, \varphi)$ and \tilde{k} replaces the “wave numbers” (ℓ, m) .

We consider the case $\tilde{k}^2 \ll M_{\text{Planck}}^2$ (This may later be relaxed).

Write the blue equations in slide 16 as the Fourier transforms

$$u^{\pm}(\tilde{x}) = \frac{1}{2\pi} \int d^2\tilde{k} e^{\pm i\tilde{k}\cdot\tilde{x}} \hat{u}^{\pm}(\tilde{k}),$$

$$p^{\pm}(\tilde{x}) = \frac{1}{2\pi} \int d^2\tilde{k} e^{\pm i\tilde{k}\cdot\tilde{x}} \hat{p}^{\pm}(\tilde{k}),$$

2-vectors are indicated by a tilde ($\tilde{}$). The inverse equations are

$$\hat{u}^{\pm}(\tilde{k}) = \frac{1}{2\pi} \int d^2\tilde{x} e^{\mp i\tilde{k}\cdot\tilde{x}} \hat{u}^{\pm}(\tilde{x}),$$

$$\hat{p}^{\pm}(\tilde{k}) = \frac{1}{2\pi} \int d^2\tilde{x} e^{\mp i\tilde{k}\cdot\tilde{x}} \hat{p}^{\pm}(\tilde{x}),$$

Signs \pm are chosen for later convenience. We omit the carets ($\hat{}$) as their locations should be clear from the notation.

$u^{\pm}(\tilde{x})$ is gravitational displacement at \tilde{x} caused by *all* p_i^{\pm} :

Important: the *momenta* p^\pm are now handled as distributions, built up from deltas where particles enter;
 the positions $u^\pm(\tilde{x})$ just mark the location of particles with respect to their horizons. Labelling the particles by an index i , we take:

$$p^\pm(\tilde{x}) \equiv \sum_i p_i^\pm \delta^2(\tilde{x} - \tilde{x}_i) \quad ; \quad u^\pm(\tilde{x}_i) = u_i^\pm ,$$

$$u^-(\tilde{x}) = 8\pi G \sum_i f(\tilde{x} - \tilde{x}_i) p_i^- = 8\pi G \int d^2\tilde{x}' f(\tilde{x} - \tilde{x}') p^-(\tilde{x}') ;$$

Green function obeys: $\tilde{\partial}^2 f(\tilde{z}) = -\delta^2(\tilde{z}) .$

Our commutation rules are now:

$$[u_i^\pm, u_j^\pm] = [u_i^\pm, p_j^\pm] = [p_i^\pm, p_j^\pm] = 0 , \quad [u_i^\pm, p_j^\mp] = i \delta_{ij} ,$$

and the algebra turns into:

$$u^-(\tilde{k}) = \frac{8\pi G}{\tilde{k}^2} p^-(\tilde{k}) \quad ; \quad [u^\pm(\tilde{x}'), p^\mp(\tilde{x})] = i \delta^2(\tilde{x}' - \tilde{x}) .$$

$$[u^\pm(\tilde{k}), p^\mp(\tilde{k}')] = i \delta^2(\tilde{k} - \tilde{k}') \quad ; \quad [p^-(\tilde{k}), p^+(\tilde{k}')] = \frac{i\tilde{k}^2}{8\pi G} \delta^2(\tilde{k} - \tilde{k}') .$$

$$u^+(\tilde{k}) = -\frac{8\pi G}{\tilde{k}^2} p^+(\tilde{k}) \quad ; \quad \tilde{\partial}^2 u^\pm(\tilde{x}) = \pm 8\pi G p^\pm(\tilde{x}) .$$

Note Newton's G in denominator. This algebra is "non-perturbative" in gravity.

For future need:

$$[u^\pm(\tilde{x}), p^\mp(\tilde{k})] = \frac{i}{2\pi} e^{\pm i\tilde{k}\cdot\tilde{x}}, \quad [u^\pm(\tilde{k}), p^\mp(\tilde{x})] = \frac{i}{2\pi} e^{\pm i\tilde{k}\cdot\tilde{x}}.$$

Because of the delta functions in the previous slide, the modes for the different harmonic waves \tilde{k} all decouple. Formally, a generic quantum state (replacing Fock space for the Standard Model + perturbative gravity) is the product of states: $|u^+(\tilde{k})\rangle$ for all allowed values of \tilde{k} .

Equivalently, one can label these states as $|u^-(\tilde{k})\rangle$ or $|p^+(\tilde{k})\rangle$ or $|p^-(\tilde{k})\rangle$. These modes are like the one-particle states in QFT, but not quite, they label *matter waves*. Even so, we just have single wave functions $\psi(u^+)$ for a single coordinate u^+ , at each value for \tilde{k} .

The wave functions $\psi(p^-)$, one for each value of \tilde{k} , are the Fourier transforms of $\psi(u^+)$, see slide 20. $\psi(p^\pm)$ is the same as $\psi(u^\pm)$, after scaling the variables p^\pm to u^\pm , see previous slide.

It is important not to confuse the *wave modes* indicated by their *mode number* \tilde{k} with the actual momentum this state might have. This will be indicated by the operator $\tilde{P} \equiv \int d^2\tilde{k} \tilde{p}(\tilde{k})$, which equals $\sum_i \tilde{p}_i$ for all particles i . Thus, like p^\pm , \tilde{p} is a distribution.

In order to calculate how each mode \tilde{k} contributes an amount $\tilde{p}(\tilde{k})$ to the total transverse momentum, let us displace all harmonics by the same (infinitesimal) distance \tilde{a} . Then:

$$p^+(\tilde{k}) e^{i\tilde{k}\cdot\tilde{x}} \rightarrow p^+(\tilde{k}) e^{i\tilde{k}\cdot(\tilde{x}+\tilde{a})}, \quad \text{or: } p^+ \rightarrow p^+ e^{i\tilde{k}\cdot\tilde{a}}; \quad \delta p^+ = i\tilde{k} \cdot \tilde{a} p^+ \equiv i\tilde{a} \cdot \tilde{P},$$

and, noting that u^\pm and p^\pm are all in the same \tilde{k} mode, we find that this mode's contribution to the momentum operator \tilde{P} is:

$$\begin{aligned} \tilde{P} &= \tilde{k} \left(p^+ \frac{\partial}{\partial p^+} + \frac{1}{2} \right) = -\tilde{k} \left(p^- \frac{\partial}{\partial p^-} + \frac{1}{2} \right) = -i\tilde{k} (p^+ u^- + \frac{1}{2}i) \\ &= \tilde{k} \left(u^+ \frac{\partial}{\partial u^+} + \frac{1}{2} \right) = -\tilde{k} \left(u^- \frac{\partial}{\partial u^-} + \frac{1}{2} \right) = -i\tilde{k} (u^- p^+ - \frac{1}{2}i) \end{aligned}$$

in agreement with the relations between u^\pm and p^\pm on slide 48.

What are the eigen values of \tilde{P} ? To understand the question, we return to Hermitean operators (the plane waves are complex, hence non Hermitean).

Split up $e^{\pm i\tilde{k}\cdot\tilde{x}} = \cos(\tilde{k}\cdot\tilde{x}) \pm i \sin(\tilde{k}\cdot\tilde{x})$. Write $p^{\pm}(-\tilde{k}) = p^{\pm*}(\tilde{k})$, and

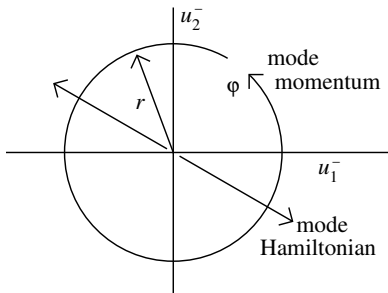
$$p^{\pm}(\tilde{k}) e^{\pm i\tilde{k}\cdot\tilde{x}} + p^{\pm}(-\tilde{k}) e^{\mp i\tilde{k}\cdot\tilde{x}} \equiv p_1^{\pm} \cos(\tilde{k}\cdot\tilde{x}) + p_2^{\pm} \sin(\tilde{k}\cdot\tilde{x}),$$

$$p^{\pm}(\tilde{k}) = p_1^{\pm} \pm ip_2^{\pm}; \quad u^{\pm}(\tilde{k}) = \frac{1}{2}(u_1^{\pm} \pm iu_2^{\pm}).$$

The $\frac{1}{2}$ is there to keep commutation rules as $[u_{\alpha}^{\pm}, p_{\beta}^{\pm}] = i\delta_{\alpha\beta}$, where $\alpha, \beta = 1, 2$. This gives the transverse momentum operator contribution of the two modes \tilde{k} and $-\tilde{k}$ together:

$$\tilde{P} = -\frac{1}{2}\tilde{k}(u_1^- - iu_2^-)(p_1^+ + ip_2^+) + h.c. = \tilde{k}(u_1^- p_2^+ - u_2^- p_1^+)$$

$$= i\tilde{k} \frac{\partial}{\partial \varphi}.$$



This generates a rotation in (u_1^-, u_2^-) space. The Hamiltonian, $H = u^- p^+ - \frac{1}{2}i = u_1^- p_1^+ + u_2^- p_2^+ - i$ generates a dilation (see slide 23)

The wave function for these two modes can be written as $\psi(u_1^-, u_2^-) = \psi(r, \varphi)$ (in polar coordinates), see Figure.

\tilde{P} generates a *rotation* while H generates a *dilation*

(actually, a Lorentz transformation, if we take u_i^+ and u_i^- together)

Note that φ is periodic, and therefore the eigenvalues of \tilde{P} are given by integers $n \in \mathbb{Z}$:

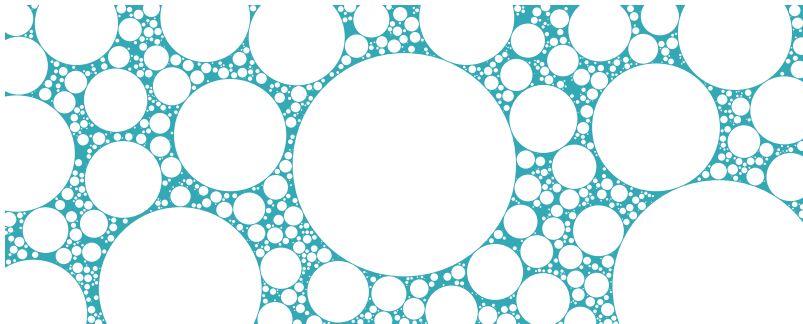
$$\tilde{P} = \tilde{k} n .$$

This illustrates that \tilde{k} itself is *not* to be interpreted as transverse momentum, and similarly, (ℓ, m) are *not* to be interpreted as angular momenta. Rather, they are spherical harmonic mode numbers.

This last part is speculation. It was part of version version 3 of these slides.
Ignore it if you only want transparent physics.

Virtual black holes and space-time foam (Summary)

Virtual black holes must be everywhere in space and time. Due to vacuum fluctuations, amounts of matter that can contract to become black holes, must occur frequently. They also evaporate frequently, since they are very small. This produces small vacuoles in the space-time fabric. How to describe multiple vacuoles is not evident. The emerging picture could be that of "space-time foam":



Work is still in progress. More than happy to discuss these ideas with like-minded colleagues.

The author benefited from discussions with P. Betzios, N. Gaddam, O. Papadoulaki, S. Mathur, L. Susskind, J. Maldacena and many others.

See: G. 't Hooft, [arxiv:1612.08640 \[gr-qc\]](https://arxiv.org/abs/1612.08640) + references there;
http://www.phys.uu.nl/~thooft/lectures/GtHBlackHole_2017.pdf

See also:

P. Betzios, N. Gaddam and O. Papadoulaki, The Black Hole S-Matrix from Quantum Mechanics, JHEP 1611, 131 (2016), [arxiv:1607.07885](https://arxiv.org/abs/1607.07885).

S.W. Hawking, M.J. Perry and A. Strominger, Superrotation Charge and Supertranslation Hair on Black Holes, [arXiv:1611.09175 \[hep-th\]](https://arxiv.org/abs/1611.09175)

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