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Quantum Black Holes, Firewalls

and the

Topology of Space-Time

A pedagogical treatment of a new approach

version #4

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This is the ‘Lecture Notes” version, and an extension, of my latest papers on black hole microstates. These notes highlight the essentials, such that they should be easier to read and understand. My plan is to keep this updated. If new insights arise, they will be added.

In version v3, a chapter, with figures, on “virtual black holes and space-time foam”, was added at the end.
In version v4 further remarks were added, as well as several slides to explain better how some apparent contradictions are to be understood, describing in more detail the generic quantum state of a black hole, and how these evolve.

These notes must be seen as a personal view of the author. Most importantly, we try to avoid any wild assumptions, as we usually see in the literature. Our claim is that everything here is assentially derived from basic laws that are well-established. Not even String Theory is used (this might change in the future).

These notes are not intended for use in talks (too much information on single “slides”) with apologies for not always following the same notation as used elsewhere.

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Black holes are an essential feature of the gravitational force. Without a thorough understanding of their properties one cannot understand quantum gravity. See: G. 't Hooft, arxiv:1612.08640 [gr-qc] + references there.

String theory could have been a good tool as well, and I don't go as far as saying that string theory would be wrong, but this lecturer does not understand string theory well enough.

The assumption that "particles are pieces of string" is just an assumption. I think we do not need to make such assumptions. If string theory is right, we will be able to derive it from other first principles.

Anyway, I here describe a path towards understanding without dubious assumptions at all. Just derivations from known principles, such as Quantum Mechanics and General Relativity.

Note, that these theories are also assumptions, but they are much more compelling. Dropping or loosening these assumptions is much more difficult than keeping them as long as we can. No theory is sacred, and some sense of taste for what to keep and what not, is a useful attire for a theoretical physicist.

We claim that topics such as microstates can be understood very well in our theory.

The tortoise coordinates

Our prototype is the Schwarzschild black hole. No serious complications are expected when generalized to Kerr-Newman or such. The *extreme* black hole would not serve our purposes, because it is a limiting case, and its horizon is fundamentally different from the more general black holes.

Consider the Schwarzschild metric:

$$\mathrm{d}s^2 = \frac{1}{1-\frac{2GM}{r}}\mathrm{d}r^2 - \left(1-\tfrac{2GM}{r}\right)\mathrm{d}t^2 + r^2\mathrm{d}\Omega^2 \; ; \qquad \begin{cases} \Omega & \equiv & (\theta,\, \varphi)\ , \\ \mathrm{d}\Omega & \equiv & (\mathrm{d}\theta,\, \sin\theta\, \mathrm{d}\varphi)\ . \end{cases}$$

Go to Kruskal-Szekeres coordinates x, y, defined by

$$\begin{aligned} x\,y &= \left(\frac{r}{2GM}-1\right) e^{r/2GM} \quad ; \\ y/x &= e^{t/2GM} \quad . \end{aligned}$$

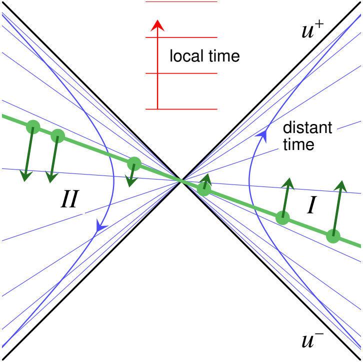
$$\mathrm{d}s^2 = \frac{32(GM)^3}{r}\, e^{-r/2GM}\, \mathrm{d}x\, \mathrm{d}y + r^2\mathrm{d}\Omega^2\ .$$

At $r = 2GM$, we have $x = 0$: past event horizon, and
$y = 0$: future event horizon.

We concentrate on the region close to the horizon: $r \approx 2GM$. There:

$$x = \frac{\sqrt{e/2}}{2GM}\, u^+ \; ; \qquad y = \frac{\sqrt{e/2}}{2GM}\, u^- \; ; \qquad 2GM \equiv R$$

$$\mathrm{d}s^2 \;\to\; 2\mathrm{d}u^+\mathrm{d}u^- + R^2\mathrm{d}\Omega^2 \, . \qquad \text{time} \;\; t/4GM = \tau$$



$$u^-(\tau) = u^-(0)e^{\tau}$$
$$u^+(\tau) = u^+(0)e^{-\tau}$$

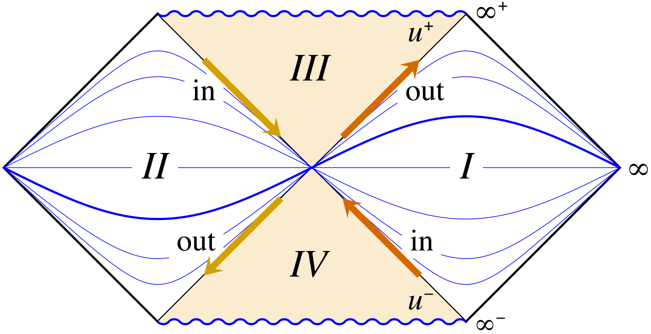
As time goes forwards, u^+ approaches the horizon asymptotically;

as time goes backwards, u^- approaches the past horizon asymptotically (tortoises).

If we redefine $u^+ \to f(u^+)$, $u^- \to g(u^-)$, then the metric keeps the form $\mathrm{d}s^2 = 2A(u)\mathrm{d}u^+\mathrm{d}u^- + r^2(u)\mathrm{d}\Omega^2$. $\to$ *Map $u^\pm$ on compact domains*:

The Penrose diagram.

For pure Schwarzschild (without matter responsible for the formation of a black hole, or representing its final decay):



Now, we begin to deviate from standard practice:

This is the "eternal black hole" (it never formed, it will never decay). We keep it. Why?

> *We put the "microstates" on a Cauchy surface. As yet, we are only interested in how they evolve for a short amount of time. For the quantum properties of the BH, the distant past, and the distant future, should be irrelevant. They are always irrelevant in quantum physics. Late in the future, or long ago in the past, the states we are looking at now, will be highly entangled states. Not suitable for generating a well-defined, curved space-time. Leave all that nonsense out.*

This picture, however, is to be considered as an intermediate state of our discussion. We use it as a starting point to describe small variations in the black hole state during small periods in time. Later, we will address the black hole's long-term time dependence more accurately.

First, consider the particles near the horizon(s).
Mass shell: $2p^+p^- + \tilde{p}^2 + \mu^2 = 0$.
Here, $\tilde{p}$ is the transverse part of the momentum, $\mu =$ mass;
$|\tilde{p}| \approx L/R$ and μ are basically constant.
But $p^-(\tau) = p^-(0)e^{\tau}$, $p^+(\tau) = p^+(0)e^{-\tau}$ (See slide 5)
Define *soft* particles: $|\vec{p}|,\ \mu \ll M_{\text{Planck}}$ Negligible effect on space-time curvature.
Define hard particles as particles that do cause appreciable space-time curvature.
As $\tau \to \infty$, $p^+ \to 0$, $p^- \to \infty$: all in-particles become *hard;*
As $\tau \to -\infty$, $p^- \to 0$, $p^+ \to \infty$: all out-particles were *hard.*

$|\tilde{p}|$ and μ stay small.

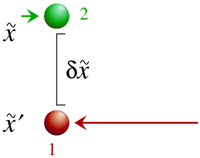
To understand what happens with the evolution at longer time intervals, we have to understand what the hard in- and out- partcles do.

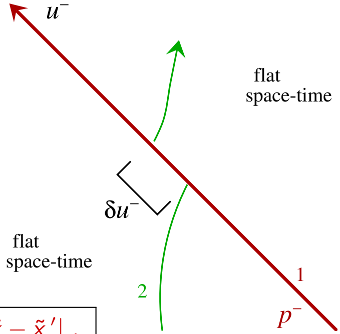
As $\tau \gg 1$, the in-particles become hard. Their interactions with other in-particles are negligible (they basically move in parallel orbits), but they do interact with the out-particles. the interaction through QFT forces stay weak, but the *gravitational forces* make that (early) in-particles interact strongly with (late) out-particles.

The gravitational force between them cannot be ignored; easy to be calculated

The gravitational backreaction:

Calculate the *Shapiro time delay* caused by the grav. field of a fast moving particle: simply Lorentz boost the field of a particle at rest:

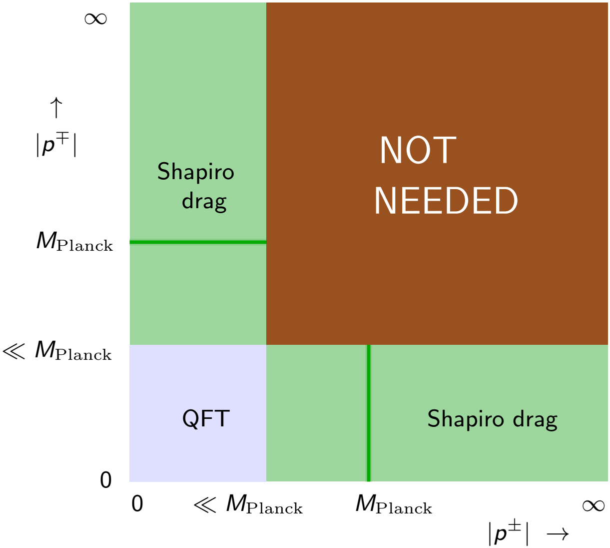




$$\boxed{\delta u^-(\tilde{x}) = -4G\, p^-(\tilde{x}\,')\, \log|\tilde{x}-\tilde{x}\,'|\ .}$$

P.C. Aichelburg and R.U. Sexl, J. Gen. Rel. Grav. **2** (1971) 303,
W.B. Bonnor, *Commun. Math. Phys.* **13** (1969) 163,
T. Dray and G. 't Hooft, *Nucl. Phys.* **B253** (1985) 173.

What we need to know is the interactions between in- and out-particles.



The fact that the mutual interactions between particles at the Planck mass or beyond, will not be needed, is a very important aspect of this work. As you will see.

We start with *only soft particles* on a Cauchy surface of the Penrose diagram. These will define *all* quantum microstates of the black hole.

Now, the question is how do these evolve with time.
The soft particles won't stay soft; their longitudinal momenta will quickly explode. To see what happens, calculate the Shapiro shift.

in-particle with momentum p^- at solid angle $\Omega' = (\theta', \varphi')$
causes a shift δu^- at solid angle $\Omega = (\theta, \varphi)$:

$$\delta u^-(\Omega) = 8\pi G\, f(\Omega, \Omega')p^- \ ; \qquad (1 - \Delta_W)f(\Omega, \Omega') = \delta^2(\Omega, \Omega') \ .$$

Many particles: $p^-(\Omega) = \sum_i p_i^- \delta^2(\Omega, \Omega_i)$.

$$\delta u^-(\Omega) = 8\pi G \int \mathrm{d}^2\Omega' f(\Omega, \Omega')p^-(\Omega') \ .$$

Now start with one unique "reference" black hole, yielding out-particles with given positions at the origin of some coordinate system. Suppose $p^-(\Omega)$ represents *all* in-particles needed to describe any *other* black hole.
Then the out-particles will be at positions $u^-(\Omega)$ given by

$$u^-_{\rm out}(\Omega) = 8\pi G \int \mathrm{d}^2\Omega' f(\Omega,\Omega') p^-_{\rm in}(\Omega') \quad .$$

Spherical wave expansion:

$$u^{\pm}(\Omega) = \sum_{\ell,m} u_{\ell m} Y_{\ell m}(\Omega) \ , \qquad p^{\pm}(\Omega) = \sum_{\ell,m} p^{\pm}_{\ell m} Y_{\ell m}(\Omega) \ ;$$

$$[u^{\pm}(\Omega),\, p^{\mp}(\Omega')] = i\delta^2(\Omega,\,\Omega') \ , \qquad [u^{\pm}_{\ell m},\, p^{\mp}_{\ell' m'}] = i\delta_{\ell\ell'}\delta_{mm'} \ ; \tag{1}$$

$$u^-_{\rm out} = \frac{8\pi G}{\ell^2+\ell+1} p^-_{\rm in} \ , \qquad u^+_{\rm in} = -\frac{8\pi G}{\ell^2+\ell+1} p^+_{\rm out} \ ,$$

$p^{\pm}_{\ell m}$ = total momentum in of ${}^{\rm out}_{\rm in}$-particles in (ℓ, m)-wave ,

$u^{\pm}_{\ell m}$ = (ℓ, m)-component of c.m. position of ${}^{\rm in}_{\rm out}$-particles .

Because we have linear equations, all different ℓ, m waves decouple, and for one (ℓ, m)-mode we have just the variables $u^{\pm}$ and $p^{\pm}$. They represent only one independent coordinate u^+, with $p^- = -i\partial/\partial u^+$.

The dynamics completely factorises in (ℓ, m) spherical harmonics – probably this remains true in the harmonics of a Kerr black hole, but that generalization is not considered here.

3 more steps to be taken:

1. Momentum density $p^{\pm}(\Omega)$ for every quantum state in a QFT is well-defined. However, our evolution law will only be unitary is we can find the QFT quantum state back from the momentum density. That's difficult.

Use vertex insertions as in string theory?

2. What is the physical interpretation of region *II* ?.

We demand *locality*. This means that commutators of space-like separated operators should vanish. Then, since in-going signals in region *I* produce signals in the out-going objects in region *II*, we must guarantee that *I* and *II* are not space-like separated.

Postulate that region *II* refers to the same black hole as region *I*, but *not at the same solid angle* $\Omega = (\theta, \varphi)$. Only one possibility:

The antipodal identification: $\Omega \rightarrow \tilde{\Omega} = (\pi - \theta, \varphi + \pi)$

3. The "firewalls". Soft particles become hard particles. Must be 'removed'.

Obvious suggestion: all information carried by the in- particles is now present in the out-particles. The in-particles are redundant ("quantum clones"). Leave the hard ones out. Hilbert space is completely specified by the coordinates $u^{-}(\Omega)$ of the out-particles. Note: as soon as $|u^{-}| > L_{\mathrm{Planck}}$, these out-particles are soft.

This is the "firewall transformation"; it removes firewalls.

The basic, explicit, calculation

The algebra (1) generates the scattering matrix, by giving us the *boundary condition* that replaces $|\mathrm{in}\rangle$-states by $|\mathrm{out}\rangle$-states. This boundary condition replaces the old *brick wall model* and it is embarassingly easy to derive

(The catch will be exposed later) .

All of this is NOT a model, or a theory, or an assumption . . .

Apart from the most basic assumption of unitary evolution, this is nothing more than applying GR and quantum mchanics !

At some points in the following derivations, it may seem that choices were made as to how to continue, such as the eternal BH Penrose diagram and the antipodal identification, and some of my choices are being criticised as they may seem illogical. I emphasize however that they will all be vindicated by the final result, a unitary evolution law. There is no other way to obtain that. None of the other approaches in the literature that I have seen produce such explicit results.

Much of the work described here has its roots in my work of the early 1990s, particularly the pioneering ideas collected in the paper on the "Scattering Matrix Ansatz". The fact that I had to discover the new alleys described here myself, suggests that those early papers may not have received enough attention. This may also hold for the new papers.

Let there be two operators, u and p, obeying the commutator equation

$$[u,\, p] = i\ , \qquad \text{so that} \qquad \langle u|p\rangle = \tfrac{1}{\sqrt{2\pi}}\, e^{ipu}\ .$$

and a wave function $|\psi\rangle$, defined by $\psi(u) \equiv \langle u|\psi\rangle$. Then

$$\hat\psi(p) \equiv \langle p|\psi\rangle = \tfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}u\, e^{-ipu}\psi(u)\ .$$

Now introduce tortoise coordinates, and split both u and p in a positive part and a negative part:

$$u \equiv \sigma_u\, e^{\varrho_u}\ , \quad p = \sigma_p\, e^{\varrho_p}\ ; \quad \sigma_u = \pm 1\ , \quad \sigma_p = \pm 1\ , \quad \text{and}$$

$$\tilde\psi_{\sigma_u}(\varrho_u) \equiv e^{\frac12 \varrho_u}\, \psi(\sigma_u\, e^{\varrho_u})\ , \quad \tilde{\hat\psi}_{\sigma_p}(\varrho_p) \equiv e^{\frac12 \varrho_p}\, \hat\psi(\sigma_p\, e^{\varrho_p})\ ;$$

normalized: $$|\psi|^2 = \sum_{\sigma_u=\pm} \int_{-\infty}^{\infty} \mathrm{d}\varrho_u |\tilde\psi_{\sigma_u}(\varrho_u)|^2 = \sum_{\sigma_p=\pm} \int_{-\infty}^{\infty} \mathrm{d}\varrho_p |\tilde{\hat\psi}_{\sigma_p}(\varrho_p)|^2\ .$$

Then $\tilde{\tilde{\psi}}_{\sigma_p}(\varrho_p) = \displaystyle\sum_{\sigma_u=\pm 1}\int_{-\infty}^{\infty} \mathrm{d}\varrho_u \, K_{\sigma_u\sigma_p}(\varrho_u+\varrho_p)\,\tilde{\psi}_{\sigma_u}(\varrho_u)$,

$$\text{with} \quad K_\sigma(\varrho) \equiv \tfrac{1}{\sqrt{2\pi}} e^{\frac{1}{2}\varrho}\, e^{-i\sigma\, e^{\varrho}} \ .$$

Notice the symmetry under $\varrho_u \to \varrho_u + \lambda$, $\varrho_p \to \varrho_p - \lambda$, which is simply the symmetry $u \to u\, e^{\lambda}$, $p \to p\, e^{-\lambda}$, a property of the Fourier transform, and an invariance of our algebra.

We now use this symmetry to write plane waves:

$\tilde{\psi}_{\sigma_u}(\varrho_u) \equiv \breve{\psi}_{\sigma_u}(\kappa)\, e^{-i\kappa\varrho_u}$ and $\tilde{\tilde{\psi}}_{\sigma_p}(\varrho_p) \equiv \breve{\tilde{\psi}}_{\sigma_p}(\kappa)\, e^{i\kappa\varrho_p}$ with

$$\breve{\tilde{\psi}}_{\sigma_p}(\kappa) = \sum_{\sigma_p=\pm 1} F_{\sigma_u\sigma_p}(\kappa)\breve{\psi}_{\sigma_u}(\kappa) \ ; \quad F_\sigma(\kappa) \equiv \int_{-\infty}^{\infty} K_\sigma(\varrho) e^{-i\kappa\varrho} \mathrm{d}\varrho \ .$$

Thus, we see left-going waves produce right-going waves. On finds (just do the integral):

$$F_\sigma(\kappa) = \int_0^\infty \frac{\mathrm{d}y}{y} y^{\frac{1}{2}-i\kappa}\, e^{-i\sigma y} \ = \ \Gamma(\tfrac{1}{2}-i\kappa)\, e^{-\frac{i\sigma\pi}{4}\ -\ \frac{\pi}{2}\kappa\sigma} \ .$$

Matrix $\begin{pmatrix} F_+ & F_- \\ F_- & F_+ \end{pmatrix}$ is unitary: $F_+F_-^* = -F_-F_+^*$ and $|F_+|^2 + |F_-|^2 = 1$.

Look at how our soft particle wave functions evolve with time τ, slide # 8. Their Hamiltonian is the dilaton operator. κ is the *energy:*

$$H = -\tfrac{1}{2}(u^+p^- + p^-u^+) \;=\; \tfrac{1}{2}(u^-p^+ + p^+u^-) =$$
$$i\frac{\partial}{\partial \varrho_{u^+}} \;=\; -i\frac{\partial}{\partial \varrho_{u^-}} \;=\; -i\frac{\partial}{\partial \varrho_{p^-}} \;=\; i\frac{\partial}{\partial \varrho_{p^+}} \;=\; \kappa\ ,$$

(if we apply the previous slide to the coordinates u^+ and p^-).

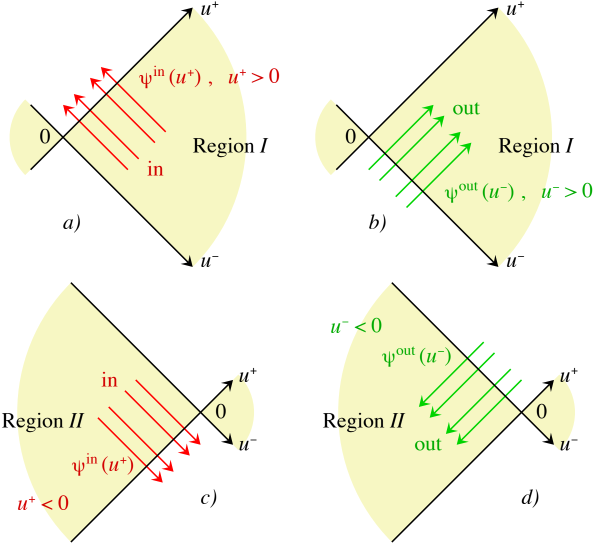
Add the scale factor $\frac{8\pi G}{\ell^2+\ell+1}$, to get, if $u^\pm = \sigma_\pm e^{\varrho^\pm}$,

$$\psi^{\rm in}_{\sigma_+}\, e^{-i\kappa\varrho^+} \;\rightarrow\; \psi^{\rm out}_{\sigma_-} e^{i\kappa\varrho^-}\ , \tag{2}$$

$$\psi^{\rm out}_{\sigma_-} \;=\; \sum_{\sigma_+} F_{\sigma_+\sigma_-}(\kappa)\, e^{-i\kappa \log\left(8\pi G/(\ell^2+\ell+1)\right)}\, \psi^{\rm in}_{\sigma_+}$$

These equations generate the contributions to the scattering matrix from all (ℓ, m) sectors of the system, where $|m| \le \ell$. At every (ℓ, m), we have a contribution to the position operators $u^\pm(\theta,\varphi)$ and momentum operators $p^\pm(\theta,\varphi)$ proportional to the partial wave function $Y_{\ell m}(\theta,\varphi)$. The signs of $u^\pm(\theta,\varphi)$ tell us whether we are in region I or region II. The signs of $p^\pm(\theta,\varphi)$ tell us whether we added or sutracted a particle from region I or region II.

a) Wave functions $\psi(u^+)$ of the in-particles live in region I, therefore $u^+ > 0$.
b) Out-particles in region I have $\psi(u^-)$ with $u^- > 0$.



The physical picture

c, d) In region II, the in-particles have $u^+ < 0$ and the out-particles $u^- < 0$.

Note that the in-particles will never get the opportunity to become truly hard particles.

Eq. (2) is to be seen as a "soft wall"-boundary condition near the origin of the Penrose diagram. Wave functions going in are reflected as wave functions going out. These again emerge as soft particles.

Thus, there is no firewall, ever.

In the previous slide, the total of the in-particles in regions *I* and *II* are transformed (basically just a Fourier transform) into out-particles in the same two regions.

Note that the regions *III* and *IV* in the Penrose diagram (see slide 6) never play much of a role, even if an observer falling in region *III* would want to assure us that (s)he is still alive.

These regions are best to be seen as lying somewhere on the time-line *where time t is somewhere beyond infinity* (thus a mere repetition of the degrees of freedom we have seen before)

The catch?

Our "boundary condition at the origin" is in terms of momentum distributions $p^{\pm}(\theta, \varphi)$ and center-of-mass positions $u^{\pm}(\theta, \varphi)$ only.

But we would need the quantum wave functions as elements of Fock space (which would be specified by positions or momenta, but also by other quantum numbers!)

Such mapping should be unitary in Hibert space. We are not certain that this can be done, but it is natural to look at how it is done in string theory. Our momentum distributions are like vertex insertions, although they are on the horizon instead of the string world sheet.

Another catch: it seems as if we get too many microstates now. Presumably we are describing black holes with variable sizes. There may well be a cut-off at some large ℓ values depending on M. Further to be investigated.

The antipodal identification

Regions *I* and *II* of the Penrose diagram are exact copies of one another. Often, it was thought that region *II* describes something like the 'inside' of a black hole. That cannot be right, since region *II*, like region *I*, has asymptotic regions. Hawking suggested that region *II* might be some other black hole, in an other universe, or far away in our universe. However, our 2×2 scattering matrix implies that the two regions are in contact with each other quantum mechanically. In ordinary branches of physics, such long-distance communication never takes place, and I don't think theories with such features make any sense.

It is far more natural to assume that region *II* describes the same black hole as region *I*. It must then represent some other part of the same black hole. Which other part? The local geometry stays the same, while the square of this $SO(3)$ operator must be the identity.

There is exactly one possibility: This is the $SO(3)$ operator $-\mathbb{I}$, which is:

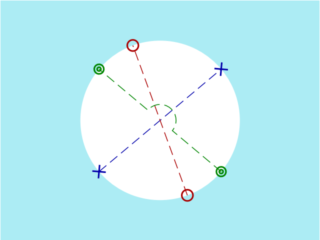
the antipodal mapping.

Antipodal *identification* only holds for the central point (origin) of the Penrose diagram. Regions *I* and *II* are different regions of the universe. But by relating region *II* to region *I* by demanding that the angular coordinates are antipodes, means that now the mapping from Schwarzschild coordinates to Kruskal Szekeres coordinates is one-to-one. This now turns out to be an essential property of our coordinate transformations. Thus, we arrive at a new restriction for all general coordinate transformations:

> *In applying general coordinate transformations for quantized fields on a curved space-time background, to use them as a valid model for a physical quantum system, one must demand that the following constraint hold: the mapping must be one-to-one and differentiable.*

The emergence of a non-trivial topology needs not be conpletely absurd, as long as no signals can be sent around. We think that this is the case at hand here. It is the absence of singularities in the physical domain of space-time that we must demand.

Black emptiness: blue regions are the accessible part of space-time; dotted lines indicate identification.



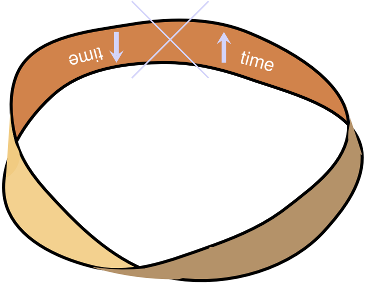
The white sphere within is *not* part of space-time. Call it a 'vacuole'.

At given time t, the black hole is a 3-dimensional vacuole. The entire life cycle of a black hole is a vacuole in 4-d Minkowski space-time.

Space coordinates change sign at the identified points
– *and also time changes sign*
(Note: time stands still at the horizon itself).

Not that all $u^{\pm}$ and $p^{\pm}$ coordinates are odd when switching between antipodes. Therefore, only odd ℓ contribute in the spherical harmonic expansion.

A timelike Möbius strip



Draw a spacelike closed curve:

Begin on the horizon at a point

$r_0 = 2GM \ , \ t_0 = 0 \ , \ (\theta_0, \varphi_0) \ .$

Move to larger r values, then travel to the antipode:

$r_0 = 2GM \ , \ t_0 = 0 \, , \ (\pi - \theta_0, \varphi_0 + \pi) \ .$

You arrived at the same point, so the (space-like) curve is closed.

Now look at the environment $\{\mathrm{d}x\}$ of this curve. Continuously transport $\mathrm{d}x$ around the curve. The identification at the horizon demands

$$\mathrm{d}x \leftrightarrow -\mathrm{d}x \ , \quad \mathrm{d}t \leftrightarrow -\mathrm{d}t \ , .$$

So this is a Möbius strip, in particular in the time direction.
Note that it makes a CPT inversion when going around the loop.

There are no direct contradictions, but take in mind that

the local Hamiltonian density switches sign as well.

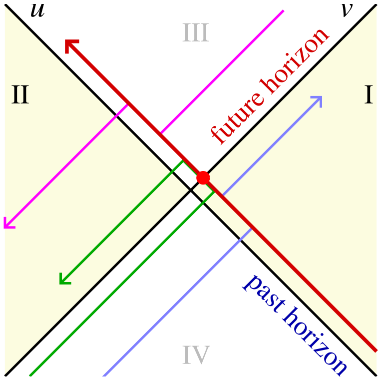
This is not true for the *total* Hamiltonian adopted by distant observers. locally, near the horizon, this is the *dilaton operator*. That operator leaves regions *I* and *II* invariant, and does not flip sign along the loop. Also, the boundary condition, our "scattering matrix", leaves this Hamiltonian invariant.

The *S*-matrix commutes with the Hamiltonian.

There are numerous treatises in the literature claiming solutions to the black hole information paradox, and about as many publications that dismiss these claims.
This author dismisses all claims, from both sides, that either ignore the gravitational back reaction of quantised excitations, or ignore the antipodal identification of points on the horizon – meaning that the horizon is a *projective* 2-sphere.

A bizarre sign switch

The effect of the gravitational footprint of an in-going particle (red line to upper left) onto the out-going particles (other coloured lines) is correctly accommodated for in this work. But there is an oddity, see our picture here of the allowed regions *I* and *II* and the unphysical regions *III* and *IV*.



The gravitational footprint moves the *pure states* of all out-particles combined in the light cone direction by an amount $\delta u^-(\theta,\varphi)$ in the same direction as the in-momentum $\delta p^-(\theta',\varphi')$.
This is a unitary transformation in the space of the combined wave functions only if all out-particles move by the same amount in the same drection, as is drawn here.

But this led to an apparently valid criticism:

This configuration does not appear to correspond to a valid solution of Einstein's equations for the local observer. What are we doing wrong?

It appears that this corresponds to a solution of Einstein's equations only if the momentum δp^- of the hard, in-particle flips its sign while passing through the future event horizon. At the same time, of course, the arrow of time switches sign.

Moreover, *those out-particles that are dragged right across the horizon (green line), do not seem to follow a geodesic at all, as regarded by the local observer.* Yet, of course, with our definition of the arrow of time, we have no choice, the particle has to continue its way as shown. What should a local observer say?

For the local observer, the diagram here does not show a classical solution, but it shows the action of an operator. The operator creates an extra in-particle (red line) that, in all states in regions *I* and *II*, shifts the geodesics of all out-particles by the same amount, in the same direction.

But if we represent the result of this operator as what happens later in time, then the fact that time runs backwards in region *II* leads to the impression that a discontinuity occurs on the other (future) horizon.

Operators should not be regarded as effects that agree with evolution equations; they represent what happens to a system if we make a change on the Cauchy surface, at some moment in time.

The change brought about here, has the effect of a positive δp^- particle in region *I* and at the same time the effect of a negative δp^- particle (an annihilayed particle) in region *II*. Note that the operators $u^\pm$ and $p^\pm$ on the horizons all switch sign when passing from region *I* to *II*, since they are each other's antipodes, on the horizon.

The BMS group.

In a development parallel to our work here, it has been argued that "conservation of information" for black holes can be understood by using the BMS group. It was claimed that this group generates an infinite class of conserved charges that must be held responsible for safegarding the information processed by the black hole. These charges are associated to supertranslations, and as such must take the form of (θ, φ)-dependent momenta in the light cone direction. To the present author, the physical interpretation of these arguments is less transparent. The arguments seem to be very formal, and thus suspect. I would like to see how the BMS group generates a unitary S-matrix.

Energy conservation

It was suggested from BMS arguments that energy might be conserved saparately at every (projective) angle (θ, φ), that is, energy on a point (θ, φ) plus energy on its antipode together should be conserved (the changes of this sum stay zero). This must be a misunderstanding. Our formalism is more transparent. What is conserved at every projective angle is the light cone momentum p^-. Since the coordinate u^- at that angle equals minus the coordinate u^- at its antipode, it follows that p^- at a point equals $-p^-$ at its antipode. However, p^- is the local energy; the energy for the distant observer is κ (see slides 16 and 17). It is conserved at every partial wave (ℓ, m), but because of the ℓ dependence of the S-matrix (the displacement δu^- due to a hard particle depends on the logarithmic Green function f, slide 11, which is non local).

The black hole mass

Other question often asked: how does the mass of the black hole respond on the absorbed and emitted particles?

This is actually very easy. The energy (the sum of the numbers κ at every value of (ℓ, m)), is exactly conserved. To see how this works, arrange all in- and out- particles into three classes:

(i) The particles coming in when still far away from the future horizon, which are the firewall transforms of the out-particles that are still very close to the past event horizon, ready to emerge at some later era,

(ii) The soft particles in the parts of regions I and II that are in the neighborhood, but not too close the any of the horizons, and

(iii) The particles moving out at a safe distance from the black hole, they are the firewall transforms of the particles that have arrived at a very close distance from the future event horizon.

The total energy, black hole plus these particles, is strictly conserved. But the particles of group (*i*) (the very late particles) and group (*iii*) (the very early particles) are so far away that we should not count those as part of the black hole mass/energy. The particles of group (*ii*) however, are so close to the horizon that these must be considered as part of the black hole.

Perhaps we have to postulate that the entire black hole mass consists of the contributions of these particles, but this we leave for later considerations.

Clearly, this mass is time dependent, as the distinction of a particle's classes is time dependent.

Note however, that a black hole is always surrounded by the cloud of entangled Hartle-Hawking particles.

The particles discussed above are the ones observed by *local observers near the horizon.*

The complete set of black hole states

When a local observer sees a vacuum, it means that the black hole is in the *Hartle-Hawking state,*

$$|\Omega\rangle = C\sum_{E,n} e^{-\frac{1}{2}\beta E}|E,n\rangle_I|E,n\rangle_{II} \ .$$

It is seen as a *vacuum* $|\Omega\rangle$ for the local observer, but contains states $|E,n\rangle$ in regions I and II for the global observer. C is a normalization constant, and n represents all quantum numbers other than the energies E of the particle states – as seen by the global observer.

This expression is obtained when we define creation operators ${a^\dagger}_{E,\,n}$ and annihilation operators $a_{E,\,n}$ as Fourier transforms of the time dependence of the local field variables $\phi(\vec{x},t)$. General coordinate transformations are assumed to act locally on these fields $\phi(\vec{x},t)$. The operators $a^\dagger$ are the positive frequency Fourier coefficients and a are the negative frequency coefficients. Since time means something different for the different observers, the transformation from global observables to local ones involves a Bogolyubov transformation between a and $a^\dagger$, and since the vacuum state $|\Omega\rangle$ is assumed to be the vacuum for the local observer, so that for him, $a_{\kappa,n}|\Omega\rangle = 0$, this is not the vacuum $|\Omega\rangle_{\rm glob}$ for the global observer. The straightforward calculation gives the above expression. $E = \sum_{\ell,m}\kappa_{\ell,m}$; $\qquad \beta = 1/kT_{\rm Hawking} \ = \ 1/2\pi$ in the units defined by our rescaled time parameter τ.

In previous slide, $|\Omega\rangle$ describes the in- and out-particles for the distant observer being in an entangled state (the antipodes are entangled). The observer cannot change that state for the out-particles, but can *choose* the in-particles any way he likes, both in region I and in regio II. But since our S-matrix relates the out-particles to the earlier in-particles, eventually the observer can choose his state any way he likes. Thus we can consider states $|E_I, n_I\rangle|E_{II}, n_{II}\rangle$ where the parameters can be anything.

The generic quantum state of a black hole can be considered in a basis where the states are not entangled.

Important point: as time τ (for distant observer) proceeds towards ∞, the out-particles move further out, causing no further difficulties, but the in-particles generate more and more momentum, so the firewall transformation must be used to replace them by out-particles. Where exactly will the point be that this transformation replaces states by softer states, and which states will be left?

Consider a given (ℓ, m) spherical mode. Let $\lambda^2 = \frac{8\pi G}{\ell^2+\ell+1}$. Then choose new coordinates $[x, p] = i$ and

$$p \equiv \lambda p_{\rm in}^- \quad \text{and} \quad x \equiv u^+/\lambda, \text{ so that } \lambda p_{\rm out}^+ = -x \quad \text{and} \quad u_{\rm out}^-/\lambda = p$$

We have $[x, p] = i$, that is, QM in one dimension. Now search for new variables such that, in these new variables, we just have a simple bounce near the horizon. For example, we can express the wave functions in the form of Gaussians. The Fourier transforms of Gaussians are again Gaussians. Narrow Gaussians in x space are wide Gaussians in p space and vice versa. So we plan to flip from x space to p space such that the widths of the Gaussians in all u space are the largest. Thus our wave functions are made as soft as possible. Goes as follows. Consider a symmetric wave function. In p space:

$\phi(p) = \phi(-p)$ and $\psi(x) = \psi(-x)$

(antisymmetric wave functions can be handled similarly). Write this wave function as a superposition of complex Gaussians:

$$\phi(p) = \int_0^\infty \mathrm{d}\alpha\, \alpha^{-\frac{3}{4}}\, \varrho(\alpha)\, e^{\frac{1}{2}i\alpha p^2} \qquad \text{then if}$$

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}p\, \phi(p) e^{ipx} \quad , \qquad \text{one derives}$$

$$\psi(x) = \int_0^\infty \mathrm{d}\beta\, \beta^{-\frac{3}{4}}\, \varrho(1/\beta)\, \frac{1+i}{\sqrt{2}}\, e^{-\frac{1}{2}i\beta x^2} \ , \quad \text{and a solution is given by:}$$

$$\varrho(\alpha) = \frac{\alpha^{3/4}}{2\pi} \int_0^\infty p\mathrm{d}p\, \phi(p)\, e^{-\frac{1}{2}i\alpha p^2} \ = \ \frac{\alpha^{-3/4}(1-i)}{2\pi\sqrt{2}} \int_0^\infty x\, \mathrm{d}x\, \psi(x)\, e^{ix^2/2\alpha}$$

By scaling our x and p cariables with the quantity λ, we achieved that the grav. backreaction just transforms p variables into x variables and back. Using the expressions of the previous slide, we can now simply keep the contributions from the variables α only when $\alpha \geq 1$ and $\beta \leq 1$. The firewall transformation is performed when $\alpha = \beta = 1$.

Note that our Hamiltonian is the dilation operator, so that not only x and p simply scale as time proceeds, but α and β do the same thing.

Thus, all our wave functions are defined at $\alpha > 1$ for the p variables so that p remains small while $\beta < 1$ ensures that contributions from small x are suppressed.

Our particles stay soft.

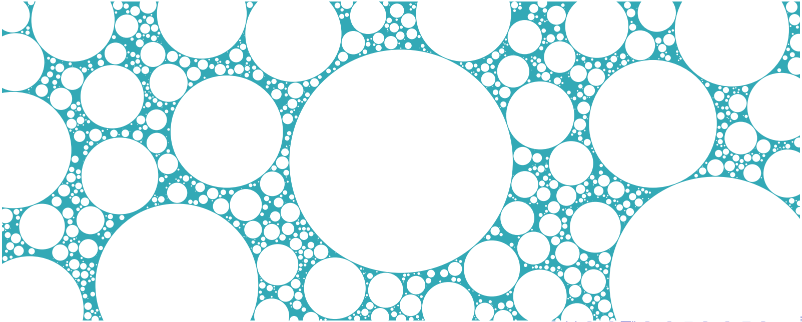
This constitutes the complete "Fock space" for the black hole.

Note that, in this scheme, we can describe a time dependent black hole. Our only limitation today is that we only allow relatively light particles enter and leave from the black hole.

This last part is speculation. It was part of version v3 of these slides.
Ignore it if you only want transparent physics.

Virtual black holes and space-time foam (Summary)

Virtual black holes must be everywhere in space and time. Due to vacuum fluctuations, amounts of matter that can contract to become black holes, must occur frequently. They also evaporate frequently, since they are very small. This produces small vacuoles in the space-time fabric.
How to describe multiple vacuoles is not evident. The emerging picture could be that of "space-time foam":



Work is still in progress. More than happy to discuss these ideas with like-minded colleagues.

See: G. 't Hooft, arxiv:1612.08640 [gr-qc] + references there;
http://www.phys.uu.nl/~thooft/lectures/GtHBlackHole_2017.pdf

See also:
P. Betzios, N. Gaddam and O. Papadoulaki, The Black Hole S-Matrix from Quantum Mechanics, JHEP 1611, 131 (2016), arxiv:1607.07885.

S.W. Hawking, M.J. Perry and A. Strominger, Superrotation Charge and Supertranslation Hair on Black Holes, arXiv:1611.09175 [hep-th]

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