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Black Hole Information and Back Reaction: The Next Step

A small step for mankind
A giant leap for me !

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This talk is about a matrix describing the unitary evolution of a black hole,

Apart from one assumption, agreed upon by almost everybody in the field now, ths is not a new theory but a factual consequence of GR. One can calculate this evolution matrix:

$$\boxed{\begin{pmatrix}\psi_+^{\rm out}\\ \psi_-^{\rm out}\end{pmatrix} = \frac{e^{-\frac{\pi i}{4}}}{\sqrt{2\pi}}\Gamma(\tfrac{1}{2}-i\kappa)\left(\frac{8\pi GR^2}{\ell^2+\ell+1}\right)^{-i\kappa}\begin{pmatrix} e^{-\frac{1}{2}\pi\kappa} & ie^{+\frac{1}{2}\pi\kappa}\\ ie^{+\frac{1}{2}\pi\kappa} & e^{-\frac{1}{2}\pi\kappa}\end{pmatrix}\begin{pmatrix}\psi_+^{\rm in}\\ \psi_-^{\rm in}\end{pmatrix}}$$

Here, ℓ refers to the quantum numbers ℓ and m of a partial wave expansion, κ refers to the momentum of in- and out- going waves in the tortoise coordinates, and the signs $\pm$ tell us, roughly, whether the wave moves in region *I* or *II* of the Penrose diagram – they actually move in both – and I shall explain what this means.

The matrix you see here, is unitary.

Consider a box with sides $L^+ \times L^-$ at the center of the black hole Penrose diagram, Let both $L^+,\ L^- \ll L_{\rm Planck}$

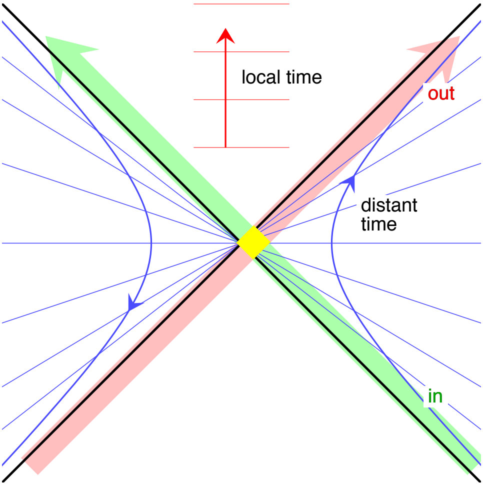
Particles going through there have either
$|p^+| \gg M_{\rm Planck}$ or $|p^-| \gg M_{\rm Planck}$.

Product $2p^+p^- = \tilde{p}^2 + m^2 < M^2_{\rm Planck}$.

p^- is in-going particles, p^+ is out-going particles.

Time dependence: $p^\pm = p^\pm(0)e^{\mp\tau}$, Black hole: $\tau = \dfrac{t}{4M}$

Classically: out-going particles are independent of in-going ones.



But not inside black holes. This used to be the BH information paradox.

In a black hole, the out-going particles are *determined* by the in-going ones (and *vice versa*). How does this work?

We **assume** a unitary relationship (S-matrix). Let *one, given,* black hole be the result of a given pure quantum state of in-going particles, and let that one evolve into some given pure out-state:

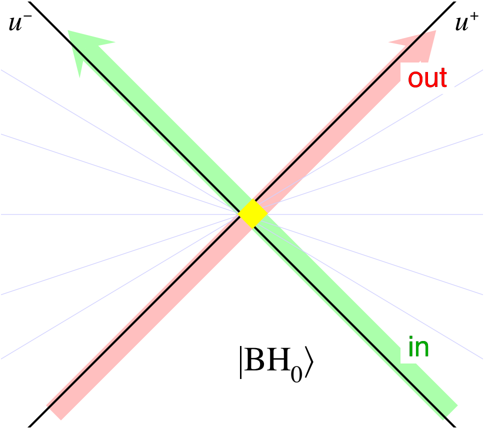
$$|\mathrm{BH}_0\rangle_{\mathrm{in}} \rightarrow |\mathrm{BH}_0\rangle_{\mathrm{out}} \ .$$

Now, use a creation (or annihilation) operator, to create (or remove) just one single particle going in:

$$\begin{aligned} |\mathrm{BH}_1\rangle_{\mathrm{in}} \quad &= \quad |\mathrm{BH}_0\rangle_{\mathrm{in}}|\delta p^-\rangle \ . \\ |\mathrm{BH}_1\rangle_{\mathrm{in}} \quad &\xrightarrow{\mathrm{def}} \quad |\mathrm{BH}_1\rangle_{\mathrm{out}} \ . \end{aligned}$$

We can <u>calculate</u> $|\mathrm{BH}_1\rangle_{\mathrm{out}}$. It is orthogonal to $|\mathrm{BH}_0\rangle_{\mathrm{out}}$.

The extra particle going in interacts gravitationally with the out-going particles:



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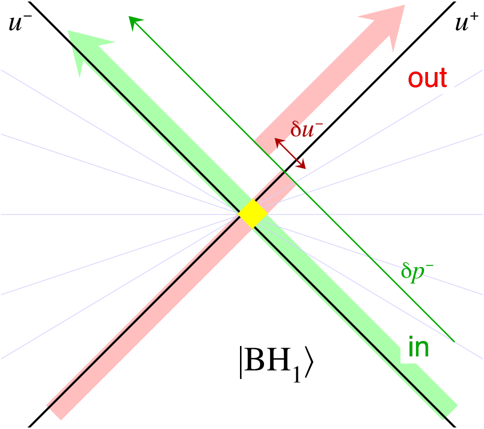

$$\delta u^-(\tilde{x}) = -4G\,\delta p^-(\tilde{x}')\,\log|\tilde{x}-\tilde{x}'|\ .$$

$$\delta u^{-}(\tilde{x}) = -4G\,\delta p^{-}(\tilde{x}')\,\log|\tilde{x} - \tilde{x}'|\;.$$

$\delta u^-(\tilde{x}) = -4G\,\delta p^-(\tilde{x}')\,\log|\tilde{x}-\tilde{x}'|$. The logarithm is the Green function for a flat (Rindler) sace-time. Obeys:

$$\tilde{\partial}^2 \log|\tilde{r}| = 2\pi\delta^2(\tilde{r}) \ .$$

In spherical black hole $(R = 2GM)$:

$$(1-\Delta_\Omega) f(\Omega-\Omega') = 8\pi G R^2 \delta^2(\Omega-\Omega') \ .$$

And now repeat: $p^-(\tilde{x}) \equiv \sum_i \delta p_i^- \delta^2(\tilde{x}-\tilde{x}_i)$,

G. Dvali: on average, there will be no more than one particle per square Planckian surface element.

In principle, the function $p^-(\tilde{x})$ *carries all information needed to represent the in-state.*

Which out-states do we get?

$$u^-(\tilde{x}) = \int \mathrm{d}^2\tilde{x}' f(\tilde{x} - \tilde{x}') p^-(\tilde{x}') \ , \qquad \tilde{\partial}^2 f(\tilde{r}) = -8\pi G \delta^2(\tilde{r})$$

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And: $u^-(\tilde{x})$ carries all information needed to represent the out-state ! How can *that* be? For the in-states, $p^-(\tilde{x})$ is an operator. $[p^-(\tilde{x}),\ p^-(\tilde{x}')] = 0$, since they are spacelike separated. Now, $u^-(\tilde{x})$ is a position operator for the particles in the out states, They also commute. Because wave functions are proportional to $e^{i(p^-u^+ + p^+u^-)}$, we have the commutator algebra

$$\begin{aligned} [u^-(\tilde{x}),\ p^+(\tilde{x}')] \ &= \ [u^+(\tilde{x}),\ p^-(\tilde{x}')] \ = \ i\delta^2(\tilde{x}-\tilde{x}') \ . \\ [u^+(\tilde{x}), u^-(\tilde{x}')] = if(\tilde{x}-\tilde{x}') \ &= \ -[u^-(\tilde{x}'), u^+(\tilde{x})] \ = \ [u^-(\tilde{x}), u^+(\tilde{x}')] \\ \tilde{\partial}^2 u^-(\tilde{x}) = -8\pi G p^-(\tilde{x}) \ ; \qquad & \tilde{\partial}^2 u^+(\tilde{x}) \ = \ +8\pi G p^+(\tilde{x}) \ ; \end{aligned}$$

This algebra is extremely simple, but also tricky. How to interpret the sign switch $\mathrm{in} \leftrightarrow \mathrm{out}$? (it *is* correct)

All states, both in the initial and the final black hole, are a representation of this algebra.

The relation between in- and out- is now not much more than a Fourier transformation: $u^{\pm} \leftrightarrow p^{\mp}$, with $p = -i\frac{\partial}{\partial u}$

Is that all ? **NO !**

There is a complication.

Let's calculate the representation:

Do the partial wave expansion

Partial waves on the spherical black hole:

$$\begin{pmatrix} u \\ p \end{pmatrix}(\theta, \varphi) \;=\; \sum_{\ell,m} \begin{pmatrix} u_{\ell,m} \\ p_{\ell,m} \end{pmatrix} Y_{\ell,m}(\theta,\varphi)$$

$$(1-\Delta) f(\Omega) \;=\; 8\pi G R^2 \delta^2(\Omega) \;\to\; (\ell^2+\ell+1) f_{\ell,m} = 8\pi G R^2 \; .$$

$$\boxed{u^{\pm}_{\ell,m} \;=\; \mp \frac{8\pi G R^2}{\ell^2+\ell+1}\, p^{\pm}_{\ell,m}}$$

In Rindler space (the limit $R \to \infty$; $\quad \ell^2 \to R^2 \tilde{k}^2$):

$$\begin{pmatrix} u \\ p \end{pmatrix}(\tilde{x}) \;=\; \frac{1}{\sqrt{(2\pi)^2}} \int \mathrm{d}^2 \tilde{k}\, e^{-\tilde{k}\cdot\tilde{x}} \begin{pmatrix} u(\tilde{k}) \\ p(\tilde{k}) \end{pmatrix}$$

$$\boxed{u^{\pm}(\tilde{k}) \;=\; \mp \frac{8\pi G}{\tilde{k}^2} p^{\pm}(\tilde{k})}$$

different ℓ, m or different $\tilde{k}$, do ***not*** *mix !*

Therefore, consider just one partial wave. Now, remember time dependence:

$$p^{\pm}(\tau) = p^{\pm}(0)e^{\mp\tau} \ ; \qquad u^{\pm}(\tau) = u^{\pm}(0)e^{\mp\tau} \ .$$

This time is a Killing vector. Therefore, we should see plane waves. In "position" space, take tortoise coordinates:

$$u^{+}_{\mathrm{in},\,\ell m}(\tau) = \pm e^{\,\varrho_{\mathrm{in}}(\tau)} \ , \quad u^{-}_{\mathrm{out},\,\ell m}(\tau) = \pm e^{\,\varrho_{\mathrm{out}}(\tau)} \ .$$

Clearly, we have not only the tortoise coordinates ϱ_{in} and ϱ_{out}, but also their signs, call them $\sigma_{\mathrm{in}} = (\pm)$ and $\sigma_{\mathrm{out}} = [\pm]$.

$$u^{+}_{\mathrm{in},\,\ell m}(\tau)=\sigma_{\mathrm{in}}e^{\,\varrho_{\mathrm{in}}(\tau)}\ ,\quad u^{-}_{\mathrm{out},\,\ell m}(\tau)=\sigma_{\mathrm{out}}e^{\,\varrho_{\mathrm{out}}(\tau)}\ .$$

We get:

$$u^{-}_{\mathrm{out},\,\ell m}(\tau)=\lambda p^{-}_{\mathrm{in},\,\ell m}(\tau)\quad ;\quad u^{+}_{in,\,\ell m}(\tau)=-\lambda p^{+}_{\mathrm{out},\,\ell,m}\ ;$$

$$\lambda\ =\ \frac{8\pi GR^2}{\ell^2+\ell+1}$$

If $\psi(u^{\pm})$ are the Fourier transforms of $\hat{\psi}(p^{\mp})$, how do the wave functions in the corresponding tortoise coordinates ϱ and σ relate? That's easy. We find, after correctly normalizing them,

$$\psi(u) = \psi(|u|, \sigma_u) = e^{-\frac{1}{2}\varrho_u}\phi(\varrho_u, \sigma_u) \ , \quad \hat{\psi}(p) = e^{-\frac{1}{2}\varrho_p}\hat{\phi}(\varrho_p, \sigma_p) \ ;$$

$$\hat{\psi}(u) = \frac{1}{\sqrt{2\pi}}\int \mathrm{d}p \, e^{ipu}\psi(p) \rightarrow$$

$$\phi(\varrho_u, \sigma_u) \ = \ \sum_{\sigma_p=\pm 1} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}\varrho_p \, e^{\frac{1}{2}(\varrho_p + \varrho_u) + i\sigma_p\sigma_u \, e^{\varrho_p+\varrho_u}} \times \phi(\varrho_p, \sigma_p) \ .$$

Then putting the factor $\lambda = \frac{8\pi G R^2}{\ell^2+\ell+1}$ in, we find the relation between the wave functions on ϱ_u^{out} and ϱ_u^{in}:

$$\psi_{\mathrm{out}}(\varrho_u^{\mathrm{out}}, \sigma_{\mathrm{out}}) \ = \ \sum_{\sigma_{\mathrm{in}}=\pm 1} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}\varrho_u^{\mathrm{in}} \, e^{\frac{1}{2}(\varrho_u^{\mathrm{in}} + \varrho_u^{\mathrm{out}}) - i\sigma_{\mathrm{in}}\sigma_{\mathrm{out}} \, e^{\varrho_u^{\mathrm{in}}+\varrho_u^{\mathrm{out}}}} \times \psi_{\mathrm{in}}(\varrho_u^{\mathrm{in}} + \log\lambda, \sigma_{\mathrm{in}}) \ .$$

Note invariance under time translations:

$$\varrho \to \varrho_u - \tau \ ; \quad \varrho_p \to \varrho_p + \tau .$$

We should use this, by writing these wave functions in plane waves for the tortoise coordinates ($\sigma = \pm 1$):

$$\psi_{\rm in}(\varrho,\sigma) = \psi_\sigma^{\rm in}\, e^{-i\kappa(\varrho+\tau)} \ ; \qquad \psi_{\rm out}(\varrho,\sigma) = \psi_\sigma^{\rm out}\, e^{i\kappa(\varrho-\tau)}$$

By Fourier transforming previous equation,

$$\boxed{\begin{pmatrix}\psi_+^{\rm out}\\ \psi_-^{\rm out}\end{pmatrix} = \frac{1}{\sqrt{2\pi}}\Gamma(\tfrac12 - i\kappa)e^{-\frac{\pi i}{4} - i\kappa\log\frac{8\pi G R^2}{\ell^2+\ell+1}}\begin{pmatrix} e^{-\frac12\pi\kappa} & ie^{+\frac12\pi\kappa}\\ ie^{+\frac12\pi\kappa} & e^{-\frac12\pi\kappa}\end{pmatrix}\begin{pmatrix}\psi_+^{\rm in}\\ \psi_-^{\rm in}\end{pmatrix}}$$

Unitarity follows from:

$$|\Gamma(\tfrac12 - i\kappa)|^2 = \frac{\pi}{\cosh \pi\kappa}$$

Homework: Euler Gamma function:

If $\sigma = \pm 1$ then

$$\int_0^\infty \frac{\mathrm{d}z}{\sqrt{z}} e^{-\sigma z} z^{-i\kappa} = \Gamma(\tfrac{1}{2} - i\kappa) e^{\frac{-i\sigma\pi}{4} - \sigma\frac{\pi}{2}\kappa} \ .$$

$$\boxed{\begin{pmatrix}\psi_+^{\text{out}}\\ \psi_-^{\text{out}}\end{pmatrix} = \frac{e^{-\frac{\pi i}{4}}}{\sqrt{2\pi}}\Gamma\left(\tfrac{1}{2} - i\kappa\right)\left(\frac{8\pi GR^2}{\ell^2+\ell+1}\right)^{-i\kappa}\begin{pmatrix} e^{-\frac{1}{2}\pi\kappa} & ie^{+\frac{1}{2}\pi\kappa}\\ ie^{+\frac{1}{2}\pi\kappa} & e^{-\frac{1}{2}\pi\kappa}\end{pmatrix}\begin{pmatrix}\psi_+^{\text{in}}\\ \psi_-^{\text{in}}\end{pmatrix}}$$

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To be read as follows:

Hilbert space is a product space for all ℓ, m; at each ℓ, m, we have a wave function $\psi_\sigma(\kappa)$, or $\psi_\sigma(\varrho,\tau)$, or $\psi(u,\tau)$ or $\psi(p,\tau)$.

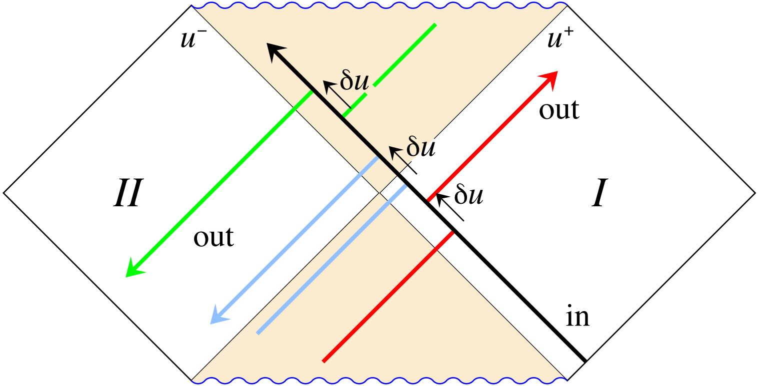
Thus, on $(\ell,\ m)$, we have a $2^{\rm nd}$ quantized 'field theory', but for every partial wave, we have a single particle at position u (or tortoise coordinate ϱ).

But note that we have the two signs, $\sigma = \pm 1$. This means that our particles fill region *I* as well as region *II* in the Penrose diagram.

The universe has two asymptotic regions.

I and *II* *talk* to each other !

What does this mean?



Note, that we have to keep time ordering w.r.t. distant time variable $\tau = t/4M$.
In-going particle in region *I* acts as *annihilated* in-going particle in region *II*.
Hawking particle, due to vacuum fluctuation, will behave as physical particle in *I* and in *II*.

Suggestion: region *II* describes the points on the horizon that are *antipodal* to the points in region *I*.

This may well be wrong, but, apart from moving these points around a bit, I see no other solution to the unitarity problem.

Remarkable phenomenological consequence of this suggestion:

Hawking particles emerging from two antipodal points on the horizon would be 100% *entangled!*

(If a rare event happens at one point, suppressed by Boltzmann factor $e^{-\beta_H E}$, then the same event will be seen at the antipodal point. The combined events would point at a temperature twice that of Hawking.)

Discussion.

We think that the apparent entanglement of regions *I* and *II* of the Penrose diagram is an important discovery. Before, we had been content with abstract functional integral expresions, which did not disclose so clearly the fact that one cannot ignore region *II*. Lesson learned: Whenever more explicit calculations are possible, we should do them; they yield much more understanding of what goes on.
Work from other authors using ‘supertranslations’ did not lead to answers as explicit as ours.

But our work is far from finished:
(1) We now have the microstates, and we can calculate the Hawking entropy, but a cut-off is needed limiting ℓ to a maximum value ($\ell < \mathcal{O}(R)$ or $|\tilde{k}| < \mathcal{O}(1)$ in Planck units). How can we understand this cut-off? L. Mersini-Houghton (8) suggests that we ignored “quantum corrections” but we disagree, as the operators $p^-(\tilde{x})$ at different $\tilde{x}$ all commute. At small values of ℓ, our expressions should be very precise, but at ℓ close to the Planck length, we do expect deviations due to shifts arising from the transverse momenta.
(2) The representation of our algebra is different from Fock space, and therefore difficult to match with the Standard Model states. Trying to do this properly will be extremely important. It could lead to constraints on the SM coming from quantum gravity.
(3) Other forces between in- and out- particles can be considered: electro-magnetism and non-Abelian forces. At the functional integral level, this was done in Ref. (4). We could try to do this more explicitly now.

Conclusion:

Antipodal identification of points on the horizon (*only* on the horizon, not in the bulk!) leads to *100% entanglement* of the Hawking particles at antipodal ponts, in principle an observable property.

We can calculate the BH microstates – except for the cut-off at $\ell \approx M_{\rm Planck}\, R_{BH}$. Taking discrete points (θ, φ) on the horizon, black holes have hair: one hair at every θ, φ, with end points on the tortoise coordinates: $u_{\rm out}$ describes exponentially growing hair, $u_{\rm in}$ describes exponentially shrinking hair.

On the scalp, there is a *fermionic degree of freedom*, the sign function $\sigma_{\rm in}$ or $\sigma_{\rm out}$. If one throws nothing in the black hole, the fermionifc field $\sigma_{\rm out}$ is conserved in time ($\sigma_{\rm in}$ would only be conserved as long as we don't see anything coming out).

There is no black hole interior; region *II* of the Penrose diagram – with causality reflected in time – describes the antipodal part. When black hole forms, the interior region seems to be physical but of course it is is invisible. Only when the black hole decays, one realises that the interior was never there.

Particles crossing the black hole interchange position with momentum and back.

The essential observation that the spherical partial waves of matter all return their quantum information independently, allows for new assessments of space-time properties that was not possible before.

All this could have been discovered decades ago.

References:

The importance of the gravitational shift (Shapiro delay) was already emphasized in:
(1) The gravitational shock wave of a massless particle (with T. Dray). Nucl. Phys. B253 (1985) 173
(2) Strings from gravity. Physica Scripta, Vol. T15 (1987) 143-150.
(3) The black hole interpretation of string theory. Nucl. Phys. B335 (1990) 138-154;
(4) The scattering matrix approach, J. Mod. Phys. A11 (1996) pp. 4623-4688, gr-qc/9607022.

Supertranslations:
(5) S.W. Hawking, M.J. Perry and A. Strominger: Soft Hair on Black Holes, arXiv: 1601.00921 [hep-th]

The partial wave expansion and antipodal entanglement:
(6) Diagonalizing the Black Hole Information Retrieval Process, arXiv:1509.01695
(7) Black hole unitarity and antipodal entanglement, arXiv:1601.03447[gr-qc]
Please wait for version 2 of this paper to appear in th ArXiv!
(8) L. Mersini-Houghton, Entropy of the Information Retrieved from Black Holes, arXiv:1511.04795 [hep-th]