Multidimensional Real Analysis
Corrigenda and Addenda

Mathematical mistakes are indicated by the symbol ∇; the majority of the corrections are minor textual changes.

Sentence preceding Definition 1.4.4 on page 3. Replace the sentence by the following:
Thus, \( \langle e_j, e_j \rangle = 1 \), while \( e_i \) and \( e_j \), for distinct \( i \) and \( j \), are mutually orthogonal vectors.

Example 1.3.10 on page 16. Insert the following words immediately after Example 1.3.10:
(Plücker’s conoid).

Lemma 2.1.1 on page 40. There exists a slightly less computational proof of this lemma. In fact, apply Lemma 1.1.7.(ii) to \( Ah = \sum_{1 \leq j \leq n} h_j Ae_j \), which follows from Formula (1.1), in order to obtain
\[
\|Ah\| \leq \sum_{1 \leq j \leq n} |h_j| \|Ae_j\| = \langle (|h_1|, \ldots, |h_n|), (\|Ae_1\|, \ldots, \|Ae_n\|) \rangle.
\]

Now the Cauchy–Schwarz inequality from Proposition 1.1.6 immediately yields
\[
\|Ah\| \leq \|h\| \sqrt{\sum_{1 \leq j \leq n} \|Ae_j\|^2} = \|h\| \|A\|_{\text{Eucl}}.
\]

Penultimate sentence preceding Proposition 2.2.1 on page 42. Add the following to the sentence:
, as will be shown in Definition 2.2.2.

Definition 2.2.2 on page 43. Replace the first part of the second sentence in the definition by the following:
Then the mapping \( f \) is said to be differentiable at \( a \)

Lemma 2.2.7 on page 45. As additional motivation for the proof of Hadamard’s Lemma one may offer the following argument.
On the one hand, for differentiable \( f \), one requires
\[
f(x) - f(a) = \phi_a(x)(x - a).
\]

On the other hand, in view of \( \|x - a\|^2 = (x - a)^t(x - a) \in \mathbb{R} \), the reformulation of differentiability in Formula (2.10) implies
\[
f(x) - f(a) = Df(a)(x - a) + \epsilon_a(x - a) \\
= Df(a)(x - a) + \frac{1}{\|x - a\|^2} \epsilon_a(x - a)(x - a)^t(x - a).
\]
A formal division of the right–hand side by \( x - a \) now suggests the formula for \( \phi_a(x) \) as given in the proof.
**Theorem 2.4.1 on page 51.** Replace the last sentence in the assertion of the chain rule by the following:

And, if $f$ is differentiable on $U$ with $f(U) \subset V$ and $g$ is differentiable on $V$,

$$D(g \circ f) = ((Dg) \circ f) \circ Df : U \to \text{Lin}(\mathbb{R}^n, \mathbb{R}^p).$$

**Proof of Corollary 2.4.3 on page 53.** Replace the first sentence by the following:

From Example 2.2.5 we know that $f : \mathbb{R}^n \to \mathbb{R}^p \times \mathbb{R}^p$ is differentiable at $a$ if $f(x) = (f_1(x), f_2(x))$, while $Df(a)h = (Df_1(a)h, Df_2(a)h)$, for $(a, h) \in \mathbb{R}^n$.

**Lemma 2.4.7 on page 54.** Replace the displayed formula by the following:

$$D(Lf)(a) = LDf(a).$$

**Corollary 2.5.5 and its proof on page 58.** Replace the assertion of the corollary and its proof by the following:

Let $K$ following:

Corollary 2.5.5 and its proof on page 58. Replace the assertion of the corollary and its proof by the following:

Let $K \subset \mathbb{R}^n$ be compact and $O \subset \mathbb{R}^n$ open with $K \subset O$. If $f : O \to \mathbb{R}^p$ is a $C^1$ mapping, then the restriction $f|_K$ of $f$ to $K$ is Lipschitz continuous.

**Proof.** Suppose $f$ is not Lipschitz continuous on $K$. Define $g : O \times O \to [0, \infty]$ by

$$g(x, x') = \begin{cases} \frac{\|f(x) - f(x')\|}{\|x - x'\|}, & x \neq x'; \\ 0, & x = x'. \end{cases}$$

Then there exist sequences $(x_l)_{l \in \mathbb{N}}$ and $(x'_l)_{l \in \mathbb{N}}$ of points in $K$ such that $\lim_{l \to \infty} g(x_l, x'_l) = \infty$. On account of Theorem 1.8.8 the sequence $(\|f(x_l) - f(x'_l)\|)_{l \in \mathbb{N}}$ is bounded, which implies that $\lim_{l \to \infty} \|x_l - x'_l\| = 0$. In turn, the sequential compactness of $K$ leads to the existence of subsequences in $K$, which will also be denoted by $(x_l)_{l \in \mathbb{N}}$ and $(x'_l)_{l \in \mathbb{N}}$, and $x \in K$ satisfying $\lim_{l \to \infty} x_l = \lim_{l \to \infty} x'_l = x$. Next select a convex open set $U \subset \mathbb{R}^n$ such that $x \in U \subset O$. Then $x_l$ and $x'_l$ belong to $U$ if $l$ is sufficiently large. For such $l$, the Mean Value Theorem 2.5.3 gives the existence of $k > 0$ having the property $g(x_l, x'_l) \leq k$, which is a contradiction. \qed

**Definition 3.1.2 on page 88.** Replace the displayed formula by the following:

$$\Psi^* f = f \circ \Psi : V \to \mathbb{R}^p,$$

that is

$$\Psi^* f(y) = f(\Psi(y)) \quad (y \in V),$$

**Subsection 3.4.(C) on page 98.** Replace the first display in this subsection by the following:

$$\mathbb{R}^p \rightrightarrows \mathbb{R}^n \times \mathbb{R}^p \rightrightarrows \mathbb{R}^n$$
given by

$$y \mapsto (\psi(y), y), f(\psi(y), y) = 0.$$

**Theorem 3.5.1 on page 100.** Replace the sentence following the first display in the theorem by the following:

Then there exist open neighborhoods $U$ of $x^0$ in $\mathbb{R}^n$ and $V$ of $y^0$ in $\mathbb{R}^p$ with the following properties: $U \times V \subset W$ and

**Application A on page 101.** Replace the last sentence (on page 102) in the application by the following:

That theorem in algebra asserts that there does not exist a formula which gives the zeros of a general polynomial function of degree $n$ in terms of the coefficients of that function by means of addition, subtraction, multiplication, division and extraction of roots, if $n \geq 5$ and one even works over $\mathbb{C}$. 2
Application C on page 103. Replace the second sentence by the following:
Consider the following equation for \( x \in \mathbb{R} \) with \( y \in \mathbb{R} \) as a parameter

Application D on page 104. Replace the sentence following the second display by the following:
Now, for all \((x; y) \in \mathbb{R}^n \times \mathbb{R}^{n^2+n} \) and \(1 \leq i, j \leq n\).

Text preceding Definition 4.1.2 on page 108. Replace the last sentence preceding the definition by the following:
There are two other common ways of specifying sets \( V \): in Definitions 4.1.2 and 4.1.3 the set \( V \) is described as an image, or inverse image, respectively, under a mapping.

Proof of Rank Lemma 4.2.7 on page 113. Replace the first part of the proof by the following:
Only \( (i) \Rightarrow (ii) \) needs verification. In view of the equality \( \dim(\ker A) + r = n \), we can find a basis \((a_{r+1}, \ldots, a_n)\) of \( \ker A \subset \mathbb{R}^n \) and vectors \( a_1, \ldots, a_r \) complementing this basis to a basis of \( \mathbb{R}^n \). Define \( \Psi \in \text{Aut}(\mathbb{R}^n) \) setting \( \Psi e_j = a_j \), for \( 1 \leq j \leq n \). Then \( (A \Psi) e_j = A a_j \), for \( 1 \leq j \leq r \), and \( (A \Psi) e_j = 0 \), for \( r < j \leq n \). The vectors \( b_j = A a_j \), for \( 1 \leq j \leq r \), form a basis of \( \text{im} A \subset \mathbb{R}^p \). Let us complement them by vectors \( b_{r+1}, \ldots, b_p \) to a basis of \( \mathbb{R}^p \). Define \( \Phi \in \text{Aut}(\mathbb{R}^p) \) by \( \Phi a_i = e'_i \), for \( 1 \leq i \leq p \). Then the operators \( \Phi \) and \( \Psi \) are the required ones, since

\[
(\Phi \circ A \circ \Psi) e_j = \begin{cases} 
  e'_j, & 1 \leq j \leq r; \\
  0, & r < j \leq n.
\end{cases}
\]

Proof of Rank Lemma 4.2.7 on page 114. Replace the final part of the proof by the following:
such that \( A a_i = e'_i \), for \( 1 \leq i \leq p \). Then \( \Phi = I \).

Proof of Rank Lemma 4.2.7 on page 114. Insert the following immediately after the proof:
Alternatively, the equality of the ranks of \( A \) and \( A^t \) may be verified as follows. We have \( \mathbb{R}^n = \ker A \oplus \text{im} A^t \). In fact,

\[
x \in \ker A \iff Ax = 0 \iff \langle Ax, y \rangle = 0 \quad (y \in \mathbb{R}^p) \iff \langle x, A^t y \rangle = 0 \quad (y \in \mathbb{R}^p) \iff x \in (\text{im} A^t)^\perp.
\]

Hence \( (\ker A)^\perp = (\text{im} A^t)^{\perp \perp} = \text{im} A^t \) and so the equality follows from \( \mathbb{R}^n = \ker A \oplus (\ker A)^\perp \). Furthermore, we know \( \dim \mathbb{R}^n = \dim \ker A + \text{rank } A \).

Theorem 4.3.1 on page 114. Replace the first sentence of the theorem by the following:
Let \( d < n \) and let \( D_0 \subset \mathbb{R}^d \) be an open subset, let \( k \in \mathbb{N}_\infty \) and let \( \phi : D_0 \to \mathbb{R}^n \) be a \( C^k \) mapping.

Theorem 4.3.1 on page 114. Replace the first part of the first sentence of (ii) in the theorem by the following:
There exist an open neighborhood \( U \) of \( x^0 \) in \( \mathbb{R}^n \) that contains \( \phi(D) \) and

Corollary 4.3.2 on page 116. Replace the initial part of the first sentence of the corollary by the following:
Let \( d \leq n \) and \( D \subset \mathbb{R}^d \) be a nonempty open subset, suppose \( k \in \mathbb{N}_\infty \),

Proof of Corollary 4.3.2 on page 116. Replace \( \phi^{-1}(\phi(D) \cap U) = D(y) \) in the fourth sentence of the proof by the following:
\( \phi^{-1}(U) = D(y) \).
Example 4.5.1 on page 121. Replace the last sentence of the first paragraph by the following:
Note that in these \((y, c)\)-coordinates a circle is locally described as the affine submanifold of \(\mathbb{R}^3\) given by \(c\) equals a constant vector.

Theorem 4.5.2 on page 121. Replace assertion (i) of the theorem by the following:
The restriction of \(g\) to \(U\) is an open surjection onto \(C\).
Replace assertion (iii) of the theorem by the following:
There exists a \(C^k\) diffeomorphism \(\Phi : U \to \Phi(U) \subset \mathbb{R}^n\) such that \(\Phi\) maps the manifold \(N(c) \cap U\) in \(\mathbb{R}^n\) into the affine submanifold of \(\mathbb{R}^n\) given by
\[
\{ (x_1, \ldots, x_n) \in \mathbb{R}^n \mid (x_{d+1}, \ldots, x_n) = c \}.
\]

Remark on page 124. Replace the last sentence by the following:
The fibers \(N(c)\) together form a fiber bundle: under the diffeomorphism \(\Phi\) from the Submersion Theorem they are locally transferred into the affine submanifolds of \(\mathbb{R}^n\) given by the last \(n - d\) coordinates being constant.

Example 4.6.2 on page 124. Add the following sentence at the end of the example:
See Exercises 4.22 and 5.58 for an explicit description of \(\text{SO}(3, \mathbb{R})\) and Exercise 4.23 for more details on \(\text{SO}(n, \mathbb{R})\), the subgroup of \(\text{O}(n, \mathbb{R})\) consisting of matrices of determinant 1.

Theorem 4.7.1 on page 126. Replace the display in assertion (iii) of the theorem by the following:
\[
V \cap U = N(g, 0) = \{ x \in U \mid g(x) = 0 \}.
\]
Replace the initial part of assertion (iv) of the theorem by the following:
There exist an open neighborhood \(U\) in \(\mathbb{R}^n\) of \(x\), a \(C^k\) diffeomorphism \(\Phi : U \to \Phi(U)\) in \(\mathbb{R}^n\) and an open subset \(Y\) of \(\mathbb{R}^d\) such that

Remark on page 128. Add the following at the end of the remark:
Furthermore, if \(V\) is not smooth it is often called an affine algebraic variety.

Remark on page 135. Replace the first sentence by the following:
In classical textbooks, and in drawings, it is more common to refer to the affine manifold \(x + T_x V\) as the tangent space of \(V\) at the point \(x\): the affine manifold which has a contact of order 1 with \(V\) at \(x\).

Example 5.3.2 on page 138. Add the following sentence at the end of the example:
Therefore the angle itself always equals \(\frac{\pi}{4}\).

Example 5.3.3 on page 138. Insert the following words immediately after Example 5.3.3: (Parametrized surface).

Example 5.3.4 on page 139. Insert the following words immediately after Example 5.3.4: (Space curve given by equations).

Example 5.3.4 on page 140. Add the following at the end of the example:
Phrased differently in terms of the cross product, we have
\[
T_x V = \mathbb{R}(\text{grad} g_1(x) \times \text{grad} g_2(x)).
\]
Example 5.3.5 on page 140. Replace the second displayed formula of the example by the following:

\[ Dg(x)h = 0 \iff \langle \text{grad } g_1(x), h \rangle = \cdots = \langle \text{grad } g_{n-d}(x), h \rangle = 0. \]

Add the following at the end of the example:
Equations for the geometric tangent space \( x + T_x V \) are obtained as follows. Consider \( h \in x + T_x V \), then \( h = x + k \) where \( k \in T_x V \). Thus, \( k = h - x \) implies

\[ 0 = Dg(x)k = Dg(x)(h - x) = Dg(x)h - Dg(x)x. \]

In other words, \( x + T_x V \) arises as the set of solutions \( h \in \mathbb{R}^n \) of a system of \( n - d \) inhomogeneous linear equations or, more precisely,

\[ x + T_x V = \{ h \in \mathbb{R}^n \mid Dg(x)h = Dg(x)x \}. \]

Example 5.3.8 on page 144. Replace the two sentences preceding the first display in the example as well as the display by:

Note that \( \gamma'(t) = (2t, 3t^2) \in \text{Lin}(\mathbb{R}, \mathbb{R}^2) \) is injective, unless \( t = 0 \). Furthermore, \( ||\gamma'(t)|| = 2|t|\sqrt{1 + (3/2t)^2}. \) For \( t \neq 0 \) we therefore have the normalized tangent vector

\[ T(t) = ||\gamma'(t)||^{-1}\gamma'(t) = \frac{\text{sgn } t}{\sqrt{1 + (3/2t)^2}} \begin{pmatrix} 1 \\ \frac{3t}{2} \end{pmatrix}. \]

The superscript \( t \) in the first formula for \( \gamma'(t) \) indicates taking the transpose, but might cause confusion.

Example 5.3.11 on page 145. Replace the first sentence following the fourth display from below on page 146 by:

Therefore, if \( e_1, \ldots, e_{n-1} \) are the standard basis vectors of \( \mathbb{R}^{n-1} \), it follows that \( T_x V \) is spanned by the vectors \( u_j := (e_j, D_j h(x')) \), for \( 1 \leq j < n \).

Example 5.5.1 on page 151. Replace the final part of the second and the third sentence by:

Recall that Theorem 1.8.8 implies that the distance attains a minimal value.

Example 5.5.3 on page 153. Insert the following at the end of the example:

Geometrically, Hadamard’s inequality follows from the following observations. The volume of the parallelepiped spanned by the vectors \( a_1, \ldots, a_n \) does not change upon replacement of the vector \( a_n \) by its component \( p_n \) perpendicular to the hyperplane spanned by the vectors \( a_1, \ldots, a_{n-1} \), because the determinant is a multilinear and antisymmetric function. Furthermore \( ||p_n|| \leq ||a_n|| \) by Pythagoras’ Theorem. Hence one obtains by downward mathematical induction on \( n \geq j \geq 1 \)

\[ ||\det(a_1 \cdots a_n)|| = \prod_{1 \leq j \leq n} ||p_j|| \leq \prod_{1 \leq j \leq n} ||a_j||. \]

Text preceding Definition 5.6.1 on page 154. Replace the first part of the third sentence by:

Our next goal is to define the Hessian of \( f|_V \) at a critical point \( x^0 \).
\begin{align*}
\textbf{Exercise 0.3 on page 177.} \quad & \text{Replace the exercise by the following:} \\
\text{We have} & \quad \arctan x + \arctan \frac{1}{x} = \pm \frac{\pi}{2} \quad (x \geq 0).
\end{align*}

Prove this by means of the following three methods.

(i) Set $\arctan x = \alpha$ and express $\frac{1}{x}$ in terms of $\alpha$.

(ii) Use differentiation.

(iii) Recall that $\arctan x = \int_0^x \frac{1}{1+t^2} \, dt$, and make a substitution of variables.

Deduce $\lim_{x \to \infty} x \left( \frac{\pi}{2} - \arctan x \right) = 1$.

(iv) More generally show, for all $x$ and $y \in \mathbb{R}$,

$$
\arctan x + \arctan y = \begin{cases}
\arctan \left( \frac{x+y}{1-xy} \right), & xy < 1; \\
\pm \frac{\pi}{2}, & xy = 1, y \geq 0; \\
\arctan \left( \frac{x+y}{1-xy} \right) \pm \pi, & xy > 1, y \geq 0.
\end{cases}
$$

\begin{align*}
\textbf{Exercise 0.5 on page 178.} \quad & \text{Replace in parts (i) and (ii) } “R_+,e_1” \text{ by the following:} \\
& \mathbb{R}_+,e_1
\end{align*}

\begin{align*}
\textbf{Exercise 0.18 on page 189.} \quad & \text{Add to the Background on page 190 the following:} \\
\end{align*}

\begin{align*}
\textbf{Exercise 0.20 on page 191.} \quad & \text{Replace the final part of the second sentence by the following:} \\
& \zeta(2n) := \sum_{k \in \mathbb{N}} \frac{1}{k^{2n}} = (-1)^{n-1} \frac{1}{2} (2\pi)^{2n} B_{2n} \frac{B_{2n}}{(2n)!}
\end{align*}

\begin{align*}
\textbf{Exercise 2.39.(v) on page 230.} \quad & \text{Replace the last part of the last sentence by the following:} \\
& (A \in O(n, \mathbb{R})).
\end{align*}

\begin{align*}
\textbf{Exercise 2.76.(ii) on page 251.} \quad & \text{Replace the initial part of the fourth sentence by the following:} \\
& \text{On the strength of Lemma 2.7.4 we obtain } \phi_{k+1} \in C^\infty(\mathcal{U}, \text{Lin}^{k+1}(\mathbb{R}^n, \mathbb{R}^p)).
\end{align*}

\begin{align*}
\textbf{Exercise 4.8.(ii) on page 296.} \quad & \text{Replace the assertion by the following:} \\
& \text{Prove that a point } x \in \mathbb{R}^3 \text{ belongs to the helicoid if and only if } x_1 \sin \frac{\pi}{2} - x_2 \cos \frac{\pi}{2} = 0.
\end{align*}

\begin{align*}
\textbf{Exercise 5.18.(iii) on page 323.} \quad & \text{Replace the assertion by the following:} \\
& \text{Show that } L = \{ (r, \alpha) \in [-1, 1] \times [-\frac{\pi}{4}, \frac{\pi}{4}] \mid r^2 = \cos 2\alpha \}.
\end{align*}

\begin{align*}
\textbf{Exercise 5.51.(ii) on page 357.} \quad & \text{Replace the first sentence by the following:} \\
& \text{Using the substitution } \sqrt{1-t^2} = y, \text{ prove}
\end{align*}
Exercise 5.71.(i) on page 393. Replace the last part of the last sentence above the last display by the following:

\[ \lambda := D TL(I) : sl(2, C) = su(2) \oplus p \rightarrow so(4) \] 
(see Exercise 5.69) with

Index on page 414. Replace “critical point of diffeomorphism” by the following:
critical point of \( C^1 \) mapping

Index on page 420. Replace “singular point of diffeomorphism” by the following:
singular point of \( C^1 \) mapping

\[ \sup_B f - \inf_B f = (\sup_B f_+ - \inf_B f_+) + (\sup_B f_- - \inf_B f_-); \]

Remark on page 436. Add the following at the end of the remark:
A simpler example is given by \( f = 1_{[0,1]} \times \{0\} \).

Example 6.6.4 on page 447. Replace the initial part of the sentence following the third display in the example by the following:
Consider \( -\pi \leq \alpha_1 < \alpha_2 \leq \pi \) and \( \phi \in C(\alpha_1, \alpha_2) \) and suppose \( \phi > 0 \),

Example 6.6.8 on page 450. Replace the fifth display in the example by the following:

\[ \det(x'(s) \ x'(s)) + \det(x(s) \ x''(s)) = \det(x(s) \ x''(s)) = 0. \]

Section 7.1 on page 487. Replace the first sentence in the second paragraph of the section by the following:
First we consider this problem locally, that is, in a sufficiently small neighborhood \( U \) in \( R^n \) of a point \( x \in V \).

Example 7.4.1 on page 498. Replace the last sentence in the first paragraph on page 499 by the following:
Indeed, the ellipse is the image under the embedding \( t \mapsto (a \sin t, b \cos t) \).

\[ \updownarrow \text{Footnote on page 504.} \]

The proof as given in the reference is erroneous, but other, correct, proofs do exist.
For more details, see Casselman, B.: The difficulties of kissing in three dimensions. Notices Amer. Math. Soc. 51 (2004), 884-885.

Notation on page 512. Replace the first sentence after the second display by the following:
For \( x = \phi(y) \in \partial \Omega \cap U \) the column vectors \( D_j \phi(y) \) in the matrix \( D \phi(y) \), for \( 1 \leq j < n \), form a basis for \( T_x(\partial \Omega) \), the tangent space to \( \partial \Omega \) at \( x \).

\[ \updownarrow \text{Notation on page 512.} \]

Replace the initial part of the first sentence after the sixth display by the following:
Note that \( \partial \Omega \cap U \supset \Psi(\{0\} \times D) \).
Third display on page 523. Replace the sentence containing this display by the following:
Begin by assuming that $S$ is a closed (and hence compact) subset of $\partial \Omega$ such that
\[ \partial' \Omega := \partial \Omega \setminus S \]
is in fact an $(n - 1)$-dimensional $C^1$ manifold, with $\Omega$ at one side of $\partial' \Omega$ at each point of $\partial' \Omega$.
Perform the substitution of $W$ by $\partial' \Omega$ systematically up till Gauss’ Divergence Theorem 7.8.5. In particular, replace Formula (7.54) by the following:
\[ \int_{\Omega} D_j ( (1 - \chi) f)(x) \, dx = \int_{\partial' \Omega} ( (1 - \chi) f \nu_j)(y) \, d_{n-1}y. \]
Replace the first sentence on page 524 by the following:
As a result, the left–hand side in (7.54) converges to $\int_{\Omega} (D_j f)(x) \, dx$, if $\epsilon \downarrow 0$; and the right–hand side in (7.54) converges to $\int_{\partial' \Omega} (f \nu_j)(y) \, d_{n-1}y$, if $U$ shrinks to $S$.
Replace the last sentence of Gauss’ Divergence Theorem 7.8.5 on page 529 by the following:
\[ \int_{\Omega} \text{div} \, f(x) \, dx = \int \langle f, \nu \rangle(y) \, d_{n-1}y, \]
where the integration on the right–hand side is performed over $\partial \Omega$ or $\partial' \Omega$, respectively.

Example 7.8.4 on pages 528. Replace the initial part of the fourth sentence by the following:
Note that if $n > 2$ (see Exercises 2.30 and 2.40.(iv))

Examples 7.9.6 and 7.9.7 on pages 534 and 535. Both examples are not applications of Gauss’ Divergence Theorem 7.8.5, but of Corollary 7.6.2. Hence they should be moved to Section 7.6.

Definition 8.3.1 on page 552. Replace the first part of the second sentence by:
Assume that $I \to \partial \Omega$ with $t \mapsto y(t)$ is a $C^1$ parametrization of $\partial \Omega$ by the disjoint union $I$ of finitely many intervals in $\mathbb{R}$,

Text following Proposition 8.5.5 on page 567. Replace the last part of the second sentence by:
whether we may choose such an $\Omega$ so that it lies inside of $U$.

\[ \nabla \text{ Exercise 6.9 on page 600. Replace the second sentence by the following:} \]
Prove $\int_B \|x\|^{-1} \, dx = 8 \log(1 + \sqrt{2})$.

Exercise 6.20 on page 602. Replace the second sentence by the following:
Prove, for all $y \in \mathbb{R}^3 \setminus \{0\}$,

Exercise 6.39 on page 612. Add to the Background on page 613 the following:
Using this functional equation one sees at once
\[ \sum_{n \in \mathbb{N}} \frac{1}{2^n n^2} = \frac{\pi^2}{12} - \frac{\log^2 2}{2}. \]
Replace the sentence on page 614 preceding the second display by the following:
Furthermore, the Clausen function $\text{Cl}_2 : \mathbb{R} \to \mathbb{R}$, and the Lobachevsky function $\text{Li} : \mathbb{R} \to \mathbb{R}$ are defined by (see Exercise 0.18.(i) and use termwise integration)
Add the following at the end of the exercise:

(vii) Use \( \int_0^\frac{\pi}{2} \log(\sin x) \, dx = -\frac{\pi}{2} \log 2 \) and the substitution \( x = \arctan \frac{1}{t} \) to derive
\[
\int_0^\infty \frac{\log(1 + t^2)}{1 + t^2} \, dt = \pi \log 2.
\]

Exercise 6.96 on page 657. Replace the second formula in the display in part (i) by the following:
\[
\mu := \int_\mathbb{R} x f_{\alpha, \lambda}(x) \, dx = \frac{\alpha}{\lambda}.
\]

Exercise 6.102 on page 665. Replace the final part of the first sentence by the following:
– needed for Exercises 7.30 and 8.20

Exercise 7.46 on page 704. Add the following sentence at the end:
Deduce hyperarea \( _{n-1}(S^n) = n \text{ vol}_n(B^n) \) (compare with Example 7.9.1 and Exercises 7.21.(iv), 7.35.(iii) and 7.45.(ii)).

Exercise 7.53 on page 706. Add the following sentence at the end of part (v):
In doing so, assume a function having the mean value property to belong to \( C^2(\Omega) \).

\[ \text{\narrowleft Exercise 8.7.(i) on page 731. Replace the assertion by the following:} \]

Prove
\[
\int_C \langle f(s), d_1 s \rangle = -3 \int_{\{x \in \mathbb{R}^2 : \|x\| \leq 1\}} \|x\|^2 \, dx = -\frac{3\pi}{2}.
\]

Exercise 8.31.(viii) on page 755. Replace the first sentence by the following:
Try to find \( G \) such that the Lorenz gauge condition \( d^* G = 0 \) is satisfied.
Replace the last sentence by the following:
In general, \( G + df \) will satisfy the Lorenz gauge condition if \( \Box f = 0 \).

Index on page 785. Replace “critical point of diffeomorphism” by the following:
critical point of \( C^1 \) mapping

Index on page 791. Replace the last entry by the following:
Lorenz gauge condition 755

Index on page 796. Replace “singular point of diffeomorphism” by the following:
singular point of \( C^1 \) mapping